CE 486
Urban Transportation Planning

Lec. 3
Trip Distribution

Dr. Mahmoud Owais

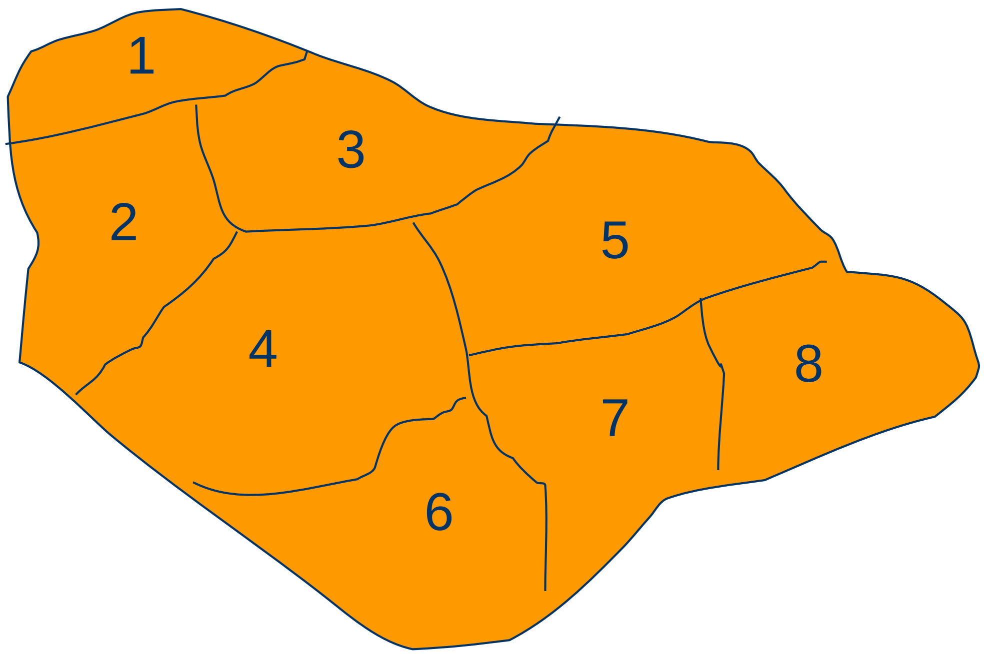
Trip Distribution

- Predicts where trips go from each TAZ
- Determines trips between pairs of zones
  - T_{ij}: trips from TAZ *i* going to TAZ *j*
- Function of attractiveness of TAZ *j*
  - Size of TAZ *j*
    - *If 2 malls are similar (in the same trip purpose), travelers will tend to go to the biggest*
  - Distance to TAZ *j*
    - *If 2 malls are similar (in the same trip purpose), travelers will tend to go to closest*

Trip Generation

Input:
- Socioeconomic Data
- Land Use Data

TAZ	Productions
1	12
2	19
3	35
4	4
5	5
6	10
7	13
8	22



TAZ	Attractions
1	9
2	12
3	4
4	38
5	45
6	6
7	4
8	2

Output:
- Trip Ends by trip purpose

Trip Generation → Trip Distribution

The question is… how do we allocate all the trips among all the potential destinations?

TAZ	Produ
1	1
2	1
3	3
4	
5	
6	1
7	1
8	2

TAZ	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

Trip Matrix

or

Trip Table

Attractions
9
12
4
38
45
6
4
2

Trip Distribution

Trip Distribution

- We link production or origin zones to attraction or destination zones
- A trip matrix is produced

TAZ	1	2	3	4	5	6	7	8
1								
2								
3								
4								
5								
6								
7								
8								

Trip Matrix

  - The cells within the trip matrix are the "trip interchanges" between zones

Basic Assumptions of Trip Distribution

- Number of trips decrease with COST between zones
- Number of trips increase with zone "attractiveness"

Methods of Trip Distribution

I. Growth Factor Models

II. Gravity Model

Growth Factor Models

- Growth Factor Models assume that we already have a basic trip matrix

TAZ	1	2	3	4
1	5	50	100	200
2	50	5	100	300
3	50	100	5	100
4	100	200	250	20

  - Usually obtained from a previous study or recent survey data

Growth Factor Models

- The goal is then to estimate the matrix at some point in the future
  - For example, what would the trip matrix look like in 2 years time?

