Faculty of Engineering
Department of Civil Engineering

Highways and airports engineering

Vertical Alignment

- A primary concern in vertical alignment is establishing the transition of roadway elevations between two grades. This transition is achieved by means of a vertical curve.
- The vertical alignment of a highway consists of straight sections known as **grades**, (or tangents) connected by **vertical curves.**

Profiles:

Curve a: Crest Vertical Curve (concave downward)

Curve b: Sag Vertical Curve (concave upward)

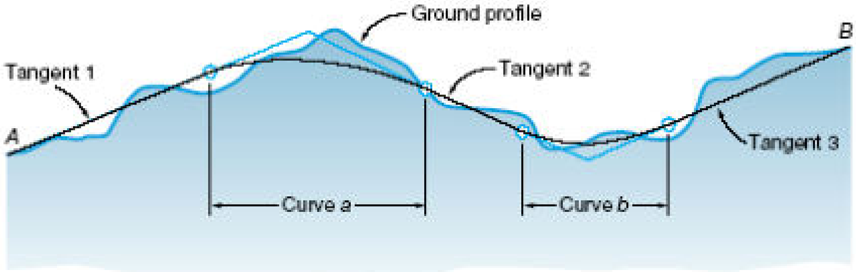
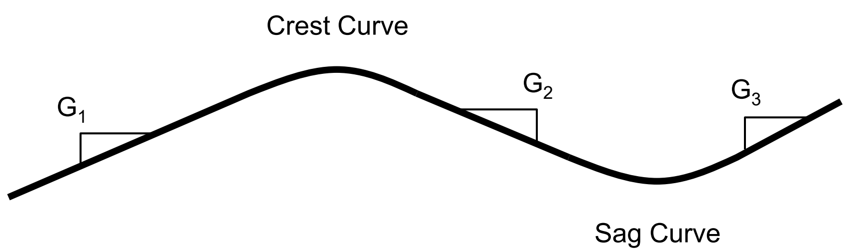


Figure 25–1 Grade line and ground profile of a proposed highway section.

Tangents: Constant Grade (Slope)

Vertical Alignment
Tangents and Curves

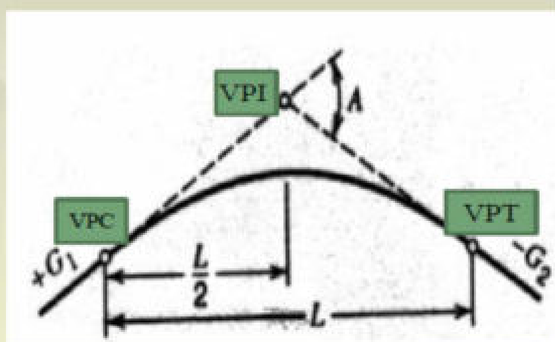
- Like the horizontal alignment, the vertical alignment is made up of tangent and curves
- In this case the curve is a parabolic curve rather than a circular or spiral curve



What is a vertical curve?

- A parabolic curve that is applied to make a smooth and safe transition between two grades on a roadway or a highway.

VPC: Vertical Point of Curvature
VPI: Vertical Point of Intersection
VPT: Vertical Point of Tangency
G1, G2: Tangent grades in percent
A: Algebraic difference in grades
L: Length of vertical curve

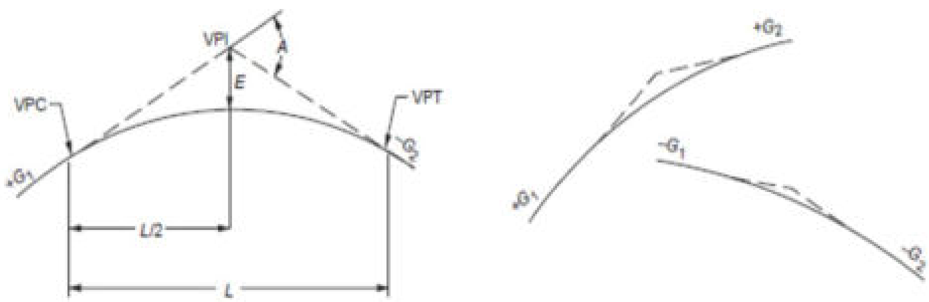


$A = G_2 - G_1$

In this case $A = -G_2 - (G_1) = -G_1 - G_2$

Crest Vertical Curves

- Type I: upgrade then downgrade
- Type II: upgrade then upgrade with less value
  : downgrade then downgrade with more value



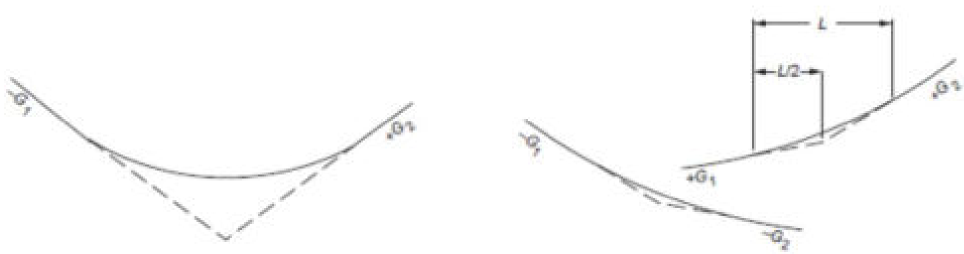
Type I   Type II

7

Sag Vertical Curves

- Type III: downgrade then upgrade

- Type IV: upgrade then upgrade with more value
  : downgrade then downgrade with less value



Type III   Type IV

8

Examples

- Are these a crest or sag curves?
- Design speed = 70 mph
- 1- G_1 = +2%, G_2 = -4%, A=-4-2=-6
- Crest curve
- 2- G_1 = -2%, G_2 = +4%, A=4+2=6
- Sag curve
- 3- G_1 = +4%, G_2 = +2%, A=2-4=-2
- Crest curve

For a crest curve, *A* is negative

For a sag curve, *A* is positive

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9

Examples

- Are these a crest or sag curves?
- Design speed = 70 mph
- 4- G_1 = -4%, G_2 = -2%, A=-2+4=2
- Sag curve
- 5- G_1 = +3%, G_2 = +4%, A=4-3=1
- Sag curve
- 6- G_1 = -2%, G_2 = -4%, A=-4+2=-2
- Crest curve

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10

Maximum Grade

www.geograph.org.uk

Harlech, Gwynedd, UK (*G* = 34%)

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11

Maximum Grade

Dee747 at picasaweb.google.com

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12

Maximum and Minimum Grade

One important design consideration is the determination of the maximum and minimum grade that can be allowed on the tangent section

The minimum grade used is typically **0.5%**

The maximum grade is generally a function of the

- Design Speed
- Terrain (Level, Rolling, Mountainous)

*On high speed facilities such as freeways the maximum grade is generally kept to **5%** where the terrain allows (**3%** is desirable since anything larger starts to affect the operations of trucks)*

*At **30 mph** design speed the acceptable maximum is in the range of 7 **to 12 %***

Maximum Grades

- Passenger vehicles can easily negotiate 4 to 5% grade without appreciable loss in speed.
- Upgrades: trucks average 7% decrease in speed.
- Downgrades: trucks average speed increase 5%

Grade Considerations

- Maximum grade – depends on terrain type, road functional class, and design speed

Rural Arterials

Terrain	60mph	70mph
Level	3%	3%
Rolling	4%	4%
Mountainous	6%	5%

Maximum Grade

TABLE 4.1
Recommended standards for maximum grades, percent

Type of terrain	Freeways	Rural highways	Urban highways
Level	3–4	3–5	5–8
Rolling	4–5	5–6	6–9
Mountainous	5–6	5–8	8–11

Source: From *A Policy on Geometric Design of Highways and Streets.* Copyright 1994 by the American Association of State Highway and Transportation Officials, Washington, DC. Used by permission.

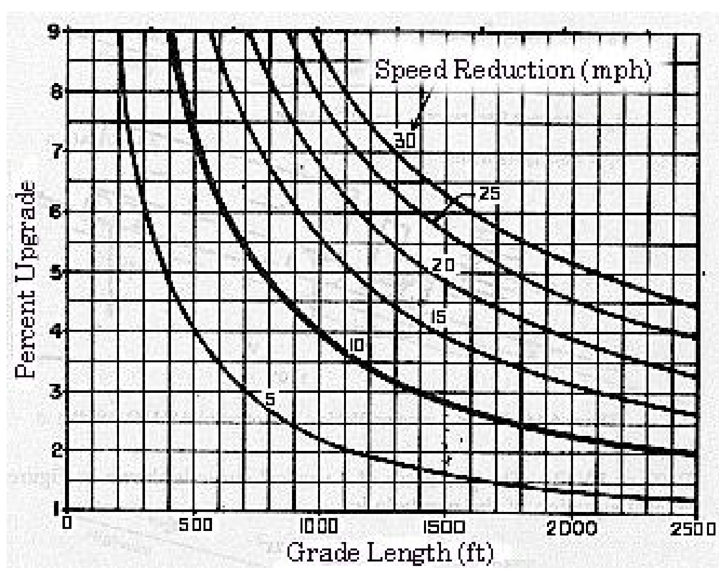
Grade Max. length

جدول رقم (٣-١٣) الطول الحرج للميول المختلفه

الميل الطولى %	الطول الحرج بالمتر
٣	٤٠٠
٤	٢٨٠
٥	٢١٠
٦	١٧٠
٧	١٥٠
٨	١٣٥
٩	١٣٠

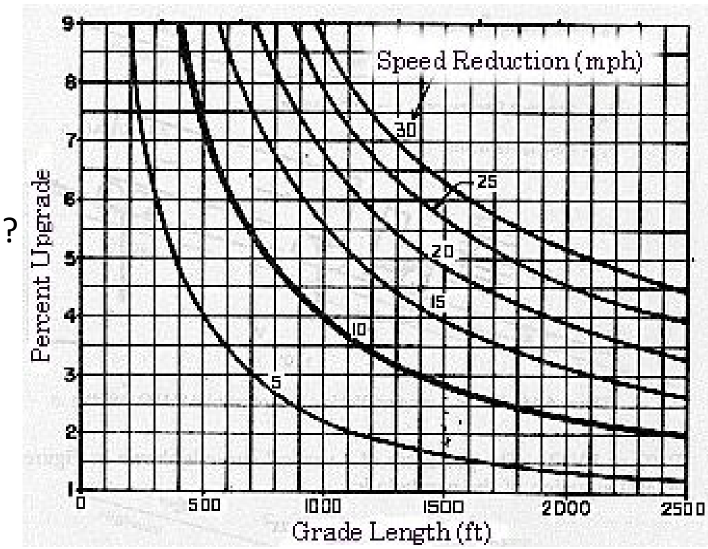
Vertical Curves

- Ascending grade:
  - Frequency of collisions increases significantly when vehicles traveling more than 10 mph below the average traffic speed are present in the traffic stream



Example

If a highway with traffic normally running at 65 mph has an inclined section with a 3.5% grade, what is the maximum length of grade that can be used before the speed of the larger vehicles is reduced to 55 mph?



