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Ω -OPEN SETS AND Ωs-OPEN SETS IN TOPOLOGICAL SPACES

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ABSTRACT. New classes of sets called Ω-open sets and Ωs-open sets are introduced and studied. Also, as applications we introduce and study Ω-compact spaces and Ωs- compact spaces.

1. INTRODUCTION AND PRELIMINARIES

Many concepts of general topology have been generalized, by considering the concept of semi-open sets due to Levine [8]. The study of generalized closed sets in the topological spaces was initiated by Levine [9] and the concept of $T_{1/2}$ spaces was introduced. In 1987, Bhattacharyya and Lahiri [3] introduced the class of semi-generalized closed sets and used them to obtain properties in $semi - T_{1/2}$ spaces. In 1990, Arya and Nour [1] defined the generalized semi- closed sets and studied some characterizations of s-normal spaces. The modified forms of generalized closed sets and generalized continuity were studied by Balachandran, Devi, Maki and Sundaram [2, 7]. Recently, Noiri and Sayed [12] introduced

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and studied new classes of sets called Ω-closed sets and Ωs-closed sets. Also, they introduced and studied Ω-continuous functions and Ωs- continuous functions and proved pasting lemma for these functions. Moreover, they introduced classes of topological spaces called $\Omega - T_{1/2}$ and $\Omega - T_s$. This paper is a continuation of their works.

Throughout this paper (X, τ) and (Y, σ) will always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of $(X, \tau), Cl(A)$, $Int(A), D[A], sCl(A), sInt(A)$ and $D_s[A]$ [4-5] denote the closure of A, the interior of A, the derived set of A, the semi-closure of A, the semi-interior of A and the semi- derived set of A, respectivily.

We recall some known definitions and properties.

DEFINITION 1.1. Let (X, τ) be a topological space. A subset $A \subseteq X$ is said to be:
(1) semi-open [8] if $A \subseteq Cl(Int(A))$ and semi-closed if $Int(Cl(A)) \subseteq A$,
(2) α-open [11] if $A \subseteq Int(Cl(Int(A)))$,
(3) regular open if $A = Int(Cl(A))$ and regular closed if $A = Cl(Int(A))$.

DEFINITION 1.2 [4]. Let (X, τ) be a topological space and $A, B \subseteq X$. Then A is semi-closed if and only if $X - A$ is semi-open and the semi-closure of B, denoted by $sCl(B)$, is the intersection of all semi-closed sets containing B.

DEFINITION 1.3 [7]. Let (X, τ) be a topological space, $A \subseteq X$ and $x \in X$. Then x is said to be a semi-limit point of A if and only if every semi-open set containing x contains a point of A diffrent from x

DEFINITION 1.4 [7]. Let (X, τ) be a topological space and $A \subseteq X$. The set of all semi-limit point of A is said to be semi-derived set A and is denoted by $D_s[A]$

DEFINITION 1.5 [6]. Let (X, τ) be a topological space and A, B be two non-void subsets of X. Then A and B are said to be semi-separated if $A \cap sCl(B) = sCl(A) \cap B = \phi$.

DEFINITION 1.6 [4]. Let (X, τ) be a topological space and $A \subseteq X$. The semi-interior of A, denoted by $sInt(A)$, is the union of all semi-open subsets of A.

DEFINITION 1.7 [12]. A subset A of (X, τ) is said to be Ω-closed if $sCl(A) \subseteq Int(U)$ whenever $A \subseteq U$ and U is semi-open.

DEFINITION 1.8 [12]. A subset A of (X, τ) is said to be Ωs-closed if $sCl(A) \subseteq Int(Cl(U)))$ whenever $A \subseteq U$ and U is semi-open.

DEFINITION 1.9 [12]. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be:
(1) Ω-continuous if $f^{-1}(V)$ is Ω -closed in (X, τ) for every closed set V of (Y, σ).
(2) Ωs-continuous if $f^{-1}(V)$ is Ωs -closed in (X, τ) for every closed set V of (Y, σ).
(3) Ω-irresolute if $f^{-1}(V)$ is Ω-closed in (X, τ) for every $\Omega-$closed set V of (Y, σ).
(4) Ωs-irresolute if $f^{-1}(V)$ is Ωs-closed in (X, τ) for every $\Omega s-$closed set V of (Y, σ).