TAZ	1	2	3	4
1	5	50	100	200
2	50	5	100	300
3	50	100	5	100
4	100	200	250	20

Trip Matrix, **t**
(2008)

→

TAZ	1	2	3	4
1	?	?	?	?
2	?	?	?	?
3	?	?	?	?
4	?	?	?	?

Trip Matrix, **T**
(2018)

Some of the More Popular Growth Factor Models

- Uniform Growth Factor
- Average Factor
- Fratar Method

Uniform Growth Factor Model

Uniform Growth Factor

$i = I$ = Production Zone

$j = J$ = Attraction Zone

$T_{ij} = \tau \, t_{ij}$ for each pair i and j

T_{ij} = Future Trip Matrix

t_{ij} = Base-year Trip Matrix

τ = General Growth Rate

Uniform Growth Factor

If we assume $\tau = 1.2$ (growth rate), then…

TAZ	1	2	3	4
1	5	50	100	200
2	50	5	100	300
3	50	100	5	100
4	100	200	250	20

Trip Matrix, **t**
(2008)

$$T_{ij} = \tau\, t_{ij} = (1.2)(5) = 6$$

TAZ	1	2	3	4
1	6	60	120	240
2	60	6	120	360
3	60	120	6	120
4	120	240	300	24

Trip Matrix, **T**
(2018)

Uniform Growth Factor

- The Uniform Growth Factor is typically used for over a 1 or 2 year horizon

  However, assuming that trips grow at a standard uniform rate is a fundamentally flawed concept

The Gravity Model

The Inspiration for the Gravity Model

The big idea behind the gravity model is Newton's law of gravitation…

$$F = k \frac{M_1 M_2}{r^2}$$

The force of attraction between 2 bodies is directly proportional to the product of masses between the two bodies and inversely proportional to the square of the distance

Some of the Variables

T_{ij} = Q_{ij} = Trips Volume between i & j

F_{ij} = $1/W^{c}_{ij}$ = Friction Factor

W_{ij} = Generalized Cost (including travel time, cost)

c = Calibration Constant

p_{ij} = Probability that trip i will be attracted to zone j

k_{ij} = Socioeconomic Adjustment Factor

The Gravity Model

$$T_{ij} = Q_{ij} = \frac{P_i \, A_j \, F_{ij} K_{ij}}{\Sigma A_j F_{ij} K_{ij}} = P_i p_{ij}$$

$$= \frac{\text{(Productions)(Attractions)(Friction Factor)}}{\text{Sum of the (Attractions x Friction Factors) of the Zones}}$$

$F_{ij} = 1 / W^c_{ij}$ & $\ln F = - c \ln W$

The bigger the friction factor,
the more trips that are encouraged

To Apply the Gravity Model

What we need…

1. Productions, $\{P_i\}$
2. Attractions, $\{A_j\}$
3. Skim Tables $\{W_{ij})$
   - Target-Year Interzonal Impedances

Gravity Model Example 8.2

- Given:
  - Target-year Productions, $\{P_i\}$
  - Relative Attractiveness of Zones, $\{A_j\}$
  - Skim Table, $\{W_{ij}\}$
  - Calibration Factor, c = 2.0
  - Socioeconomic Adjustment Factor, K = 1.0
- Find:
  - Trip Interchanges, $\{Q_{ij}\}$

Given…

TAZ	Productions
1	1500
2	0
3	2600
4	0
Σ	**4100**

TAZ	"Attractiveness"
1	0
2	3
3	2
4	5
Σ	**10**

Target-Year Inter-zonal Impedances, $\{W_{ij}\}$

TAZ	1	2	3	4
1	5	10	15	20
2	10	5	10	15
3	15	10	5	10
4	20	15	10	5

Calibration Factor

$c = 2.0$

Socioeconomic Adj. Factor

$K = 1.0$

Calculate Friction Factors, $\{F_{ij}\}$

TAZ	1	2	3	4
1	0.0400	0.0100	0.0044	0.0025
2	0.0100	0.0400	0.0100	0.0044
3	0.0044	0.0100	0.0400	0.0100
4	0.0025	0.0044	0.0100	0.0400

$$F_{ij} = \frac{1}{W^{c}_{ij}} = F_{11} = \frac{1}{5^2} = 0.04$$

Find Denominator of Gravity Model Equation $\{A_jF_{ij}K_{ij}\}$

TAZ	1	2	3	4	Σ
1	0.0000	0.0300	0.0089	0.0125	**0.0514**
2	0.0000	0.1200	0.0200	0.0222	**0.1622**
3	0.0000	0.0300	0.0800	0.0500	**0.1600**
4	0.0000	0.0133	0.0200	0.2000	**0.2333**