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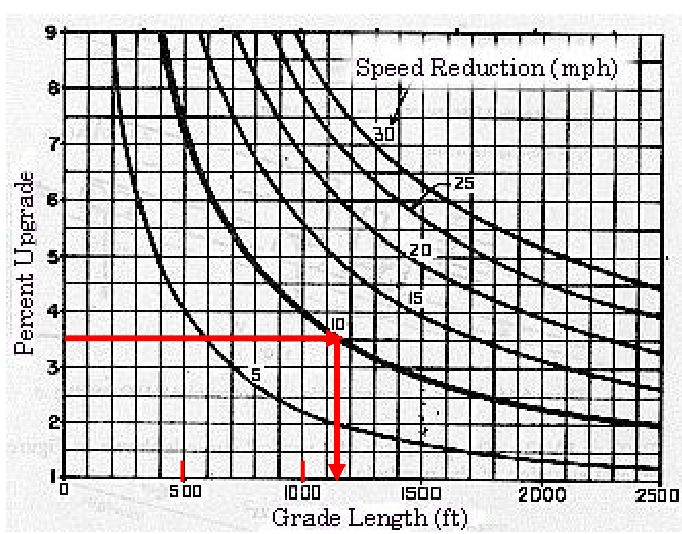
19

Example 1

- a 3.5% grade causes a reduction in speed of 10 mph after 1150 feet



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20

Example 2

- a 4.5% grade causes a reduction in speed of 10 mph after 900 feet

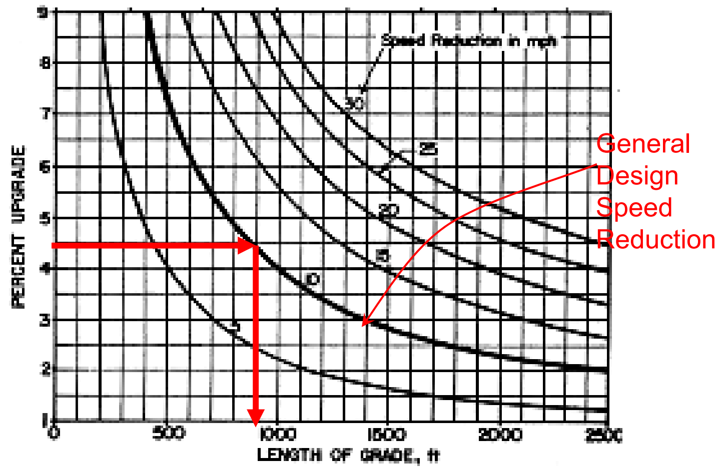


Figure III-30. Critical lengths of grade for design, assumed typical heavy truck of 300 lb/hp, entering speed = 55 mph.

Maximum Change of Grade Permitted Without Use of a Vertical Curve, and Min. length of vertical curve for good appearance

Table 6.2 Vertical Curve Appearance Criteria

Design Speed (km/h)	Maximum Change of Grade Permitted Without Use of a Vertical Curve (%)	Minimum Length of Vertical Curve for Good Appearance (m)
30	1.5	15
40	1.2	20
50	1.0	30
65	0.8	40
80	0.6	50
100	0.5	60

Source: adapted from Table 7.42 & 7.43, RMSS, Vol. V11A

Vertical Curves

- To provide transition between two grades
- Consider
  - Drainage (rainfall)
  - Driver safety (SSD)
  - Driver comfort
- Use parabolic curves
- Crest vs Sag curves

Vertical Curves

- ***G1* , *G2*** = grades of tangents (%)
- ***A*** = algebraic difference = $G_2 - G_1$
- ***L*** = length of vertical curve
- ***PVC*** = point of vertical curve
- ***PVI*** = point of vertical intersection
- ***PVT*** = point of vertical tangent

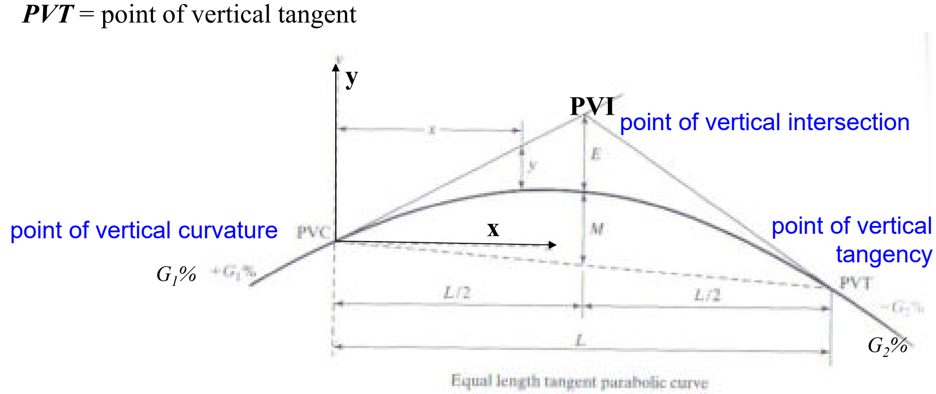


Figure 6-17 Geometry of the Vertical Curve (Carter and Homburger, 1978).

Examples

- What is A?
- 1- G_1 = +2%, G_2 = -4%
- A = $G_2 - G_1$ = -4-2 = -6%
- 2- G_1 = -2%, G_2 = +4%
- A = $G_2 - G_1$ = 4+2 = +6%
- 3- G_1 = +4%, G_2 = +2%
- A = $G_2 - G_1$ = 2-4= -2%

Examples

- What is A?
- 4- G_1 = -4%, G_2 = -2%
- A = $G_2 - G_1$ = -2+4 = +2%
- 5- G_1 = +3%, G_2 = +4%
- A = $G_2 - G_1$ = 4-3 = +1%
- 6- G_1 = -2%, G_2 = -4%
- A = $G_2 - G_1$ = -4- (-2) = -2%

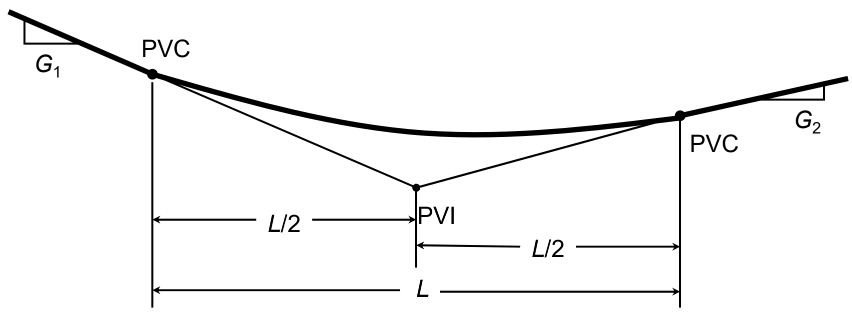
Parabolic Curves

- *The general form of the parabolic equation, as applied to vertical curves, is:*
- $y = ax^2 + bx + c$
- *where*
- *y = roadway elevation at distance x from the beginning of the vertical curve (the PVC) in stations or ft (m) ,*
- *x = distance from the beginning of the vertical curve in stations or ft (m) ,*
- *a, b, c = coefficients defined below*
- *At x = 0, y = PVC*
- *Then PVC = c*
- *c = elevation of the PVC*

Parabolic Curves

- Note that the first derivative gives the slope.
- $dy/dx = G_1$, at $x = 0$
- $dy/dx = 2ax + b$
- $b = G_1$
- $dy/dx = G_2$, at $x = L$
- $dy/dx = 2ax + b$
- $G_2 = 2a*L + G1$
- $a = (G_2 - G_1) / 2L$
- $y = (G_2 - G_1)\, x^2 / 2L + G_1 x + PVC$
- $y = A x^2 / 2L + G_1 x + PVC$
- $dy/dx = (G_2 - G_1)\, x / L + G_1 = A x / L + G_1$

Properties of Vertical Curves



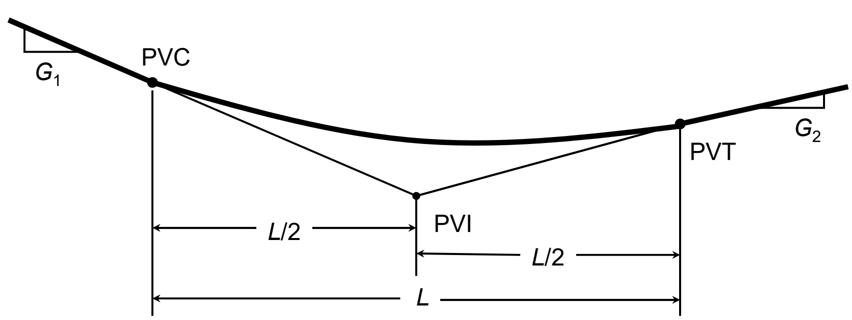
Change in grade: $A = G_2 - G_1$

where G is expressed as % (positive /, negative \)

For a crest curve, A is negative

For a sag curve, A is positive

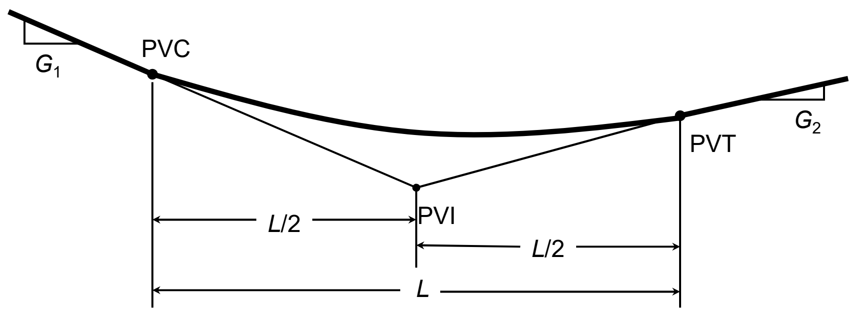
Properties of Vertical Curves



Rate of change of curvature: $K = L / |A|$

Which is a gentler curve - small K or large K?

Properties of Vertical Curves



Rate of change of grade: $r = (g_2 - g_1) / L$

where,

g is expressed as a ratio (positive /, negative \)

L is expressed in feet or meters

Note – K and r are both measuring the same characteristic of the curve but in different ways

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31

Examples

- What is K, r, if L = 150 m?
- 1- G_1 = +2%, G_2 = -4%, crest curve
- K = L/|A| = 150 / |-0.04-0.02| = 2500 m
- r = A / L= (-0.04-0.02) / 150 = - 0.0004 / m
- 2- G_1 = +4%, G_2 = +2%, crest curve
- K = L/|A| = 150 / |0.02-0.04| = 7500 m
- r = A / L= (0.02-0.04) / 150 = - 0.00013 / m
- 3- G_1 = -6%, G_2 = -2%, sag curve
- K = L/|A| = 150 / |-0.02+0.06| = 3750 m
- r = A / L= (-0.02+0.06) / 150 = + 0.00027 / m

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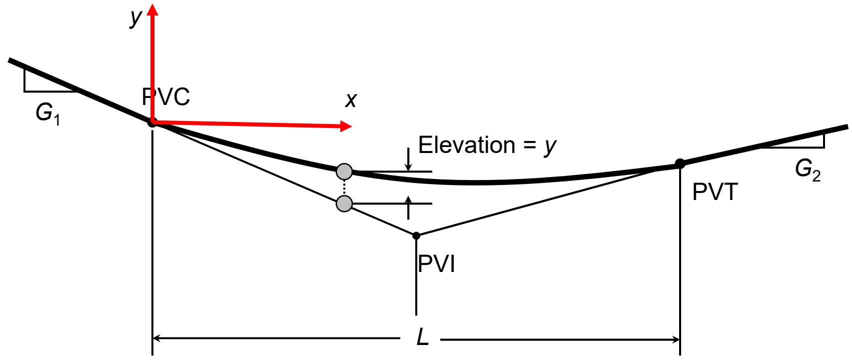
32

Properties of Vertical Curves



Equation for determining the elevation at any point on the curve

$$y = (G_2 - G_1)\,x^2\ /\ 2L\ + G_1\,x + PVC$$

$$y = 1/2\ r\ x^2\ +\ G_1 x + y_0 \qquad \text{Where } r = A / L$$

where,

y_0 = elevation at the PVC
G = grade expressed as a ratio
x = horizontal distance from PVC
r = rate of change of grade expressed as ratio

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33

High/Low point on curve

Distance PVC to the turning point (high/low point on curve)

$$y = A\,x^2\ /\ 2L\ + G_1\,x + PVC$$

$$dy/dx = A\,x / L + G_1\ = 0,$$

$$X = -\ (G_1 * L/A) = -(\ G_1 * 1/r)$$

$$x = -(G_1/r)$$

Where r = A / L

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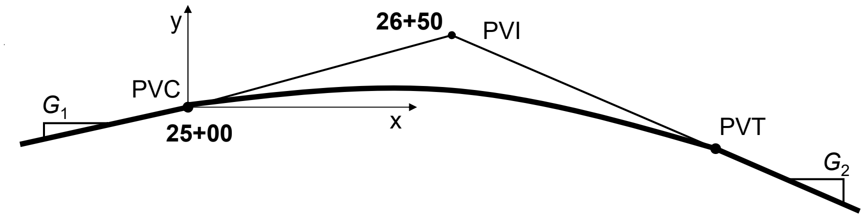
34

Properties of Crest Curves



Example:

G_1 = 2 % G_2 = - 3 %

Elevation of PVI = 150.00 m

Station of PVC = 25+00

Station of PVI = 26+50

Length of curve?