PROPOSITION 1.10 [12]. If for any subset E of (X, τ) , $D[E] \subseteq D_s[E]$, then the union of two Ω-closed sets (resp. Ωs-closed sets) is Ω-closed (resp. Ωs -closed).

2. Ω -OPEN SETS AND Ωs-OPEN SETS

DEFINITION 2.1. A subset A of (X, τ) is said to be Ω-open in (X, τ) if its complement $X - A$ is Ω -closed in $(X, \tau$.

DEFINITION 2.2. A subset A of (X, τ) is said to be Ωs-open in (X, τ) if its complement $X - A$ is Ωs -closed in (X, τ).

PROPOSITION 2.3. Let (X, τ) be a topological space and $A \subseteq X$.
(1) A is an Ω-open set if and only if $Cl(F) \subseteq sInt(A)$ whenever $F \subseteq A$ and F is semi-closed.
(2) A is an Ωs-open set if and only if $Cl(Int(F)) \subseteq sInt(A)$ whenever $F \subseteq A$ and F is semi-closed.
(3) If A is Ω-open, then A is Ωs-open.

PROOF. (1) *Necessity.* Let A be an Ω-open set in (X, τ) and suppose $F \subseteq A$, where F is semi-closed. Then, $X - A$ is Ω-closed and it is contained in the semi-open set $X - F$. Therefore, $sCl(X - A) \subseteq Int(X - F)$. Hence, $Cl(F) \subseteq sInt(A)$.

Sufficiency. If F is a semi-closed set such that $Cl(F) \subseteq sInt(A)$ whenever $F \subseteq A$, it follows that $X - A \subseteq X - F$ and $X - sInt(A) \subseteq X - Cl(F)$. Therefore, $sCl(X - A) \subseteq Int(X - F)$. Hence, $X - A$ is Ω-closed and A becomes an Ω-open set. This proves (1).
(2) *Necessity.* Let A be an Ωs-open set in (X, τ) and suppose $F \subseteq A$, where F is semi-closed. By definition $X - A$ is Ωs-closed and it is contained in the semi-open set $X - F$. Therefore, $sCl(X - A) \subseteq Int(Cl(X - F))$. Hence, $Cl(Int(F)) \subseteq sInt(A)$.
Sufficiency. If F is a semi-closed set such that $Cl(Int(F)) \subseteq sInt(A)$ whenever $F \subseteq A$, it follows that $X - A \subseteq X - F$ and $X - sInt(A) \subseteq X - Cl(Int(F))$. Therefore, $sCl(X - A) \subseteq Int(Cl(X - F))$. Hence, $X - A$ is Ωs-closed and A becomes an Ωs-open set.
(3) Obvious.

LEMMA 2.4. (1) Let $A_j (1 \leq j \leq n)$ be Ω-open sets of (X, τ) and semi-separated , i.e., $sCl(A_j) \cap A_i = \phi$ for $i \neq j$.Then, $\bigcup_{j=1}^{n} A_j$ is Ω-open.

PROOF. Assume that $F \subseteq \bigcup_{j=1}^{n} A_j$, where F is a semi-closed set in (X,τ). Then, for each i we have $F \cap sCl(A_i) \subseteq (\bigcup_{j=1}^{n} A_j) \cap sCl(A_i) = \bigcup_{j=1}^{n}(A_j \cap sCl(A_i)) = A_i$. Since $F \cap sCl(A_i)$ is semi-closed and A_i is Ω-open, then $Cl(F \cap sCl(A_i)) \subseteq sInt(A_i)$ for each i. Therefore, we have $Cl(F) \subseteq Cl(\bigcup_{i=1}^{n}(F \cap sCl(A_i))) \subseteq \bigcup_{i=1}^{n}(Cl(F \cap sCl(A_i))))$ $\subseteq \bigcup_{i=1}^{n} sInt(A_i) \subseteq sInt(\bigcup_{i=1}^{n} A_i)$. Hence, $\bigcup_{i=1}^{n} A_i$ is an Ω-open set.