$$A_jF_{ij}K_{ij} = A_4F_{34}K_{34}$$
$$(5)(0.01)(1.0) = 0.05$$

Find Probability that Trip i will be attracted to Zone j, $\{p_{ij}\}$

TAZ	1	2	3	4
1	0.0000	0.5838	0.1730	0.2432
2	0.0000	0.7397	0.1233	0.1370
3	0.0000	0.1875	0.5000	0.3125
4	0.0000	0.0571	0.0857	0.8571

$$p_{ij} = \frac{A_jF_{ij}K_{ij}}{\Sigma(A_jF_{ij}K_{ij})} = \frac{0.05}{0.16} = 0.3125$$

Find Trip Interchanges, $\{Q_{ij}\}$

TAZ	1	2	3	4	Σ
1	0	876	259	365	**1500**
2	0	0	0	0	**0**
3	0	488	1300	813	**2600**
4	0	0	0	0	**0**
Σ	**0**	**1363**	**1559**	**1177**	**4100**

$$Q_{ij} = P_ip_{ij} = (2600)(0.3125) = 813$$

Keep in mind that the socioeconomic factor, K, can be a matrix of values rather than just one value

TAZ	1	2	3	4
1	1.4	1.2	1.7	1.9
2	1.2	1.1	1.1	1.4
3	1.7	1.1	1.5	1.3
4	1.9	1.4	1.3	1.6

The Problem with K-Factors

- Although K-Factors may improve the model in the base year, they assume that these special conditions will carry over to future years and scenarios
  - This limits model sensitivity and undermines the model's ability to predict future travel behavior
- The need for K-factors often is a symptom of other model problems.
  - Additionally, the use of K-factors makes it more difficult to figure out the real problems

Limitations of the Gravity Model

- Too much of a reliance on K-Factors in calibration
- External trips and intrazonal trips cause difficulties
- The skim table impedance factors are often too simplistic to be realistic
  - Typically based solely upon vehicle travel times
    - At most, this might include tolls and parking costs
  - Almost always fails to take into account how things such as good transit and walkable neighborhoods affect trip distribution
  - No obvious connection to behavioral decision-making

Calibrating a Gravity Model

- Calibrating of a gravity model is accomplished by developing friction factors and developing socioeconomic adjustment factors

- Friction factors reflect the effect travel time of impedance has on trip making