L/2 = Sta. PVI – Sta. PVC

$L/2$ = 2650 m - 2500 m = 150 m

L = 300 m

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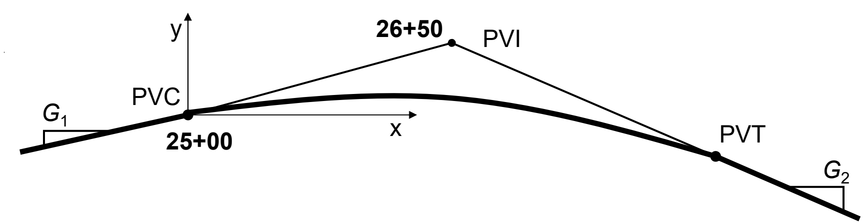
35

Properties of Crest Curves



Example:

G_1 = 2 % G_2 = - 3 %

Elevation of PVI = 150.00 m

Station of PVC = 25+00

Station of PVI = 26+50

***r* - value?**

$r = (G_2 - G_1)/L$

$r = |(-0.03 - 0.02)|/300$ m

r = - 0.000167 / meter

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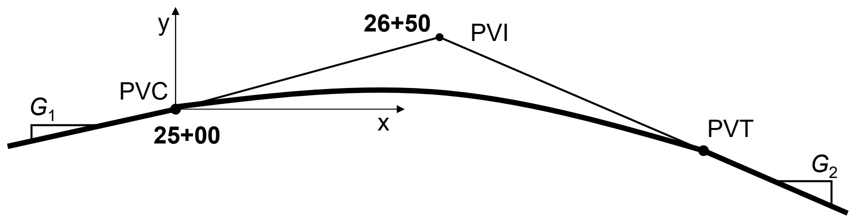
36

Properties of Crest Curves



Example:
G_1 = 2 % G_2 = - 3 %
Elevation of PVI = 150.00 m
Station of PVC = 25+00
Station of PVI = 26+50

Station of high point?

$x = -(G_1/r)$

x = -([0.02] / [-0.000167/m])

x = 120 m

Station = [25+00] + 120 m

Station = 26+20

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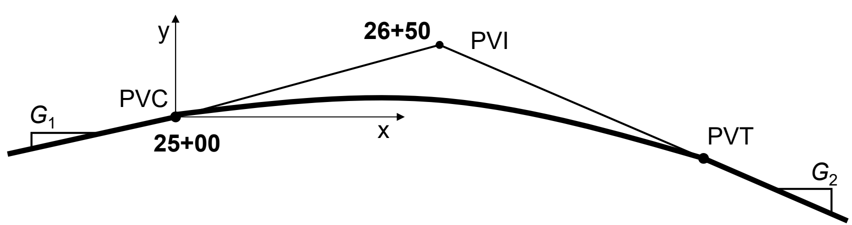
37

Properties of Crest Curves



Example:
G_1 = 2 % G_2 = - 3 %
Elevation of PVI = 150.00 m
Station of PVC = 25+00
Station of PVI = 26+50

Elevation at high point?

$y = y_0 + G_1x + 1/2\ rx^2$

y_0 = Elev. PVC

Elev. PVC = Elev. PVI - $G_1L/2$

Elev. PVC = 150 m - [0.02][150 m]

Elev. PVC = 147 m

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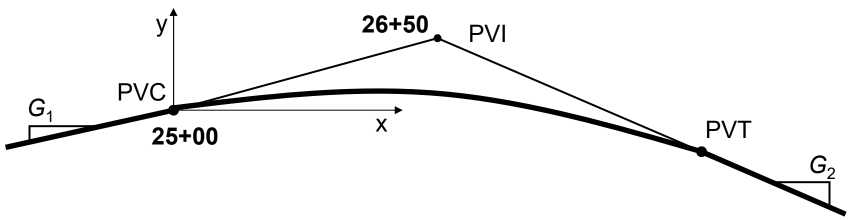
38

Properties of Crest Curves



Example:

G_1 = 2 % G_2 = - 3 %

Elevation of PVI = 150.00 m

Station of PVC = 25+00

Station of PVI = 26+50

Elevation at high point?

$y = y_0 + G_1x + 1/2\ rx^2$

y = 147 m + [0.02][120 m] + 1/2 [-0.000167/m][120 m]^2

y= 147 + 2.4 – 1.2

y = 148.2 m

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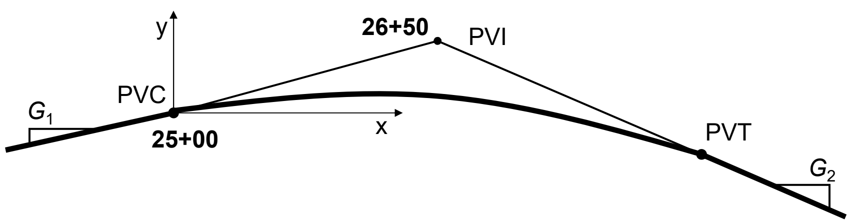
39

Properties of Crest Curves



Example:

G_1 = 2 % G_2 = - 3 %

Elevation of PVI = 150.00 m

Station of PVC = 25+00

Station of PVI = 26+50

Elevation at station 25+75?

y = 147 m + [0.02][75 m] + 1/2 [-0.000167/m][75 m]^2

y= 147 + 1.5 – 0.47

y = 148.03 m

Elevation at station 27+25?

y = 147 m + [0.02][225 m] + 1/2 [-0.000167/m][225 m]^2

y= 147 + 4.5 – 4.22

y = 147.28 m

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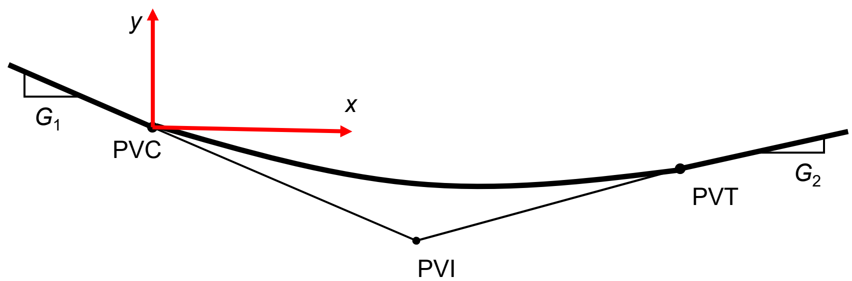
40

Properties of Sag Curves



Example:
G_1 = -1% G_2 = +3%
Elevation of PVI = 125.00 m
Station of PVT = 25+00
Station of PVI = 24+00

Length of curve?

L/2 = Sta. PVT – Sta. PVI
L/2 = 2500 m - 2400 m = 100 m
L = 200 m

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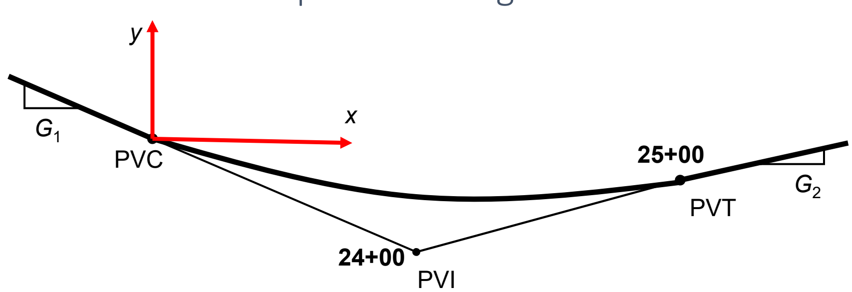
41

Properties of Sag Curves



Example:
G_1 = -1% G_2 = +3%
Elevation of PVI = 125.00 m
Station of PVT = 25+00
Station of PVI = 24+00

***r* - value?**

$r = (G_2 - G_1)/L$
r = (0.03 - [-0.01])/200 m
r = 0.0002 / meter

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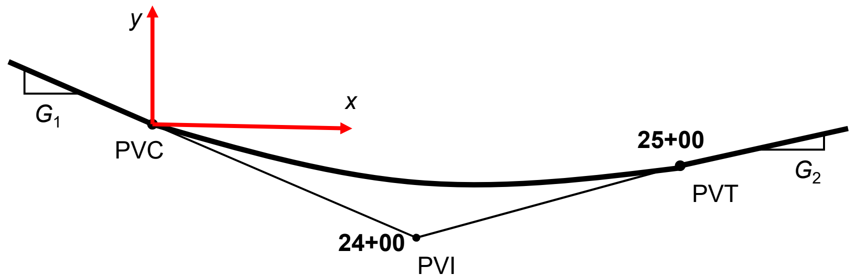
42

Properties of Sag Curves



Example:

G_1 = -1% G_2 = +3%

Elevation of PVI = 125.00 m

Station of PVT = 25+00

Station of PVI = 24+00

Station of low point?

$x = -(G_1/r)$

x = -([-0.01] / [0.0002/m])

x = 50 m

Station PVC = 24+00 - 1+00 = 23+00

Station = [23+00] + 50 m

Station 23+50

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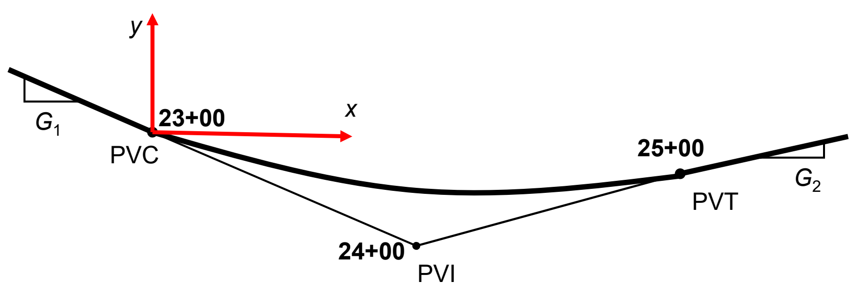
43

Properties of Sag Curves



Example:

G_1 = -1% G_2 = +3%

Elevation of PVI = 125.00 m

Station of PVT = 25+00

Station of PVI = 24+00

Elevation at low point?

Elevation PVC

$y = y_0 + G_1x + 1/2\ rx^2$

y_0 = Elev. PVC

Elev. PVC = Elev. PVI - $G_1L/2$

Elev. PVC = 125 m - [-0.01][100 m]

Elev. PVC = 126 m

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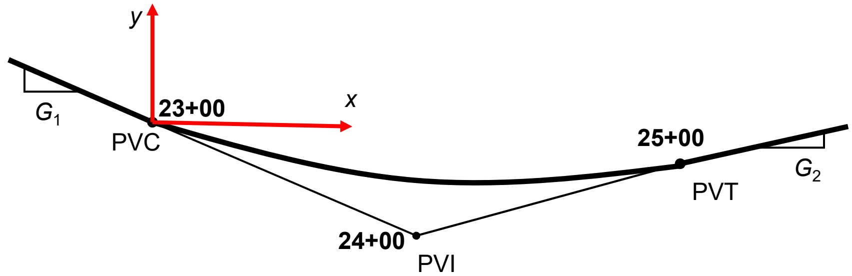
44

Properties of Sag Curves



Example:
G_1 = -1% G_2 = +3%
Elevation of PVI = 125.00 m
Station of PVT = 25+00
Station of PVI = 24+00

Elevation at low point?