DEFINITION 2.5 [10]. (1) A subset A of a topological space (X,τ) is said to be dominated by the interior of a semi-open set if $A \subseteq Int(W)$ whenever $A \subseteq W$ and W is semi-open.

(2) A subset A of a topological space (X,τ) is said to be dominated by the closure of a semi-closed set if its complement $X - A$ is dominated by the interior of a semi-open set.

LEMMA 2.6 [10]. Let A and B be subests of (X,τ) and (Y,σ) respectively such that $A \times B$ *is dominated by the interior of a semi-open set.* If A and B are both compact, then for a semi-open set U containing $A \times B$, there exist two open sets S and T such that $A \subseteq S$, $B \subseteq T$ and $S \times T \subseteq U$.

THEOREM 2.7. Let A and B be subests of (X,τ) and (Y,σ) respectively.

(i) If $A \times B$ is dominated by the interior of a semi-open set and if A and B are both compact and Ω-closed sets, then $A \times B$ is an Ω-closed set.

(ii) If $A \times B$ dominates the closure of a semi-closed set and if A and B are both Ω-open sets, then $A \times B$ is an Ω-open set.

(iii) If $A \times Y$ is Ω-open, then A is Ω-open.

PROOF. (i) Assume that $A \times B \subseteq U$, where U is a semi-open set, then by Lemma 2.4 there exist two open sets S and T such that $A \subseteq S$, $B \subseteq T$ and $S \times T \subseteq U$. Since A and B are Ω-closed sets, then $sCl(A \times B) \subseteq sCl(A) \times sCl(B) \subseteq Int(S) \times Int(T) \subseteq Int(S \times T) \subseteq Int(U)$. Hence, $A \times B$ is an Ω-closed set.

(ii) Let F be a semi-closed set of $(X \times Y, \tau \times \sigma)$ such that $F \subseteq A \times B$. Using assumption, fo each $(x,y) \in F$, $sCl(\{x\} \times \{y\}) \subseteq sCl(\{x\}) \times sCl(\{y\}) \subseteq Cl(\{x\}) \times Cl(\{y\}) = Cl(\{(x,y)\}) \subseteq Cl(F) \subseteq A \times B$. Two semi-closed sets $sCl(\{x\})$ and $sCl(\{y\})$ are contained in A and B respectively. Since A and B are Ω-open, then $Cl(sCl(\{x\})) \subseteq sInt(A)$ and $Cl(sCl(\{y\})) \subseteq sInt(B)$ hold. This implies for each $(x,y) \in F$, $Cl((x,y)) \subset Cl(sCl(\{x\})) \times Cl(sCl(\{y\})) \subset sInt(A) \times sInt(B) \subseteq sInt(A \times B)$. So, $Cl(F) \subseteq sInt(A \times B)$. Hence $A \times B$ is Ω -open.

(iii) Let $W \subseteq A$ be a semi-closed set. Since $W \times Y$ is a semi-closed set such that $W \times Y \subseteq A \times Y$, then $Cl(W \times Y) \subseteq sInt(A \times Y)$. So, $Cl(W) \times Y \subseteq sInt(A) \times Y$. Therefore, $Cl(W) \subseteq sInt(A)$.

3. Ω-OPEN SETS AND Ωs-OPEN SETS FOR SUBSPACES

LEMMA 3.1. Let A and B be subsets of (X,τ) such that $A \subseteq B$. Suppose that B is open and semi-closed. Then, $A \in SC(B)$ (A is semi-closed with respect to the subspace B) if and only if $A \in SC(X)$(A is semi-closed with respect to X).

PROOF. Let $A \in SC(B)$. Then, $B - A \in SO(B)$ ($B - A$ is semi-open with respect to the subspace B). Since $B - A \subseteq, B \subseteq X$ and $B \in SO(X)$, then $B - A \in SO(X)$. Therefore. $(X - A) = (X - B) \cup (B - A) \in SO(X)$. Thus $A \in SC(X)$. Conversely, let $A \in SC(X)$ Then, $B - A = (X - A) \cap B \in SO(X)$. Therefore, $B - A \in SO(B)$. Thus, $A \in SC(B)$.