- A trial-and-error adjustment process is generally adopted

- One other way is to use the factors from a past study in a similar urban area

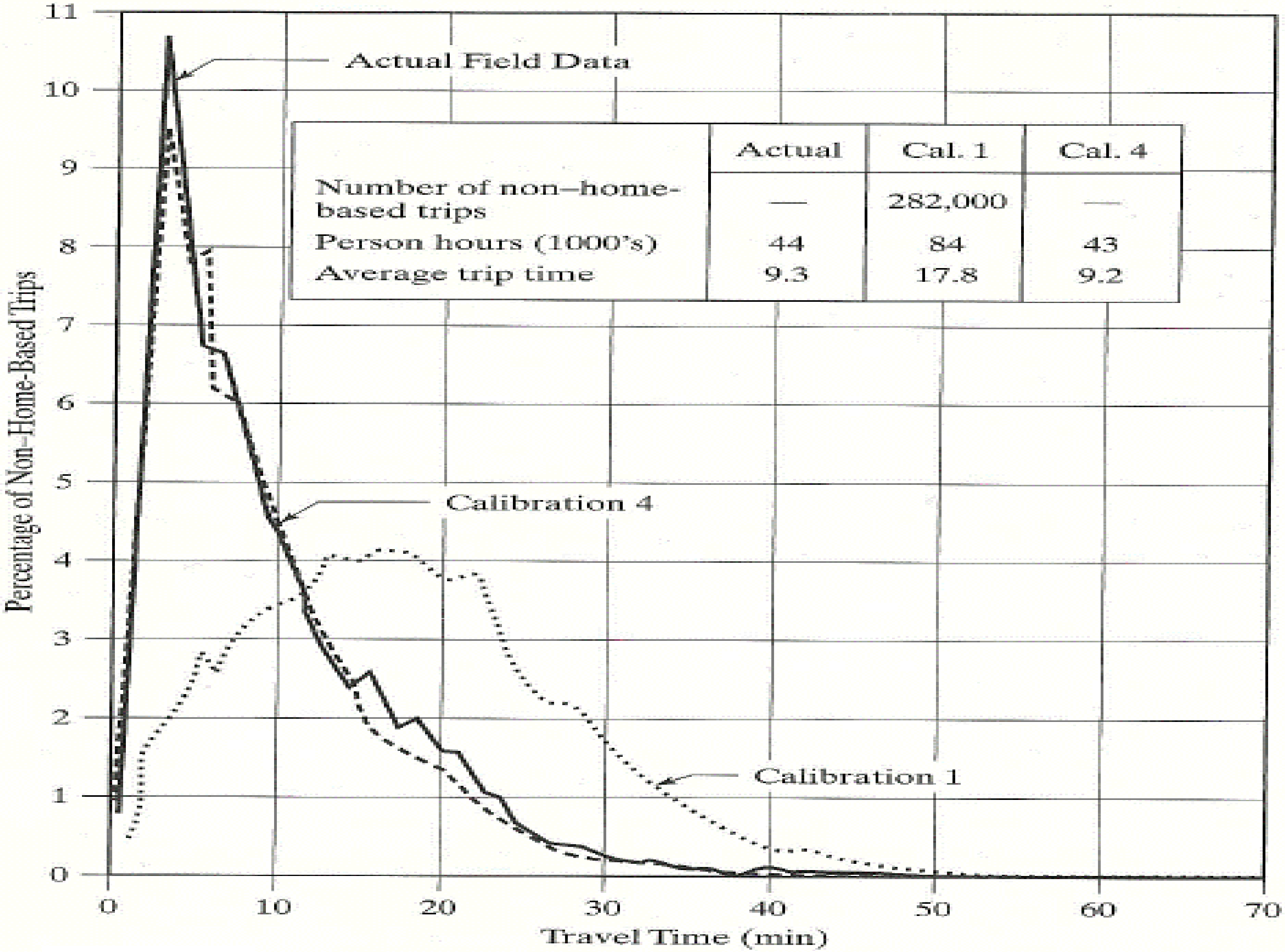
Three items are used as input to the gravity model for calibration:

1. Production-attraction trip table for each purpose
2. Travel times for all zone pairs, including intrazonal times
3. Initial friction factors for each increment of travel time

The calibration process involves adjusting the friction factor parameter until the planner is satisfied that the model adequately reproduces the trip distribution as represented by the input trip table – from the survey data such as the trip-time frequency distribution and the average trip time.