$y = y_0 + G_1 x + 1/2\ rx^2$

y = 126 m + [-0.01][50 m] + 1/2 [0.0002/m][50 m]^2

$y = 126 - 0.5 + 0.25$

y = 125.75 m

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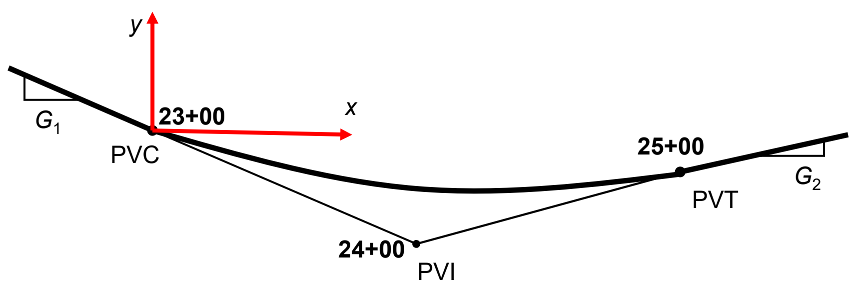
45

Properties of Sag Curves



Example:
G_1 = -1% G_2 = +3%
Elevation of PVI = 125.00 m
Station of PVT = 25+00
Station of PVI = 24+00

Elevation at station 23+50?

y = 126 m + [-0.01][50 m] + 1/2 [0.0002/m][50 m]^2

y = 126 -0.5+0.25= 125.75 m

Elevation at station 24+50?

y = 126 m + [-0.01][150 m] + 1/2 [0.0002/m][150 m]^2

y = 126 -1.5+2.25= 126.75 m

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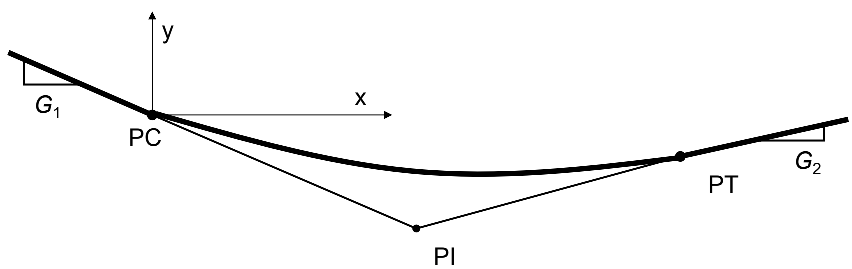
46

Example

A -2.5% grade is connected to a +1.0% grade by means of a 180 m vertical curve. The P.I. station is 100+00 and the P.I. elevation is 100.0 m above sea level. What are the station and elevation of the lowest point on the vertical curve?



Solution

Rate of change of grade:

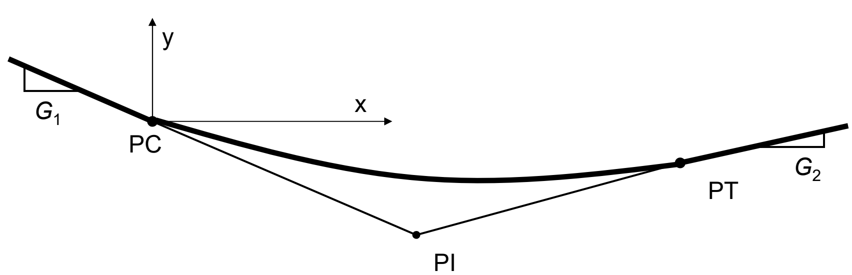
$r = (g_2 – g_1)/L = (+1\% – (-2.5\%)) / 1.8$ sta $= 1.944\%$ / sta

Station of the low point:

At low point, slope = 0

Slope $= g_1 + r\,x = 0$

$x = - (g_1 / r) = - (-2.5/1.944\text{Sta}) = 1.29$ sta $= 1 + 29$ sta



Solution

Station of PC = (100 +00) -(0 +90) =99 +10
Station of low point =(99 +10) +(1 +29) =100 +39
Elevation of PC:
y_0 =100.0 m +(-0.9 sta)(-2.5%) =102.25 m
Elevation of low point:
$y = y_0 + g_1 x + [(r x^2)/2]$
$y = 102.25 + (-2.5\%)\ (1.29\ sta) + [(1.944\%sta*(1.29sta^2)/2]$
y = 102.25 -3.225 + 1.617
y = 100.64

15-11-2023 Prof. Mahmoud Enieb 49

49

Example: A crest vertical curve joins a +3% and –4% grade. Design speed is 75 mph. Length = 2184.0 ft. Station at VPI is 345+ 60.00, elevation at VPI = 250 feet. Find elevations and station for VPC (BVC) and VPT (EVC).

L/2 = 2184/2 = 1092.0 ft

Station at VPC = [345 + 60.00] - [10 + 92.00] = **334 + 68.00**

Vertical Diff VPI to VPC: 0.03 x (2184/2) = 32.76 feet

Elevation $_{VPC}$ = 250 – 32.76 = **217.24 feet**

Station at VPT = [345 + 60.00] + [10 + 92.00] = **356 + 52.00**

Vertical Diff VPI to VPT = 0.04 x (2184/2) = 43.68 feet

Elevation $_{VPT}$ = 250 – 43.68 = **206.32 feet**

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50

Example: A crest vertical curve joins a +3% and –4% grade. Design speed is 75 mph. Length = 2184.0 ft. Station at VPI is 345+ 60.00, elevation at VPI = 250 feet.

Calculate points along the vertical curve.

Station at VPC (PVC) is = 345+60 – (2+184/2) = 334 + 68.00 ft

Elevation at VPC is = 250.0 – 0.03*(2184/2) = 217.24 feet.

x = distance from VPC

$Y = (Ax^2) / 200 L$

$Elevation_{tangent}$ = elevation at VPC + distance * grade (x * G1)

$Elevation_{curve} = Elevation_{tangent} + Y$

Find elevation on the curve at a point 400 feet from VPC (217.24).

A = -4 – (+3) = - 7

$Y = (Ax^2) / 200 L = (-7*(400)^2)/200*2184 = -2.56$ ft

Elevation at tangent = 217.24 + (400 x 0.03) = 229.24 ft

Elevation on curve = 229.24 – 2.56 feet = 226.68 ft

Example: Equal-Tangent Vertical Curve

Given the information show below, compute and tabulate the curve for stakeout at full 100 ft stations.

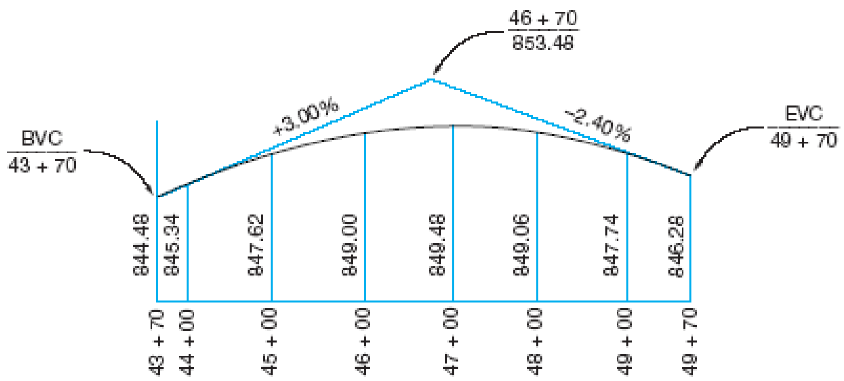


Figure 25–4 Crest curve of Example 25–1.

15-11-2023 Prof. Mahmoud Enieb 53

53

Solution:

$L = STA_{EVC} - STA_{BVC}$

L = 4970 – 4370 = 600'

or 6 *full* stations

$r = (g_2 - g_1) / L$

$r = (-2.4 - 3) / 6$

r = -0.90,

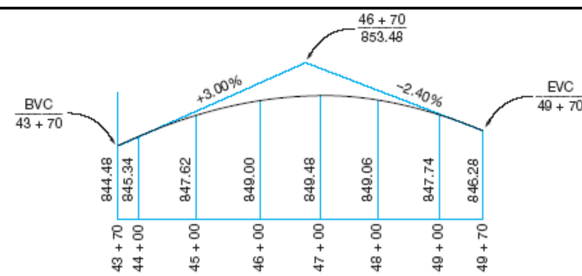


Figure 25–4 Crest curve of Example 25–1.

r/2 = -0.45 % per station

$STA_{BVC} = STA_{Vertex} - L / 2 = 4670 - 600/2 = STA_{BVC} = STA\ 43 + 70$

$STA_{EVC} = STA_{Vertex} + L / 2 = 4670 + 600/2 = STA_{EVC} = STA\ 49 + 70$

$Elev_{BVC} = Elev_{vertex} - g_1 (L/2) = 853.48 - 3.00 (3) = 844.48'$

$Elev_{EVC} = Elev_{vertex} - g_2 (L/2) = 853.48 - 2.40 (3) = 846.28'$

15-11-2023 Prof. Mahmoud Enieb 54

54

Solution: (continued)

$r/2 = -0.45$ % per station

$Elev_x = Elev_{BVC} + g_1x + (r/2)x^2$

$Elev_{44+00} = 844.48 + 3.00(0.30) - 0.45(0.30)^2 = 845.34'$

$Elev_{45+00} = 844.48 + 3.00(1.30) - 0.45(1.30)^2 = 847.62'$

$Elev_{46+00} = 844.48 + 3.00(2.30) - 0.45(2.30)^2 = 849.00'$

etc.

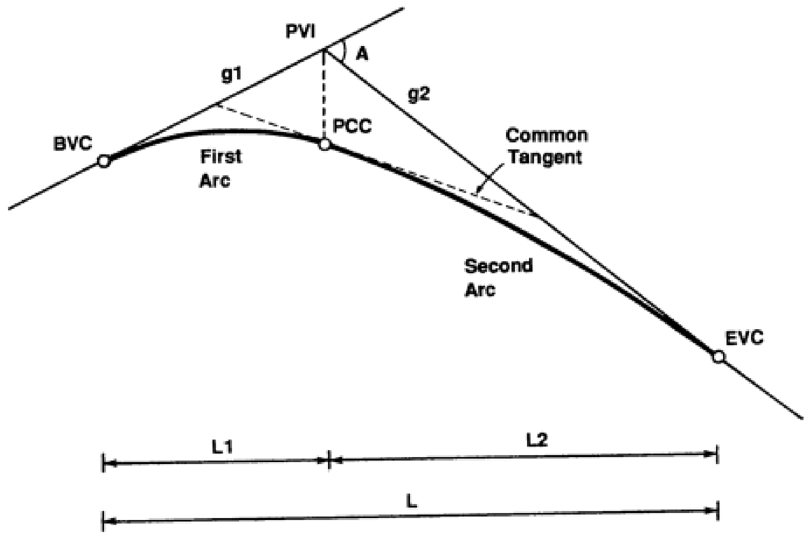
$Elev_{49+00} = 844.48 + 3.00(5.30) - 0.45(5.30)^2 = 847.74'$

$Elev_{49+70} = 844.48 + 3.00(6.00) - 0.45(6.00)^2 = 846.28'$ (CHECKS, Okay)

Solution: (continued)

Station	x (stations)	g_1x	$r/2*x^2$	Curve Elevation
43 + 70 BVC	0.0	0.00	0.00	844.48
44 + 00	0.3	0.90	-0.04	845.34
45 + 00	1.3	3.90	-0.76	847.62
46 + 00	2.3	6.90	-2.38	849.00
47 + 00	3.3	9.90	-4.90	849.48
48 + 00	4.3	12.90	-8.32	849.06
49 + 00	5.3	15.90	-12.64	847.74
49 + 70 EVC	6.0	18.00	-16.20	846.28

Traditional unsymmetric vertical curve.