PROPOSITION 3.2. Suppose that for any subset E of (X, τ), $D[E] \subseteq D_s[E]$ and both A and B are Ω-open (resp. Ωs-open) sets. Then, $A \cap B$ is Ω-open (resp. Ωs-open) set in (X, τ).

PROOF. Since A and B are both Ω-open (resp. Ωs -open) sets, then $X - A$ and $X - B$ are both Ω-closed (resp. Ωs-closed) sets. Therefore, by Proposition 2.3 $(X - A) \cup (X - B)$ is Ω-closed (resp. Ωs-closed). Since $X - (A \cap B) = (X - A) \cup (X - B)$, then $A \cap B$ is Ω -open (resp. Ωs-open).

THEOREM 3.3. Let B and Y be subsets of (X, τ) such that $B \subseteq Y$ and Y is open and closed set.

(1) If B is an Ω-open set relative to Y, then B is Ω -open in X;

(2) If B is an Ωs-open set relative to Y, then B is Ωs -open in X.

PROOF. (1) Let B be an Ω-open set relative to Y such that $F \subseteq B$, where $F \in SC(X)$. From Lemma 3.1, we have $F \in SC(Y)$. Therefore, $Cl_Y(F) \subseteq sInt_Y(B)$. Hence, $Y \cap Cl(F) \subseteq sInt_Y(B)$. Since $sInt_Y(B) \in SO(Y)$ and Y is open, then $sInt_Y(B) = sInt(sInt_Y(B)) \subseteq sInt(B)$. Thus, we have $Y \cap Cl(F) \subseteq Y \cap sInt(B)$. Hence, $Y \subseteq sInt(B) \cup (X - Cl(F))$. Hence, $Cl(F) \subseteq, sInt(B) \cup (X - Cl(F)) \subseteq sInt(B)$. Thus B is Ω-open relative to X.

(2) Let B be an Ωs-open set relative to Y such that $F \subseteq B$, where $F \in SC(X)$. From Lemma 3.1, we have $F \in SC(Y)$. Therefore, $Cl_Y(Int_Y(F)) \subseteq sInt_Y(B)$. Hence, we have that $Y \cap Cl(Int(F)) \subseteq sInt_Y(B)$ and therefore $Y \cap Cl(Int(F)) \subseteq Y \cap sInt(B)$. Therefore, $Y \subseteq sInt(B) \cup (X - Cl(Int(F))$. Hence, $Cl(Int(F)) \subseteq Cl(F) \subseteq Cl(Y) \subseteq sInt(B) \cup (X - Cl(Int(F)))$. Therefore, $Cl(Int(F)) \subseteq sInt(B)$ and hence B is Ωs-open relative to X

THEOREM 3.4. Let $X_\circ$ be an open and semi-closed subset of (X, τ).

(1) If V is Ω-open relative to X. Then, $X_\circ \cap V$ is Ω-open relative to $X_\circ$.

(2) If V is Ωs-open relative to X. Then, $X_\circ \cap V$ is Ωs-open relative to $X_\circ$.

PROOF. (1) Suppose that $F \subseteq X_\circ \cap V$, where $F \in SC(X_\circ)$. By Lemma 3.1, $F \in SC(X)$ and hence $\overline{F} \subseteq sInt(V)$. Thus $\overline{F} \cap X_\circ \subseteq sInt(V) \cap X_\circ$. Since $sInt(V) \cap X_\circ \in SO(X)$, then $sInt(V) \cap X_\circ \in SO(X_\circ)$. So, $sInt(V) \cap X_\circ = sInt_{X_\circ}(sInt(V) \cap X_\circ) \subseteq sInt_{X_\circ}(V \cap X_\circ)$. Therefore, $\overline{F} \cap X_\circ = Cl_{X_\circ}(F) \subseteq sInt_{X_\circ}(V \cap X_\circ)$ and so $X_\circ \cap V$ is Ω-open relative to $X_\circ$.