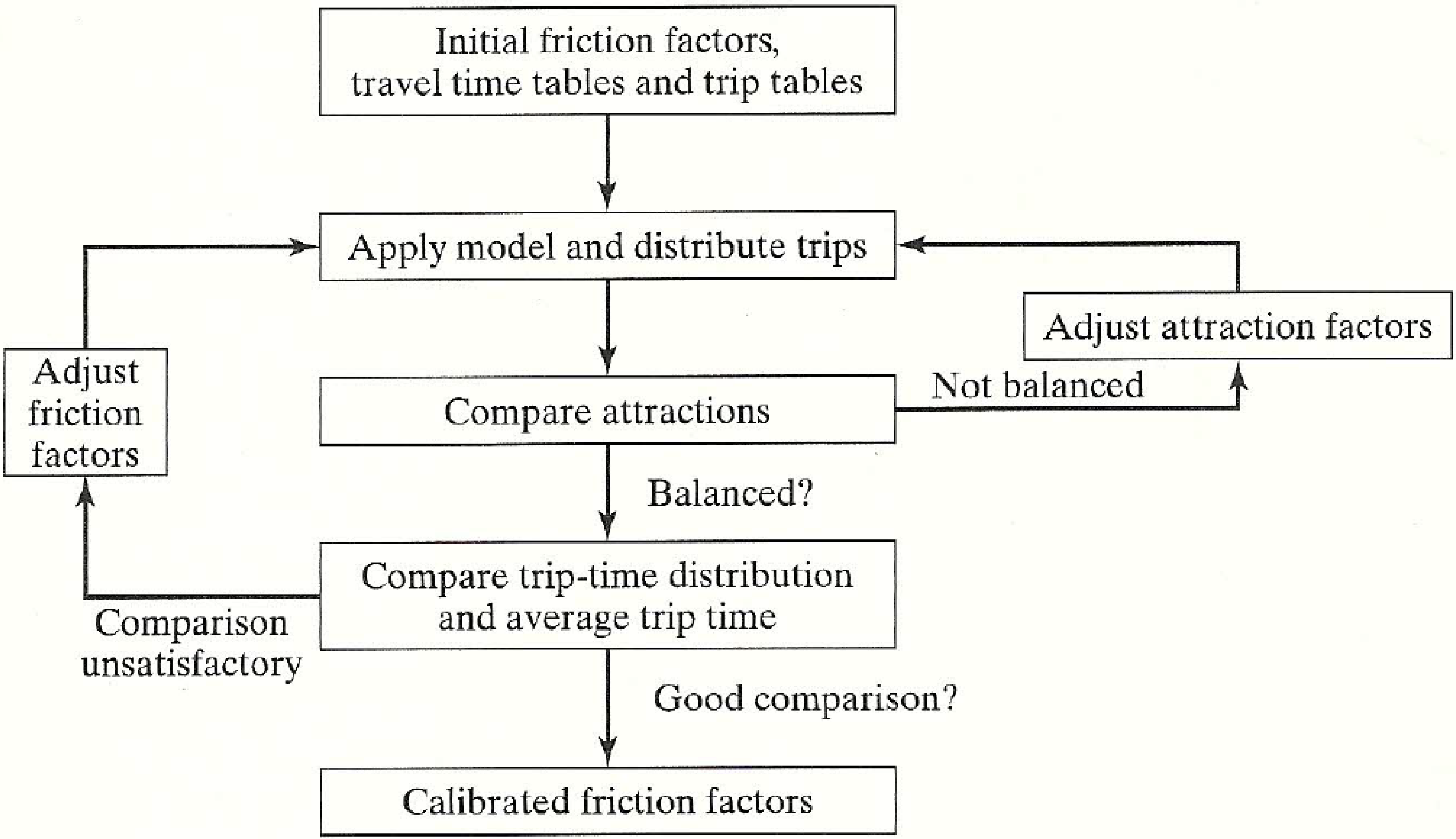
The Calibration Process

1. Use the gravity model to distribute trips based on initial inputs.
2. Total trip attractions at all zones j, as calculated by the model, are compared to those obtained from the input “observed” trip table.
3. If this comparison shows significant differences, the attraction Aj is adjusted for each zone, where a difference is observed.
4. The model is rerun until the calculated and observed attractions are reasonably balanced.
5. The model’s trip table and the input travel time table can be used for two comparisons: the trip-time frequency distribution and the average trip time. If there are significant differences, the process begins again.