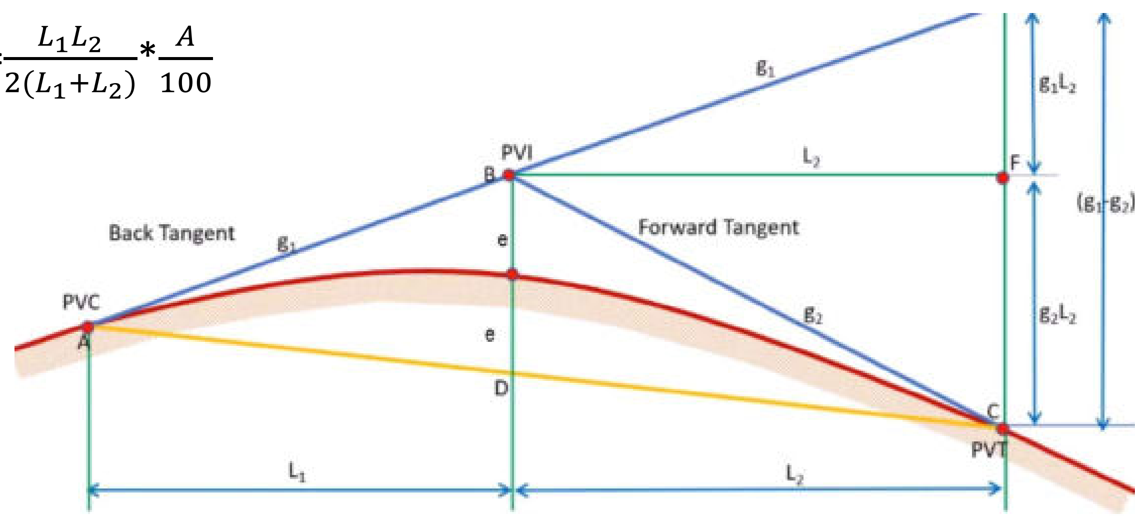


15-11-2023 Prof. Mahmoud Enieb 57

57

Parameters of traditional unsymmetric vertical curve

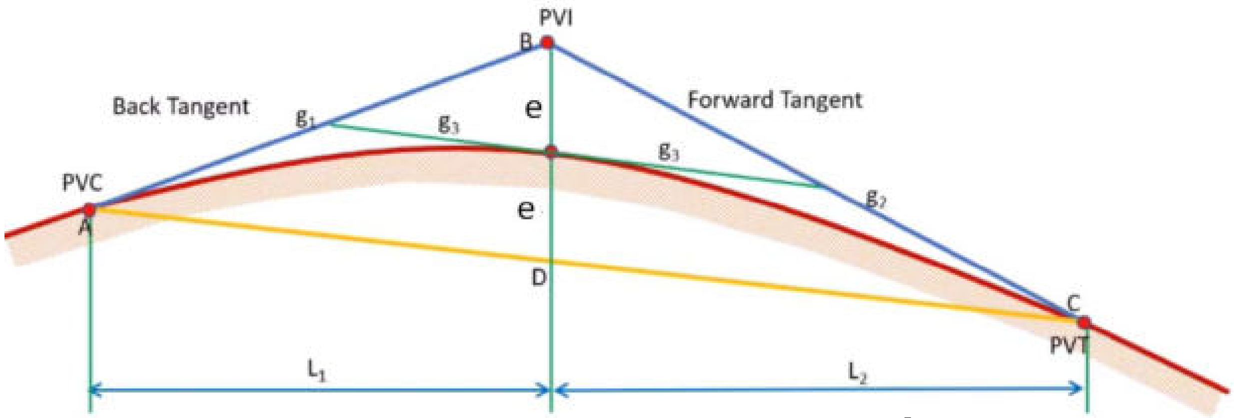
$$e=\frac{L_1 L_2}{2(L_1+L_2)}*\frac{A}{100}$$



15-11-2023 Prof. Mahmoud Enieb 58

58

Parameters of traditional unsymmetric vertical curve



$$e = \frac{L_1 L_2}{2(L_1 + L_2)} * \frac{A}{100}$$

$$If\ L_1 = L_2 = \frac{L}{2}$$

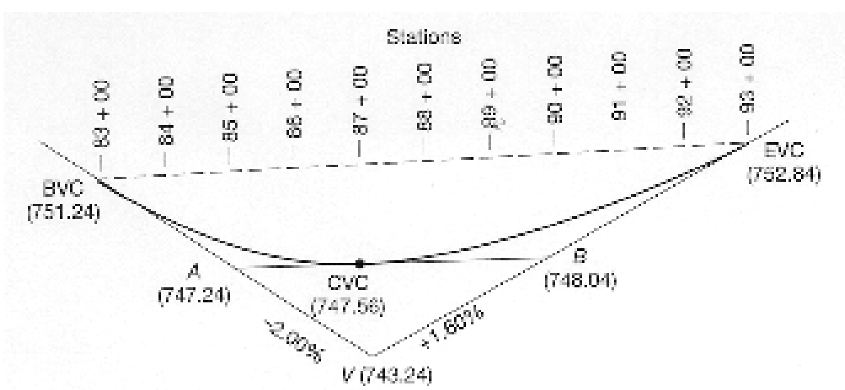
$$e = \frac{AL}{800}$$

15-11-2023 Prof. Mahmoud Enieb 59

59

Unequal-Tangent Parabolic Curve

A grade g_1 of **-2%** intersects g_2 of **+1.6%** at a vertex whose station and elevation are **87+00** and **743.24**, respectively. A **400** ft vertical curve is to be extended back from the vertex, and a **600** ft vertical curve forward to closely fit ground conditions. Compute and tabulate the curve for stakeout at full stations.



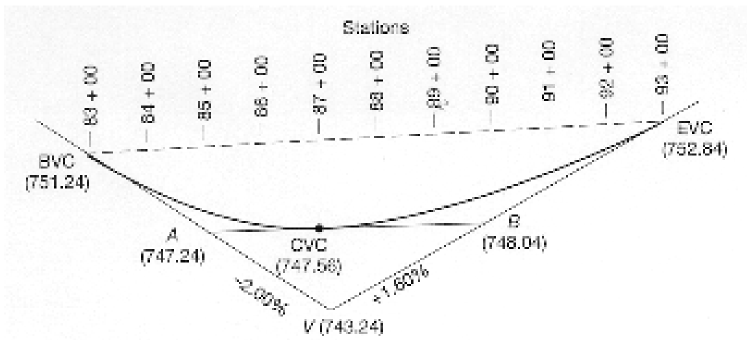
15-11-2023 60
Prof. Mahmoud Enieb

60

Solution:

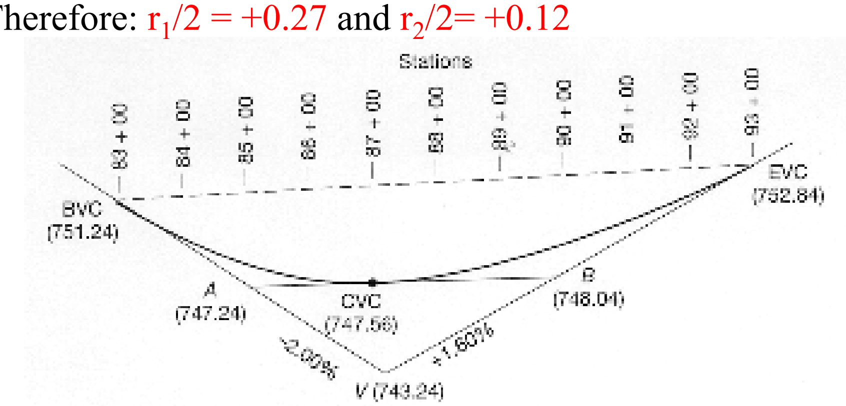
The CVC is defined as a point of compound vertical curvature. We can determine the station and elevation of points A and B by The CVC is defined as a point of compound vertical curvature. We can determine the station and elevation of points A and B by reducing this unequal tangent problem to two equal tangent problems. Point A is located 200' from the BVC and Point B is located 300' from the EVC. Knowing this we can compute the elevation of points A and B. Once A and B are known we can compute the grade from A to B thus allowing us to solve this problem as two equal tangent curves.

Pt. A STA 85 + 00, Elev. = 743.24 + 2 (2) = 747.24'
Pt. B STA 90 + 00, Elev. = 743.24 + 1.6 (3) = 748.04'



Solution (continued):

The grade between points A and B can be calculated as:
q_{A-B} = (748.04 -747.24) / 5 = + 0.16 %
and the rate of curvature for the two equal tangent curves can be computed as:
r_1 = (0.16+2.0)/4 = + 0.54 and r_2 = (1.6 -0.16) / 6 = +0.24
Therefore: $r_1/2$ = +0.27 and $r_2/2$= +0.12

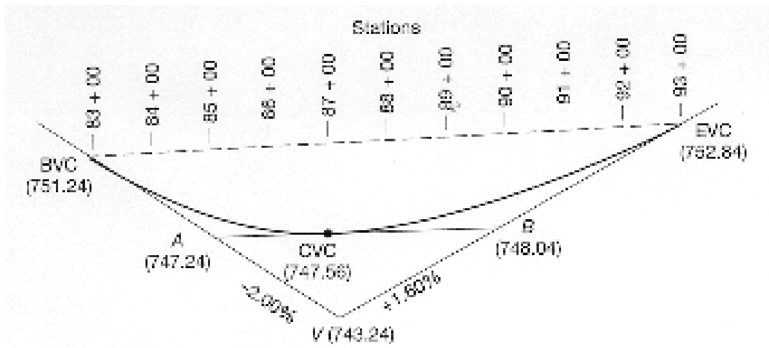


Solution (continued):

The station and elevations of the BVC, CVC and EVC are computed as:

BVC STA 83 + 00, Elev. 743.24 + 2 (4) = 751.24’
EVC STA 93 + 00, Elev. 743.24 + 1.6 (6) = 752.84’
CVC STA 87 + 00, Elev. 747.24 + 0.16 (2) = 747.56’

Please note that the CVC is the EVC for the first equal tangent curve and the BVC for the second equal tangent curve.