(2) Suppose that $F \subseteq X_\circ \cap V$, where $F \in SC(X_\circ)$. By Lemma 3.1, $F \in SC(X)$ and hence $F^{\circ -} \subseteq sInt(V)$. Thus $F^{\circ -} \cap X_\circ \subseteq sInt(V) \cap X_\circ$. Since $sInt(V) \cap X_\circ \in SO(X)$, then $sInt(V) \cap X_\circ \in SO(X_\circ)$. So, $sInt(V) \cap X_\circ = sInt_{X_\circ}(sInt(V) \cap X_\circ) \subseteq sInt_{X_\circ}(V \cap X_\circ)$. Therefore, $Cl_{X_\circ}(Int_{X_\circ}(F)) \subseteq F^{\circ -} \cap X_\circ \subseteq sInt_{X_\circ}(V \cap X_\circ)$ and so $X_\circ \cap V$ is Ωs-open relative to $X_\circ$.

LEMMA 3.5. Let B and Y be subsets of (X, τ) such that $B \subseteq Y$.

(1) If B is Ω-open in (X,τ) and Y is open and semi-closed, then B is Ω-open relative to Y,

(2) If Y is open and closed, then B is Ωs-open in (X,τ) if and only if B is Ωs-open relative to Y.

PROOF. (1) From Theorem 3.4 (1) $B = B \cap Y$ is Ω-open relative to Y.
(2) From Theorem 3.4 (2) $B = B \cap Y$ is Ωs-open relative to Y.

4. Ω-COMPACT SPACES AND Ωs-COMPACT SPACES

DEFINITION 4.1.
(1) A topological space (X,τ) is said to be Ω- compact if every cover of X by Ω-open sets has a finite subcover.
(2) A topological space (X,τ) is said to be Ωs- compact if every cover of X by Ωs-open sets has a finite subcover.

DEFINITION 4.2. (1) A subset Y of a topological space (X,τ) is said to be Ω- compact relative to X if every cover of Y by Ω-open sets of X has a finite subcover.
(2) A subset Y of a topological space (X,τ) is said to be Ωs- compact relative to X if every cover of Y by Ωs-open sets of X has a finite subcover.

DEFINITION 4.3. (1) A subset Y of a topological space (X,τ) is said to be $\mathbf{\Omega}$-compact if the space (Y,τ_Y) is $\mathbf{\Omega}$- compact.
(2) A subset Y of a topological space (X,τ) is said to be $\mathbf{\Omega s}$- compact if the space (Y,τ_Y) is $\mathbf{\Omega s}$- compact.

PROPOSITION 4.4. (1) An Ω-closed subset K of an $\mathbf{\Omega}$- compact space (X,τ) is $\mathbf{\Omega}$-compact relative to X.
(2) An Ωs-closed subset K of an $\mathbf{\Omega s}$- compact space (X,τ) is $\mathbf{\Omega s}$- compact relative to X.

PROOF. (1) Let Φ be a cover of K by Ω-open sets of X. Since K is an Ω-closed subset of X, then $X-K$ is Ω-open. Hence, the member $\Phi \cup (X-K)$ form an Ω -open cover of X. Since X is $\mathbf{\Omega}$- compact, then $\Phi \cup (X-K)$ contains a finite subcover of X. In other words, there are a finite number of Ω-open sets $U_1, U_2, ..., U_n$ of Φ such that $U_1 \cup U_2 ... \cup U_n \cup (X-K) = X$. Hence, $U_1, U_2, ..., U_n$ covers K and K is $\mathbf{\Omega}$- compact relative to X.
(2) The proof is similar to (1).

THEOREM 4.5. Let (X,τ) be a topological space and let G be an open and closed subset of X.
(1) G is Ω-compact if and only if it is Ω-compact relative to X.
(2) G is Ωs-compact if and only if it is Ωs-compact relative to X.

PROOF. Let $\Phi = \{U_\alpha : \alpha \in \nabla\}$ be a cover of G, where U_α is Ω-open relative to X for each $\alpha \in \nabla$. Since G is open and closed, then by Theorem 3.4 (1) we have $G \cap U_\alpha$ is Ω-open relative to G for each $\alpha \in \nabla$. Since G is Ω -compact, then there exists a finite subfamily

$\nabla_\circ$ of ∇ such that $G = \bigcup\{G \cap U_\alpha : \alpha \in \nabla_\circ\}$. Therefore, $G \subseteq \bigcup\{U_\alpha : \alpha \in \nabla_\circ\}$ and G is Ω-compact relative to X.