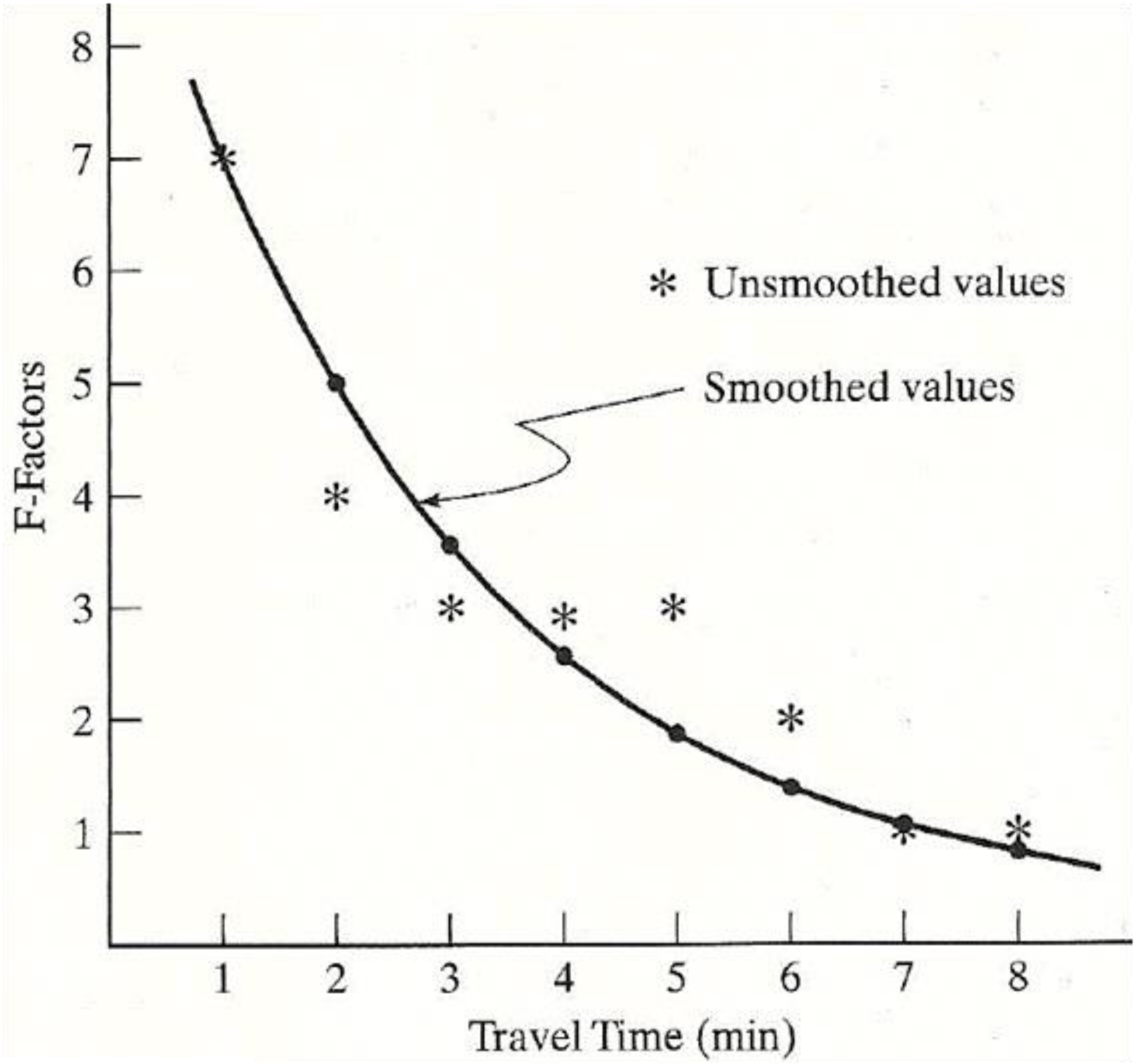


Travel Times Versus Percentage of Non–Home-Based Trips (FHWA/UMTA, 1977).

the results of four iterations comparing travel-time frequency.



Sequence of Steps for the Gravity Model Calibration (FHWA/UMTA, 1977).



Smoothed Adjusted Factors, Calibration 2

- An example of smoothed values of F factors
- In general, values of F decreases as travel time increases, and may take the form F varies as t^{-1}, t^{-2}, or e^{-t}.

- A more general term used for representing travel time (or a measure of separation between zones) is impedance, and the relationship between a set of impedance (W) and friction factors (F) can be written as:

$$F_{ij} = 1/W_{ij}^{c}$$

Example:

A gravity model was calibrated with the following results:

Impedance (travel time, mins), W	4	6	8	11	15
Friction factors, F	.035	.029	.025	.021	.019

Using the f as the dependent variable, calculate parameter A and c of the equation.

$$F = A/W^{c}$$

Solution:

The equation can be written as

$\ln F = \ln A - c \ln W$

ln W	1.39	1.79	2.08	2.40	2.71
ln F	-3.35	-3.54	-3.69	-3.86	-3.96

These figures yield the following values of A = .07 and c = .48.

Hence, $F = 0.07/W^{0.48}$

Example: Trip Distribution

A 3-by-3 trip table representing a total of 2500 trips is shown in the following table, which is for the base year.

Origin \ Destination	1	2	3	Total
1	1	4	2	7
2	6	2	3	11
3	4	1	2	7
Total	11	7	7	25

The next table indicates the origin and destination growth factors for the horizon year.

Zone	1	2	3
Origin factor (production)	2.0	3.0	4.0
Destination factor (attraction)	3.0	4.0	2.0

Use the Fratar technique to distribute the trips in the horizon year.

Solution:

In the horizon year, the desired trip table should resemble the following matrix, where the row and column total equal the corresponding base-year totals multiplied by the origin and destination growth factors.