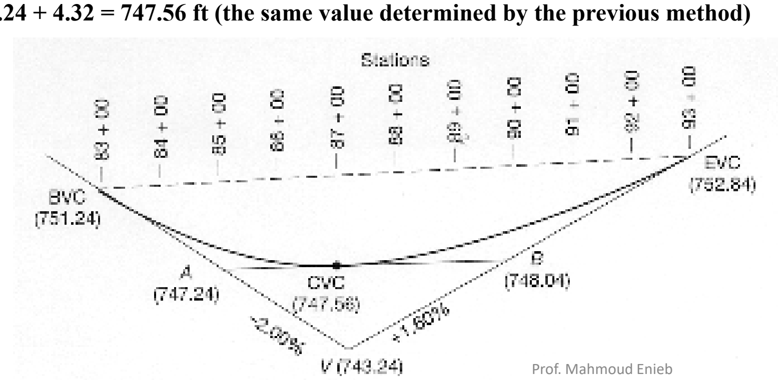
Solution (continued):

In case using (e) to determine the CVC elevation:

$$e = \frac{l1 * l2}{2(l1 + l2)} * \frac{A}{100}$$

$$e = \frac{400 * 600}{2(400 + 600)} * \frac{(1.6 + 2)}{100} = 4.32\ ft$$

Elev. (CVC) = 743.24 + 4.32 = 747.56 ft (the same value determined by the previous method)



Computation of values for g_1x and g_2x

	STATION	x	g_1x	$(r/2)x^2$	Curve Elevation
BVC	83 + 00	0	0	0	751.24'
	84 + 00	1	-2.00		
	85 + 00	2			
	86 + 00	3			
CVC	87 + 00	4			747.56'
	88 + 00	1	0.16		
	89 + 00	2			
	90 + 00	3			
	91 + 00	4			
	92 + 00	5			
EVC	93 + 00	6			

$g_1x = -2\ (1) = -2.00$

$g_2x = .16(1) = 0.16$

Computation of values for $(r_1/2)x^2$ and $(r_2/2)x^2$

	STATION	x	g_1x	$(r/2)x^2$	Curve Elevation
BVC	83 + 00	0	0	0	751.24'
	84 + 00	1	-2.00	0.27	
	85 + 00	2	-4.00		
	86 + 00	3	-6.00		
CVC	87 + 00	4	-8.00		747.56'
	88 + 00	1	0.16	0.12	
	89 + 00	2	0.32		
	90 + 00	3	0.48		
	91 + 00	4	0.64		
	92 + 00	5	0.80		
EVC	93 + 00	6	0.96		

$(r_1/2)x^2 = (0.27)(1)^2 = 0.27$

$(r_2/2)x^2 = (0.12)(1)^2 = 0.12$

Elevation Computations for both Vertical Curves

	STATION	**x**	**g_1x**	**$(r/2)x^2$**	**Curve Elevation**
BVC	83 + 00	0	0	0	751.24'
	84 + 00	1	-2.00	0.27	
	85 + 00	2	-4.00	1.08	
	86 + 00	3	-6.00	2.43	
CVC	87 + 00	4	-8.00	4.32	747.56'
	88 + 00	1	0.16	0.12	
	89 + 00	2	0.32	0.48	
	90 + 00	3	0.48	1.08	
	91 + 00	4	0.64	1.92	
	92 + 00	5	0.80	3.00	
EVC	93 + 00	6	0.96	4.32	

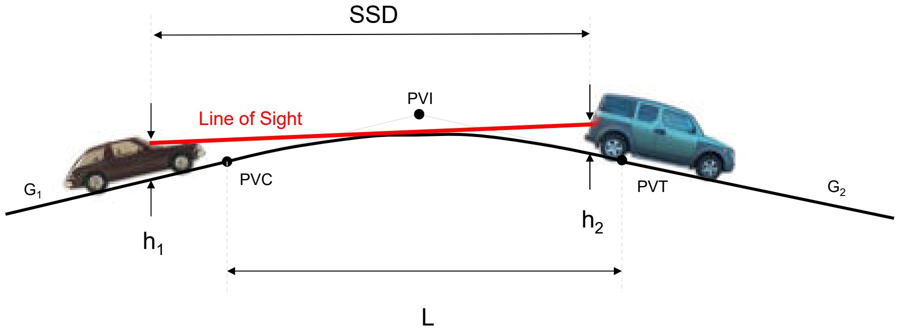
Y_1 = 751.24 - 2.00 + 0.27 = 749.51'

Y_2 = 747.56 + 0.16 + 0.12 = 747.84'

Computed Elevations for Stakeout at Full Stations

	STATION	**x**	**g_1x**	**$(r/2)x^2$**	**Curve Elevation**
BVC	83 + 00	0	0	0	751.24'
	84 + 00	1	-2.00	0.27	749.51'
	85 + 00	2	-4.00	1.08	748.32'
	86 + 00	3	-6.00	2.43	747.67'
CVC	87 + 00	4	-8.00	4.32	747.56'
	88 + 00	1	0.16	0.12	747.84'
	89 + 00	2	0.32	0.48	748.36'
	90 + 00	3	0.48	1.08	749.12'
	91 + 00	4	0.64	1.92	750.12
	92 + 00	5	0.80	3.00	751.36'
EVC	93 + 00	6	0.96	4.32	752.84' (OK)

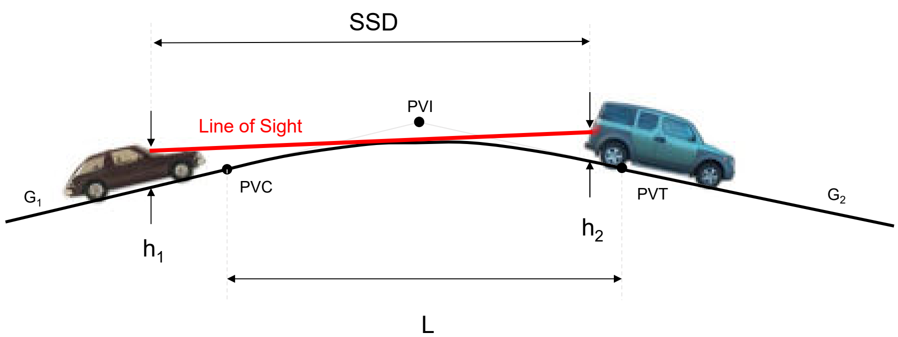
Crest Vertical Curves



15-11-2023 Prof. Mahmoud Enieb 69

69

Crest Vertical Curves



For SSD ≤ L

$$L = \frac{A(SSD)^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$$

For SSD > L

$$L = 2(SSD) - \frac{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}{A}$$

15-11-2023 Prof. Mahmoud Enieb 70

70

Crest Vertical Curves

- Assumptions for design
  - h_1 = driver's eye height = 3.5 ft.
  - h_2 = taillight height = 2.0 ft.
- Simplified Equations

For SSD ≤ L

$$L = \frac{A(SSD)^2}{2158}$$

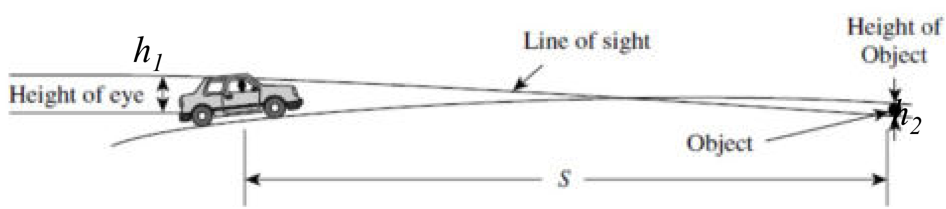
For SSD > L

$$L = 2(SSD) - \frac{2158}{A}$$

15-11-2023 Prof. Mahmoud Enieb 71

71

Sight Distance on Vertical Curve



Stopping sight distance diagram for crest vertical curve

$$L_{\min} = \begin{cases} \dfrac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} & \text{when } \mathbf{SSD \le L} \\ 2S - \dfrac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} & \text{when } \mathbf{SSD > L} \end{cases}$$

15-11-2023 Prof. Mahmoud Enieb 72

72

- where *S* sight distance (from Green book)
- *L* vertical curve length
- *A* absolute value of the algebraic difference in grades, in percent, $|g2- g1|$
- h_1 height of eye
- h_2 height of object
- Height of eye is assumed to be h_1 = 3.5 ft. =1.07 m.
- For stopping sight distance, the height of object is normally taken to be h_2 = 2.0 ft. = 0.60 m.
- For passing sight distance, the height of object used by AASHTO is h_2 = 4.25 ft. = 1.300 m.

AASHTO design tables

- Vertical curve length can also be found in design tables
- Assumptions for design
  - h_1 = driver's eye height = 3.5 ft.
  - h_2 = taillight height = 2.0 ft.

L = K *A

Where

K = length of curve per percent algebraic difference in intersecting grade

Charts from Green Book

Metric				US Customary			
Design speed (km/h)	Stopping sight distance (m)	Rate of vertical curvature, K[a] Calculated	Rate of vertical curvature, K[a] Design	Design speed (mph)	Stopping sight distance (ft)	Rate of vertical curvature, K[a] Calculated	Rate of vertical curvature, K[a] Design
20	20	0.6	1	15	80	3.0	3
30	35	1.9	2	20	115	6.1	7
40	50	3.8	4	25	155	11.1	12
50	65	6.4	7	30	200	18.5	19
60	85	11.0	11	35	250	29.0	29
70	105	16.8	17	40	305	43.1	44
80	130	25.7	26	45	360	60.1	61
90	160	38.9	39	50	425	83.7	84
100	185	52.0	52	55	495	113.5	114
110	220	73.6	74	60	570	150.6	151
120	250	95.0	95	65	645	192.8	193
130	285	123.4	124	70	730	246.9	247
				75	820	311.6	312
				80	910	383.7	384

[a] Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A). K = L/A

Exhibit 3-76. Design Controls for Stopping Sight Distance and for Crest and Sag Vertical Curves

From Green book

15-11-2023 Prof. Mahmoud Enieb

75

Metric			US Customary		
Design speed (km/h)	Passing sight distance (m)	Rate of vertical curvature, K* design	Design speed (mph)	Passing sight distance (ft)	Rate of vertical curvature, K* design
30	200	46	20	710	180
40	270	84	25	900	289
50	345	138	30	1090	424
60	410	195	35	1280	585
70	485	272	40	1470	772
80	540	338	45	1625	943
90	615	438	50	1835	1203
100	670	520	55	1985	1407
110	730	617	60	2135	1628
120	775	695	65	2285	1865
130	815	769	70	2480	2197
			75	2580	2377
			80	2680	2565

Note: *Rate of vertical curvature, K, is the length of curve per percent algebraic difference in intersecting grades (A). K=L/A

Exhibit 3-77. Design Controls for Crest Vertical Curves Based on Passing Sight Distance

From Green book

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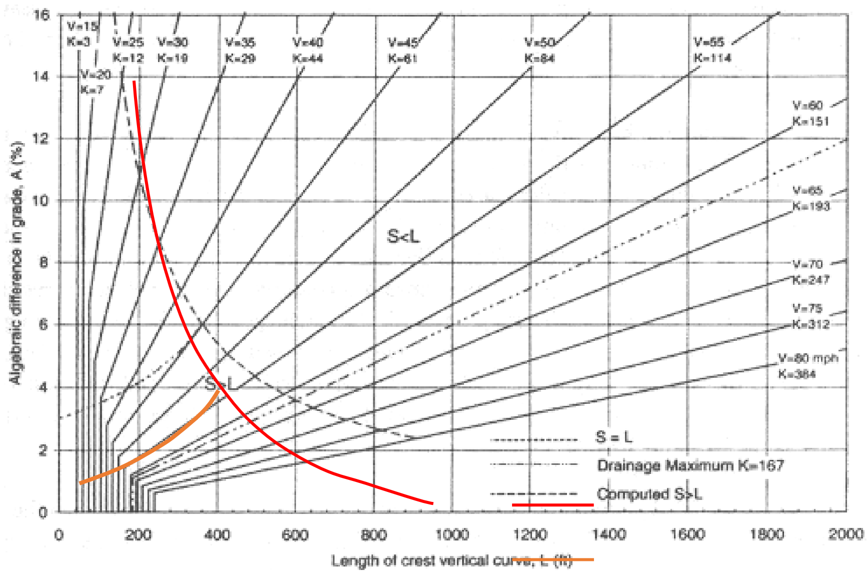
76