Conversely, let G be Ω-compact relative to X and $\Phi = \{U_\alpha : \alpha \in \nabla\}$ be a cover of G, where U_α is Ω-open relative to G for each $\alpha \in \nabla$. Then by Theorem 3.3 (1) we have U_α is Ω-open relative to X for each $\alpha \in \nabla$. Since G is Ω-compact relative to X, then there exists a finite subfamily $\nabla_\circ$ of ∇ such that $G = \bigcup\{U_\alpha : \alpha \in \nabla_\circ\}$. Therefore, G is Ω-compact.

(2) The proof is similar to (1).

THEOREM 4.6. Let (X, τ) be a topological space and let A and G be subsets of X such that $A \subseteq G$ and G be open and closed.

(1) A is Ω-compact relative to G if and only if it is Ω -compact relative to X.

(2) A is Ωs-compact relative to G if and only if it is Ωs -compact relative to X.

PROOF. (1) Let $\Phi = \{U_\alpha : \alpha \in \nabla\}$ be a cover of A, where U_α is Ω-open relative to X for each $\alpha \in \nabla$. Since G is open and closed, then by Theorem 3.4 (1) we have $G \cap U_\alpha$ is Ω-open relative to G for each $\alpha \in \nabla$ and $A \subseteq \{G \cap U_\alpha : \alpha \in \nabla\}$ Since A is Ω -compact relative to G, then there exists a finite subfamily $\nabla_\circ$ of ∇ such that $A \subseteq \bigcup\{G \cap U_\alpha : \alpha \in \nabla_\circ\}$. Therefore, $A \subseteq \bigcup\{U_\alpha : \alpha \in \nabla_\circ\}$ and A is Ω-compact relative to X.

Conversely, let A be Ω-compact relative to X and $\Phi = \{U_\alpha : \alpha \in \nabla\}$ be a cover of A, where U_α is Ω-open relative to G for each $\alpha \in \nabla$. Then by Theorem 3.3 (1) we have U_α is Ω-open relative to X for each $\alpha \in \nabla$. Since A is Ω-compact relative to X, then there exists a finite subfamily $\nabla_\circ$ of ∇ such that $A \subseteq \bigcup\{U_\alpha : \alpha \in \nabla_\circ\}$. Therefore, A is Ω-compact relative to G.

(2) The proof is similar to (1).

PROPOSITION 4.7. (1) Let $f : (X, \tau) \to (Y, \sigma)$ be Ω-continuous (resp. Ω-irresolute) and let the subset B of (X, τ) be Ω-compact relative to X. Then, $f(B)$ is compact in (Y, σ) (resp. Ω-compact relative to Y).

(2) Let $f : (X, \tau) \to (Y, \sigma)$ be Ωs-continuous (resp. Ωs-irresolute) and the subset B of (X, τ) be Ωs-compact relative to X. Then, $f(B)$ is compact in (Y, σ) (resp. Ωs-compact relative to Y).

PROOF.(1) Let $\Phi = \{U_\alpha : \alpha \in \nabla\}$ be a collection of open (resp. Ω-open) subsets of (Y, σ) such that $f(B) \subseteq \bigcup\{U_\alpha : \alpha \in \nabla\}$. Then, $B \subseteq \bigcup\{f^{-1}(U_\alpha) : \alpha \in \nabla\}$ and so there exists a finite subfamily $\nabla_\circ$ of ∇ such that $B \subseteq \bigcup\{f^{-1}(U_\alpha) : \alpha \in \nabla_\circ\}$. Therefore, $f(B) \subseteq \bigcup\{U_\alpha : \alpha \in \nabla_\circ\}$. Thus, $f(B)$ is compact in (Y, σ) (resp. Ω-compact relative to Y).

(2) The proof is similar to (1). [illegible] (resp. Ωs-closed) function if $f(V)$ is Ω-closed (resp. Ωs-closed) in (Y, σ) for every closed set V in (X, τ).