Origin \ Destination	1	2	3	Total
1	x_{ij}	x_{ij}	x_{ij}	14
2	x_{ij}	x_{ij}	x_{ij}	33
3	x_{ij}	x_{ij}	x_{ij}	28
Total	33	28	14	75

The next step is to multiply the destination growth factors (DGF) by the cell numbers, giving the following matrix:

Destination / Origin	1	2	3	Actual Total	Desired Total	Row factors
1	3.00	16.00	4.00	23.00	14.00	0.61
2	18.00	8.00	6.00	32.00	33.00	1.03
3	12.00	4.00	4.00	20.00	28.00	1.40
Total	33.00	28.00	14.00	75.00		

However, the actual row totals and the desired row total do not match and a set of row factors to correct them is calculated. Now we multiply the row factors by the cell figures in the preceding matrix to obtain cell values as follows:

Origin \ Destination	1	2	3	Actual Total
1	1.83	9.76	2.44	14.03
2	18.54	8.24	6.18	32.96
3	16.80	5.60	5.60	28.00
Actual total	37.17	23.60	14.22	74.99
Desired total	33	28	14	
Total	0.89	1.19	0.98	

Again, the column totals do not match the desired column totals, and therefore a set of column factors are derived that will possibly correct the situation. The column factors are multiplied by the cell figures of matrix, giving us a new matrix:

Origin \ Destination	1	2	3	Actual Total	Desired Total	Row factors
1	1.78	11.61	2.39	15.78	14.00	0.89
2	16.50	9.81	6.06	32.37	33.00	1.02
3	14.95	6.66	5.49	27.10	28.00	1.03
Total	33.23	28.08	13.94	75.00		

Once again, the row totals and column totals are calculated and the process goes through for a second time, producing a matrix that is good enough for planning purposes.

Destination / Origin	1	2	3	Actual Total
1	1.58	10.33	2.13	14.04
2	16.83	10.01	6.18	33.02
3	15.40	6.86	5.65	27.91
Actual total	33.81	27.20	13.96	74.97
Desired total	33	28	14	
Total	0.98	1.03	1.00	

Another iteration,

Destination / Origin	1	2	3	Actual Total	Desired Total	Row factors
1	1.55	10.64	2.13	14.32	14.00	0.98
2	16.49	10.31	6.18	32.98	33.00	1.00
3	15.09	7.07	5.65	27.81	28.00	1.01
Total	33.13	28.02	13.96	75.00		

Origin \ Destination	1	2	3	Actual Total
1	1.52	10.43	2.09	14.04
2	16.49	10.31	6.18	33.02
3	15.24	7.14	5.71	27.91
Actual total	33.25	27.88	13.98	74.97
Desired total	33	28	14	
Total	0.99	1.00	1.00	

Hence, the final O-D table would be

Origin \ Destination	1	2	3	Actual Total
1	2	10	2	14
2	16	11	6	33
3	15	7	6	28
Actual total	33	28	14	75

Gravity Model Example

Example:
A small town has been divided into three traffic zones. An origin-destination survey was conducted earlier this year and yielded the number of trips between each zone as shown in the table below. Travel times between zones were also determined. Provide a trip distribution calculation using the gravity model for two iterations. Assume $K_{ij} = 1$.

The following table shows the number of productions and attractions in each zone:

Zone	1	2	3	Total
Productions	250	450	300	1000
Attractions	395	180	425	1000

The survey's results for the zones' travel time in minutes were as follows:

Zone	1	2	3
1	6	4	2
2	2	8	3
3	1	3	5

The following table shows travel time versus friction factor.

Time (min)	1	2	3	4	5	6	7	8
Friction factor	82	52	50	41	39	26	20	13

Solution:

The mathematical formulation for the gravity model as provided as Equation 12.3:

$$T_{ij} = P_i \frac{(A_j F_{ij} K_{ij})}{\sum (A_j F_{ij} K_{ij})}$$

Since $K_{ij} = 1$, this factor does not affect calculations. The iterative application of Equation 12.3 is as follows:

Iteration 1 :

$T_{11} = 250 * ((395 * 26) / ((395 * 26) + (180 * 41) + (425 * 52)))$
$T_{11} = 250 * (10,270 / 39,750)$
$T_{11} =$ **65**

$T_{12} = 250 * ((180 * 41) / ((395 * 26) + (180 * 41) + (425 * 52)))$
$T_{12} = 250 * (7,380 / 39,750)$
$T_{12} =$ **46**