Design Controls for Crest Vertical Curves for SSD



from AASHTO's *A Policy on Geometric Design of Highways and Streets 2004*

15-11-2023 Prof. Mahmoud Enieb 77

77

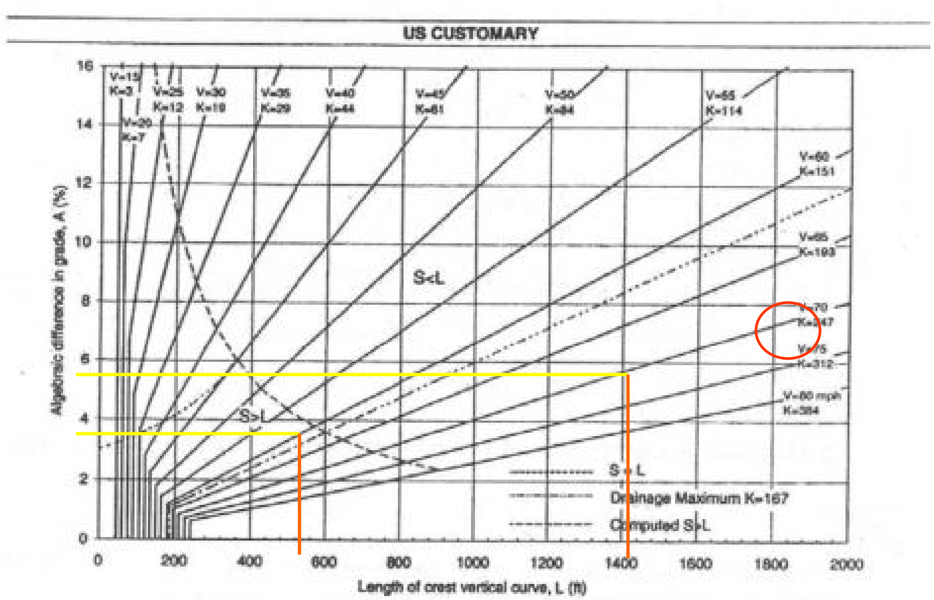


Exhibit 3-75. Design Controls for Crest Vertical Curves—Open Road Conditions

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78

Maximum Change of Grade Permitted Without Use of a Vertical Curve, and Min. length of vertical curve for good appearance

Table 6.2 Vertical Curve Appearance Criteria

Design Speed (km/h)	Maximum Change of Grade Permitted Without Use of a Vertical Curve (%)	Minimum Length of Vertical Curve for Good Appearance (m)
30	1.5	15
40	1.2	20
50	1.0	30
65	0.8	40
80	0.6	50
100	0.5	60

Source: adapted from Table 7.42 & 7.43, RMSS, Vol. V11A

Maximum Change of Grade Permitted Without Use of a Vertical Curve (Ahmed Hassan, 2023)

Design speed (Km/h)	20	30	40	50	60	70	80	90	100	110	120	130
Maximum change in Grade in percent	2	1.5	1	0.75	0.6	0.5	0.45	0.4	0.35	0.3	0.25	0.2

Chart vs computed

From Exhibit: 3.76 of stopping sight distance:

V = 60 mph K = 151 ft / % change

For $g_1 = 3$ $g_2 = -1$

$A = (g_2 - g_1) = (-1 - 3) = -4$

L = (K * | A |) = 151 * 4 = 604

From chart of stopping sight distance:

| A | = 4

L ≈ 600 ft

Example 1

For $g_1 = 4$ $g_2 = -2$, V = 70 mph, K = 247

$A = (g_2 - g_1) = (-2 - 4) = -6$

L = (K * | A |) = 247 * 6 = 1482 ft

From chart of stopping sight distance:

| A | = 6

L ≈ 1480 ft

Example 2

Determine the minimum length of a crest vertical curve between a +0.5% grade and a -1.0% grade for a road with a 100 km/h design speed. The vertical curve must provide 190 m stopping sight distance and meet the California appearance criteria (h1 =1.07, h2 = 0.15)m. Round up to the next greatest 20 m interval.

Solution:

Stopping sight distance criterion: assume S < L

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{[0.5 - (-1.0)](190^2)}{200(\sqrt{1.070} + \sqrt{0.150})^2} = 134.0 \text{ m}$$

134.0 m < 190 m, so S >L

$$L = 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} = 2(190) - \frac{200(\sqrt{1.070} + \sqrt{0.150})^2}{[0.5 - (-1.0)]}$$

Example 2

L = 380.0 - 269.5 =110.5 m, Okay S >L

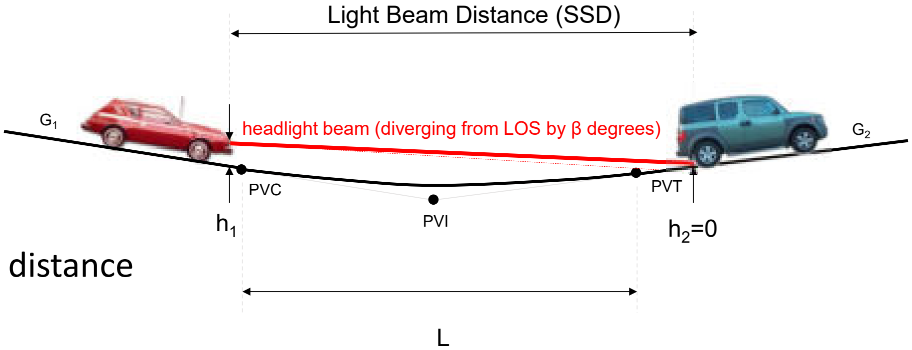
Appearance criterion:

Design speed =100 km/hr but grade break = 1.5% < 2.0%, curve length should be more than 60 m.

Conclusion:

Sight distance criterion governs. Use 120 m vertical curve

Sag Vertical Curves
Headlight Sight Distance



Daytime sight distance unrestricted

For SSD ≤ L

$$L = \frac{A(SSD)^2}{200(h_1 + S\tan\beta)}$$

For SSD > L

$$L = 2(SSD) - \frac{200(h_1 + (SSD)\tan\beta)}{A}$$

15-11-2023 Prof. Mahmoud Enieb 85

85

Headlight Sight Distance
English units

- Assumptions for design
  - h_1 = headlight height = 2.0 ft.
  - β = 1 degree

- Simplified Equations

For SSD ≤ L

$$L = \frac{A(SSD)^2}{400 + 3.5(SSD)}$$

For SSD > L

$$L = 2(SSD) - \left(\frac{400 + 3.5(SSD)}{A}\right)$$

15-11-2023 Prof. Mahmoud Enieb 86

86

Headlight Sight Distance (Metric units)

- Assumptions for design
  - h_1 = headlight height = 0.6 m.
  - β = 1 degree
- Simplified Equations

For SSD ≤ L

$$L = \frac{AS^2}{120 + 3.5S}$$

For SSD > L

$$L = 2S - \left(\frac{120 + 3.5S}{A}\right)$$

Headlight and stopping sight distance K values

Table 3: K Values for Sag Vertical Curves

English units

design speed (mph)	stopping sight distance (ft.)	K
30	200	37
35	250	49
40	305	64
45	360	79
50	425	96
55	495	115
60	570	136
65	645	157
70	730	181

Headlight and stopping sight distance K values

metric units

design speed (km/h)	stopping sight distance (m)	K
50	65	13
60	85	18
70	105	23
80	130	30
90	160	38
100	185	45
110	220	55

Sag Vertical Curves

- Sight distance is governed by night- time conditions
  - Distance on curve illuminated by headlights need to be considered
- Driver comfort
- Drainage
- General appearance

Sag Vertical Curve, **Passenger Comfort**

Centripetal acceleration does not exceed 1 ft./s^2 (0.3 m/s^2)

$$L = \frac{AV^2}{46.5} \text{ English units} \qquad L = \frac{AV^2}{395} \text{ metric units}$$

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91

Sag Vertical Curve, Drainage

- 1. The same drainage criterion used for crest vertical curves on curbed roadways applies to sag
- 2. vertical curves: K = 167 for English units and
- K = 51 metric units :
- 3. A minimum longitudinal grade of at least 0.5% is reached at a point about 15 m from either side of the low point.
- 4. There is at least a 100-mm elevation differential between the low point in the sag and the two points 15 m to either side of the low point. (Slope = 0.1*100/15 = 0.67%).

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92

Sag Vertical Curve, Appearance

- The general controlling factor for appearance of a sag curve is **K ≥ 100** for English units and
- **K ≥ 30** for metric units for small to intermediate values of A, which corresponds to speeds of 50 mph (80 km/h) and greater.
- Thus, for design speeds less than 50 mph (80km/h) the designer will want to strongly consider using K values greater than presented in previse tables whenever site conditions allow
- In general, the larger the K value, the better the appearance of a curve.

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93

Sag Vertical Curve, Appearance

- Min L = 2 V (m) when (V > 60 km/h) and grade break >2% or
- As in crest, use min L = 3V ft (V mph)

Grade break < 2% and V > 60 km/h take L = 60 m

- L = 60 m for V < 60 km/h, Grade break < 2%

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94

Sag Vertical Curve: Example

A sag vertical curve is to be designed to join a -3% to a +3% grade. Design speed is 40 mph. What is L?

Skipping steps: SSD = 313.67 feet

S > L

Determine whether S<L or S>L

$$L = 2S - \left(\frac{400 + 3.5S}{A}\right)$$

$$L = 2(313.67 \text{ ft}) - \frac{(400 + 3.5 \times 313.67)}{[3 - (-3)]} = \mathbf{377.70 \text{ ft}}$$

313.67 < 377.70, so condition does not apply

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95

Sag Vertical Curve: Example

SSD = 313.67 feet

$$L = \frac{AS^2}{400 + 3.5S}$$

$$L = \frac{6 \times (313.67)^2}{400 + 3.5 \times 313.67} = \underline{\mathbf{394.12 \text{ ft}}}$$

313.67 < 394.12, so condition applies

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96

Design Controls for Sag Vertical Curves

Metric				US Customary			
Design speed (km/h)	Stopping sight distance (m)	Rate of vertical curvature, K[a] Calculated	Rate of vertical curvature, K[a] Design	Design speed (mph)	Stopping sight distance (ft)	Rate of vertical curvature, K[a] Calculated	Rate of vertical curvature, K[a] Design
20	20	2.1	3	15	80	9.4	10
30	35	5.1	6	20	115	16.5	17
40	50	8.5	9	25	155	25.5	26
50	65	12.2	13	30	200	36.4	37
60	85	17.3	18	35	250	49.0	49
70	105	22.6	23	40	305	63.4	64
80	130	29.4	30	45	360	78.1	79
90	160	37.6	38	50	425	95.7	96
100	185	44.6	45	55	495	114.9	115
110	220	54.4	55	60	570	135.7	136
120	250	62.8	63	65	645	156.5	157
130	285	72.7	73	70	730	180.3	181
				75	820	205.6	206
				80	910	231.0	231

[a] Rate of vertical curvature, K, is the length of curve (m) per percent algebraic difference intersecting grades (A). K = L/A

L = K*A=
=64*6
= 384 ft

from AASHTO's *A Policy on Geometric Design of Highways and Streets 2004*

Sag Vertical Curve: Example

A sag vertical curve is to be designed to join a -3% to a +3% grade. Design speed is 40 mph. What is L?

Skipping steps: SSD = 313.67 feet

Testing for comfort:

$$L = \frac{AV^2}{46.5} = \frac{(6 \times [40\ \text{mph}]^2)}{46.5} = \mathbf{206.5\ feet}$$

Testing for appearance:

L = 3* 40 = **120 feet**

<u>Use 400 ft vertical curve</u>

Example 1

Determine the minimum length of a sag vertical curve between a -0.7% grade and a +0.5% grade for a road with a 110 km/h design speed. The vertical curve must provide 220 m stopping sight distance and meet the California appearance criteria and the AASHTO comfort standard. Round up to the next greatest 20 m interval.

Solution:

Stopping sight distance criterion: assume S < L

$$L = \frac{AS^2}{120 + 3.5S} = \frac{[0.5 - (-0.7)](220^2)}{120 + 3.5(220)} = 65.3 \text{ m}$$

65.3 m < 220 m, so S >L

$$L = 2S - \frac{120 + 3.5S}{A} = 2(220) - \frac{120 + 3.5(220)}{[0.5 - (-0.7)]}$$

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99

Solution 1

65.3 m < 220 m, so S >L

$$L = 2S - \frac{120 + 3.5S}{A} = 2(220) - \frac{120 + 3.5(220)}{[0.5 - (-0.7)]}$$

L = 440 - 741.7 = -301.7 m

Since L < 0, no vertical curve is needed to provide stopping sight distance.