DEFINITION 4.8. [illegible] closed (resp. Ωs-closed) in (Y, σ) for every Ω-closed (resp. Ωs -closed) set V in (X, τ).

PROPOSITION 4.9. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then

(1) f is pre-Ω-closed if and only if for every $S \subseteq Y$ and every Ω-open set $U \subseteq X$ containing $f^{-1}(S)$, there exists an Ω-open V containing S such that $f^{-1}(V) \subseteq U$.

(2) f is pre-Ωs-closed if and only if for every $S \subseteq Y$ and every Ωs-open set $U \subseteq X$ containing $f^{-1}(S)$, there exists an Ωs-open V containing S such that $f^{-1}(V) \subseteq U$.

PROOF. (1) Assume f is pre-Ω-closed. For any subset S of Y and any Ω-open set $U \subseteq X$ containing $f^{-1}(S)$ we put $V = Y - f(X-U)$. Since $f^{-1}(S) \subseteq U$, then $f(X - f^{-1}(S)) \supseteq f(X-U)$. Therefore, $V = Y - f(X-U) \supseteq Y - f(X - f^{-1}(S)) = Y - f(f^{-1}(Y-S)) \supseteq Y-(Y-S) = S$. Since U is Ω-open, $X-U$ is Ω-closed and by hypothesis f is pre-Ω-closed we see that V is Ω-open in Y. Moreover, we have $f^{-1}(V) = X - f^{-1}(f(X-U)) \subseteq X-(X-U) = U$.

Conversely, for every Ω-closed subset F of X, we have $f^{-1}(Y - f(F)) = X - f^{-1}(f(F)) \subseteq X - F$. Let $U = X - F$ and $S = Y - f(F)$. Then, by hypothesis there exists an Ω -open subset V of Y such that $f^{-1}(V) \subseteq X - F$ and $Y - f(F) \subseteq V$. Therefore, we have $F \subseteq X - f^{-1}(V) = f^{-1}(Y-V)$. Hence, $Y - V \subseteq f(F) \subseteq f(f^{-1}(Y-V)) \subseteq Y - V$. Then, $Y - V = f(F)$ is Ω-closed.

(2) The proof is similar to (1).

THEOREM 4.10. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then

(1) Let f be pre-Ω-closed and $f^{-1}(y)$ is Ω-compact relative to X for each $y \in Y$. Suppose that G is Ω -compact relative to Y. If for each finite Ω-open sets are semi-separated, then $f^{-1}(G)$ is Ω-compact relative to X.

(2) Let f be pre-Ωs-closed and $f^{-1}(y)$ is Ωs-compact relative to X for each $y \in Y$. Suppose that G is Ωs -compact relative to Y. If for each finite Ωs-open sets are semi-separated, then $f^{-1}(G)$ is Ωs-compact relative to X.

PROOF. (1) Let $\Phi = \{F_\alpha : \alpha \in \nabla\}$ be a cover of $f^{-1}(G)$ by Ω-open subsets of X. Then, for each $y \in G$ there exists finite subfamily $\nabla(y)$ of ∇ such that $f^{-1}(y) \subseteq \bigcup\{F_\alpha : \alpha \in \nabla(y)\}$. Put $U(y) \subseteq \bigcup\{F_\alpha : \alpha \in \nabla(y)\}$. Since f is pre-Ω-closed, then by Proposition 4.9 there exists an Ω-open subset $V(y)$ of Y such that $y \in V(y)$ and $f^{-1}(V(y)) \subseteq U(y)$. Since $\{V(y) : y \in G\}$ is a cover of G by Ω-open subsets of Y, there exists a finite number of points $y_1, y_2, ..., y_n$ in G such that $G \subseteq \bigcup\{V(y_j) : j = 1, ..., n\}$. Then, we have $f^{-1}(G) \subseteq \bigcup_{j=1}^n f^{-1}(V(y_j)) \subseteq \bigcup_{j=1}^n U(y_j) = \bigcup_{j=1}^n \bigcup_{\alpha\in\nabla(y_j)} F_\alpha$.

(2) The proof is similar to (1).

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