$T_{13} = 250 * ((425 * 52) / ((395 * 26) + (180 * 41) + (425 * 52)))$
$T_{13} = 250 * (22,100 / 39,750)$
$T_{13} =$ **139**

$$T_{21} = 450 * ((395 * 52) / ((395 * 52) + (180 * 13) + (425 * 50)))$$
$$T_{21} = 450 * (20{,}540 / 44{,}130)$$
$$T_{21} = \mathbf{209}$$

$$T_{22} = 450 * ((180 * 13) / ((395 * 52) + (180 * 13) + (425 * 50)))$$
$$T_{22} = 450 * (2{,}340 / 44{,}130)$$
$$T_{22} = \mathbf{24}$$

$$T_{23} = 450 * ((425 * 50) / ((395 * 52) + (180 * 13) + (425 * 50)))$$
$$T_{23} = 450 * (21{,}250 / 44{,}130)$$
$$T_{23} = \mathbf{217}$$

$$T_{31} = 300 * ((395 * 82) / ((395 * 82) + (180 * 50) + (425 * 39)))$$
$$T_{31} = 300 * (32{,}390 / 57{,}965)$$
$$T_{31} = \mathbf{168}$$

$$T_{32} = 300 * ((180 * 50) / ((395 * 82) + (180 * 50) + (425 * 39)))$$
$$T_{32} = 300 * (9{,}000 / 57{,}965)$$
$$T_{32} = \mathbf{46}$$

$$T_{33} = 300 * ((425 * 39) / ((395 * 82) + (180 * 50) + (425 * 39)))$$
$$T_{33} = 300 * (16{,}575 / 57{,}965)$$
$$T_{33} = \mathbf{86}$$

Trip Matrix for Iteration 1

Zone	1	2	3	Productions
1	65	46	139	250
2	209	24	217	450
3	168	46	86	300
Computed Attractions	442	116	442	
Given Attractions	395	180	425	

Next, calculate the adjusted attraction factors using Equation 12.4.

$$A_{jk} = \frac{A_j}{C_{j(k-1)}} A_{j(k-1)}$$

<u>Zone 1</u>

A_{jk} = (395 / 442) * 395
A_{jk} = **353**

<u>Zone 2</u>

A_{jk} = (180 / 116) * 180
A_{jk} = **279**

<u>Zone 3</u>

A_{jk} = (425 / 442) * 425
A_{jk} = **409**

Now apply the gravity model formula for Iteration 2 using the above adjusted attraction factors.

Iteration 2

T_{11} = 250 * ((353 * 26) / ((353 * 26) + (279 * 41) + (409 * 52)))
T_{11} = 250 * (9,178 / 41,885)
T_{11} = **55**

T_{12} = 250 * ((279 * 41) / ((353 * 26) + (279 * 41) + (409 * 52)))
T_{12} = 250 * (11,439 / 41,885)
T_{12} = **68**

T_{13} = 250 * ((409 * 52) / ((353 * 26) + (279 * 41) + (409 * 52)))
T_{13} = 250 * (21,268 / 41,885)
T_{13} = **127**

$T_{21} = 450 * ((353 * 52) / ((353 * 52) + (279 * 13) + (409 * 50)))$
$T_{21} = 450 * (18{,}356 / 42{,}433)$
$T_{21} = \mathbf{195}$

$T_{22} = 450 * ((279 * 13) / ((353 * 52) + (279 * 13) + (409 * 50)))$
$T_{22} = 450 * (3{,}627 / 42{,}433)$
$T_{22} = \mathbf{38}$

$T_{23} = 450 * ((409 * 50) / ((353 * 52) + (279 * 13) + (409 * 50)))$
$T_{23} = 450 * (20{,}450 / 42{,}433)$
$T_{23} = \mathbf{217}$

$T_{31} = 300 * ((353 * 82) / ((353 * 82) + (279 * 50) + (409 * 39)))$
$T_{31} = 300 * (28{,}946 / 58{,}847)$
$T_{31} = \mathbf{148}$

$T_{32} = 300 * ((279 * 50) / ((353 * 82) + (279 * 50) + (409 * 39)))$
$T_{32} = 300 * (13{,}950 / 58{,}847)$
$T_{32} = \mathbf{71}$

$T_{33} = 300 * ((409 * 39) / ((353 * 82) + (279 * 50) + (409 * 39)))$
$T_{33} = 300 * (15{,}951 / 58{,}847)$
$T_{33} = \mathbf{81}$

Trip Matrix for Iteration 2

Zone	**1**	**2**	**3**	**Productions**
1	55	68	127	250
2	195	38	217	450
3	148	71	81	300
Computed Attractions	398	177	425	
Given Attractions	395	180	425	

Observe that the computed attractions approximately equal the given attractions. A total convergence would be expected in another iteration.