Comfort criterion:

$$L = \frac{AV^2}{395} = \frac{[0.5 - (-0.7)](110^2)}{395} = 36.8 \text{ m}$$

Appearance criterion:

Design speed =110 km/h >60 km/h but grade break =1.2% <2%. Use 60 m.

Conclusion:

Appearance criterion governs. Use 60 m vertical curve.

100

100

Solution 1

General appearance criterion:
Minimum curve length of 30A [100A], K = 30 m [K = 100 ft].
Min. L = 30*1.2 = 36 m

Exhibit 3-78, are equal to 0.6 times the design speed in km/h [three times the design speed in mph]. L = 0.6*110 = 66 m
Conclusion:
Appearance criterion governs. Use 60 m vertical curve.

Drainage affects
A minimum grade of 0.30 percent should be provided within 15 m [50 ft] of the level point). This criterion corresponds to K of 51 m [167 ft] per percent change in grade

101

101

Design Controls for Sag Vertical Curves, Open Road Conditions, economic reasons

L = K*A=
=51*1.2
= 66 m

Metric				US Customary			
Design speed (km/h)	Stopping sight distance (m)	Rate of vertical curvature, K[a] Calculated	Rate of vertical curvature, K[a] Design	Design speed (mph)	Stopping sight distance (ft)	Rate of vertical curvature, K[a] Calculated	Rate of vertical curvature, K[a] Design
20	20	2.1	3	15	80	9.4	10
30	35	5.1	6	20	115	16.5	17
40	50	8.5	9	25	155	25.5	26
50	65	12.2	13	30	200	36.4	37
60	85	17.3	18	35	250	49.0	49
70	105	22.6	23	40	305	63.4	64
80	130	29.4	30	45	360	78.1	79
90	160	37.6	38	50	425	95.7	96
100	185	44.6	45	55	495	114.9	115
110	220	54.4	55	60	570	135.7	136
120	250	62.8	63	65	645	156.5	157
130	285	72.7	73	70	730	180.3	181
				75	820	205.6	206
				80	910	231.0	231

[a] Rate of vertical curvature, K, is the length of curve (m) per percent algebraic difference intersecting grades (A). K = L/A

from AASHTO's *A Policy on Geometric Design of Highways and Streets 2004*

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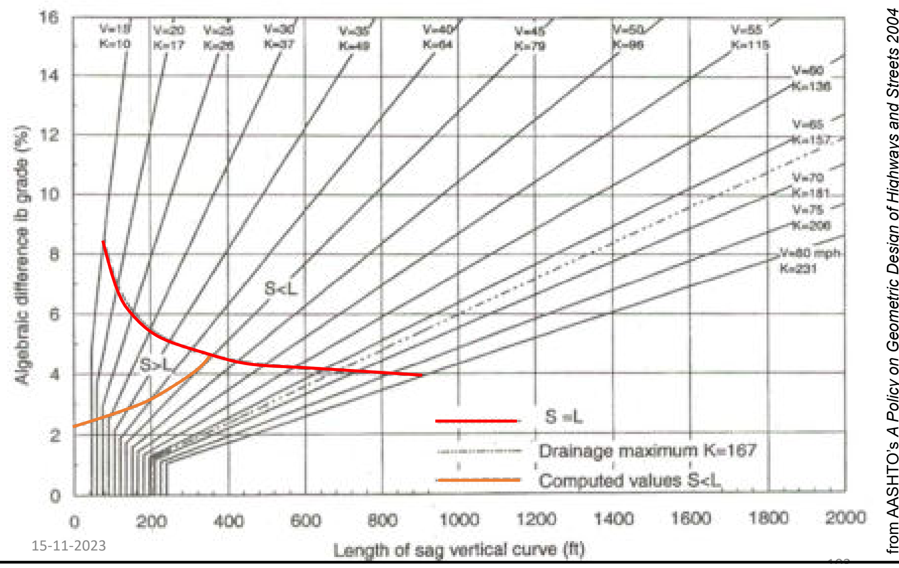
102

Design Controls for Sag Vertical Curves, Open Road Conditions, Drainage

L = K*A=
=167*1.2
= 200 ft
≈ 60 m

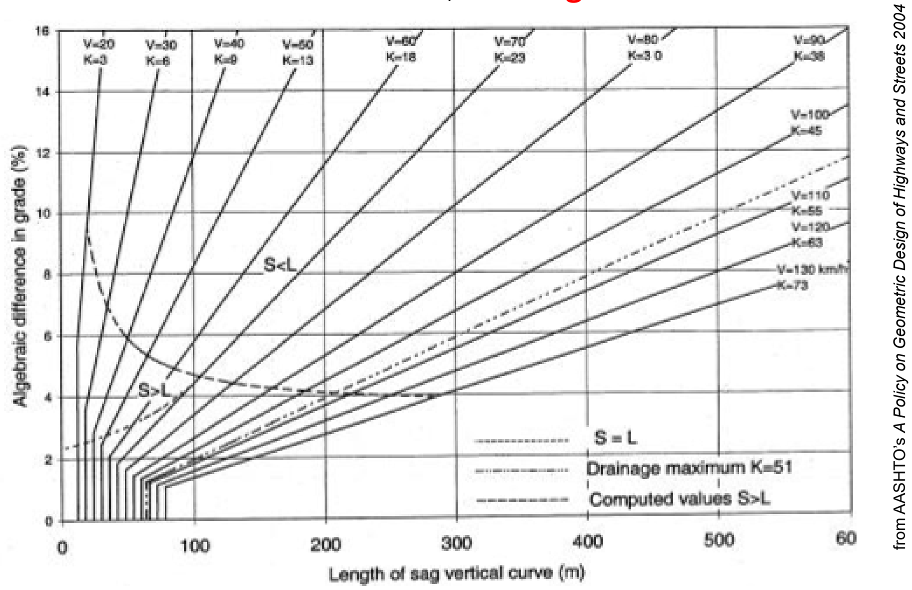


15-11-2023

103

Design Controls for Sag Vertical Curves, Open Road Conditions, Drainage

L = K*A=
=51*1.2
≈ 60 m



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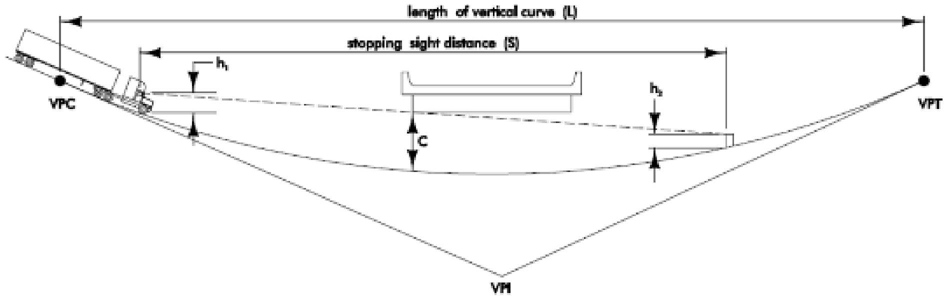
104

Sag Vertical Curve Undercrossing

Sight distance through undercrossing usually does not present a problem in the design of vertical curves.



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105

Sag Vertical Curve Undercrossing

The equations for sag vertical curve length at an undercrossing are:

$$L = \frac{AS^2}{800\left(C - \frac{h_1 + h_2}{2}\right)} \qquad S < L$$

$$L = 2S - \frac{800\left(C - \frac{h_1 + h_2}{2}\right)}{A} \qquad S > L$$

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106

Stopping Sight Distance

With the height of the eye of the driver (h_1) set at 8 feet (2.4 meters) for a truck driver and the height of the object (h_2) set at 2.0 ft. (0.6 m.) for stopping sight distance, these equations simplify to:

$L = \dfrac{AS^2}{800(C - 5)}$	$L = \dfrac{AS^2}{800(C - 1.5)}$ metric units	S < L
$L = 2S - \dfrac{800(C - 5)}{A}$	$L = 2S - \dfrac{800(C - 1.5)}{A}$ metric units	S > L

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107

Passing Sight Distance

With the height of the eye of the driver (h_1) set at 8 feet (2.4 meters) for a truck driver and the height of the object (h_2) set at 3.5 feet (1.07 m) for passing sight distance, these equations simplify to:

$L = \dfrac{AS^2}{800(C - 5.75)}$	$L = \dfrac{AS^2}{800(C - 1.740)}$	S < L
$L = 2S - \dfrac{800(C - 5.75)}{A}$	$L = 2S - \dfrac{800(C - 1.740)}{A}$ metric units	S > L

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108

Example

- A bridge is being designed to pass over a rural two-lane highway with a design speed of 60 mph. The section of the two-lane highway where the bridge crosses over is a 1740 feet vertical sag curve with A = 3.15. The bridge clearance is 16.8 feet. Does adequate passing sight distance exist on the two-lane highway or does the bridge clearance need to be adjusted?

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109

Example

- Start by assuming S<L and solve the appropriate equation for PSD
- $S = \sqrt{\frac{800L(C-5.75)}{A}}$ √ (800* 1740(16.8 – 5.75)/ 3.15
- S = 2210 ft. > L

$$S = \frac{L}{2} + \frac{400(C-5.75)}{A} = 2{,}273 \text{ feet}$$

- Passing sight distance for a design speed of 60 mph is 2273 feet, so adequate sight distance exists with a bridge clearance of 16.8 feet.

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110

Problem 1

A car is traveling at 30 mph in the country at night on a wet road through a 150 ft. long sag vertical curve. The entering grade is -2.4 percent, and the exiting grade is 4.0 percent. A tree has fallen across the road at approximately the PVT. Assuming the driver cannot see the tree until it is lit by her headlights, is it reasonable to expect the driver to be able to stop before hitting the tree?

Problem 2

Similar to Problem 1 but for a crest curve.

A car is traveling at 30 mph in the country at night on a wet road through a 150 ft. long crest vertical curve. The entering grade is 3.0 percent and the exiting grade is -3.4 percent. A tree has fallen across the road at approximately the PVT. Is it reasonable to expect the driver to be able to stop before hitting the tree?

Solution Problem 1

Assume S < L

$$L = \frac{6.4(S)^2}{400 + 3.5(S)} = 150, S = 146.17\, ft$$

From Table *f* =0.36 at V = 30 mph
SSD for this speed can be calculated:

$$\boldsymbol{SSD = 1.47Vt + \frac{V^2}{30(f \pm G)}}$$

$$\boldsymbol{SSD = 1.47 * 30 * 2.5 + \frac{30^2}{30(0.36 \pm 0)}} = 193.58 \text{ ft}$$

Or for SSD = 146.17ft, V = 24.6 mph
Hence the driver can't to stop before hitting the tree

15-11-2023 Prof. Mahmoud Enieb 113

113

Solution Problem 2

For SSD ≤ L

$$L = \frac{A(SSD)^2}{200\left(\sqrt{h_1} + \sqrt{h_2}\right)^2}$$

For SSD > L

$$L = 2(SSD) - \frac{200\left(\sqrt{3.5} + \sqrt{0.5}\right)^2}{A}$$

For SSD h1 = 3.5 ft, h2 = 0.5 ft.
assume S< L

$$L = \frac{6.4(SSD)^2}{200\left(\sqrt{3.5} + \sqrt{0.5}\right)^2} = 150, SSD = 176.5\, ft > L$$

$$150 = 2(SSD) - \frac{200\left(\sqrt{3.5} + \sqrt{0.5}\right)^2}{6.4}, SSD = 178.84\ ft\ Okay$$

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114

Problem 2

SSD for this speed can be calculated:

$$\boldsymbol{SSD = 1.47Vt + \frac{V^2}{30(f \pm G)}}$$

$$\boldsymbol{SSD = 1.47 * 30 * 2.5 + \frac{30^2}{30(0.36 \pm 0)}} = 193.58 \text{ ft}$$

Hence the driver can't to stop before hitting the tree
Speed = 28.38 mph