International Journal of Pure and Applied Mathematics
Volume 106 No. 6 2016, 57-73
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v106i6.7

ijpam.eu

AN EXTENSIVE STUDY OF SUPRA GENERALIZED PRE-REGULAR CLOSED SETS

O.R. Sayed[1], Gnanambal Ilango[2], Vidhya Menon[3]

[1]Department of Mathematics
Assiut University
71516, Assiut, EGYPT

[2]Department of Mathematics
Govt. Arts College
Coimbatore, INDIA

[3]Department of Mathematics
CMS College of Science and Commerce
Coimbatore, INDIA

Abstract: The purpose of this paper is to study the concept of gpr^{μ} - closure and gpr^{μ} - interior. Also some more results of gpr^{μ} - continuous functions are investigated.

AMS Subject Classification: 54A05, 54F65
Key Words: supra topological space, supra closed set, gpr^{μ}-closure, gpr $^{\mu}$-interior, gpr^{μ}-continuous function

1. Introduction

The notion of g-continuous functions was introduced and studied by Balachandran, Sundaram and Maki [2]. The research work in the field of continuity was further developed and many topologists introduced and investigated different types of continuous functions in general topology. The study of gpr-continuous functions in topological spaces was initiated by Gnanambal and Balachandran [3] in 1999. Also, in supra topological spaces, the study on continuity was discussed by many researchers. In 1983, Mashour et al [7] initiated the study of S-continuous maps and S*-continuous maps in supra topological spaces. This

Received: February 15, 2016
Published: April 2, 2016

made the other topologists to inculcate various types of continuous functions in supra topological spaces. In this paper, we shall continue the investigation carried out in [11] and study the notion of gpr^{μ}-closure and gpr^{μ}-interior. Throughout this paper, (X,τ), (Y,σ) and (Z,η) represents topological spaces on which no separation axioms are assumed unless explicitly stated. A sub collection $\mu \subset P(X)$ is called a supra topology [7] on X if $X \in \mu$ and μ is closed under arbitrary union. (X,μ) is called a supra topological space. The elements of μ are said to be supra open in (X,μ) and the complement of a supra open set is called supra closed set. The supra closure of a set A, denoted by $cl^{\mu}(A)$, is the intersection of supra closed sets including A. The supra interior of a set A, denoted by $int^{\mu}(A)$, is the union of supra open sets included in A. We call μ a supra topology associated with the topology τ if $\tau \subset \mu$.

2. Preliminaries

Definition 1. A subset A of a supra topological space (X,μ) is called:

(i) supra pre-closed [11] if $cl^{\mu}(int^{\mu}(A)) \subseteq A$.

(ii) supra α-closed [1] cl^{μ} $(int^{\mu}$ $(cl^{\mu}(\text{A}))) \subseteq A$.

(iii) supra semi-closed [1] if int^{μ} $(cl^{\mu}(A)) \subseteq A$.

(iv)supra regular closed [1] $A = int^{\mu}$ $(cl^{\mu}(A))$.

The complements of above mentioned closed sets are called their respective open sets.

The collection of all supra pre-open, supra pre-closed, supra semi-closed, supra regular open, supra generalized pre-regular closed and supra generalized pre-regular open subsets of X will be denoted by $PO^{\mu}(X)$, $PC^{\mu}(X)$, $SC^{\mu}(X)$, $RO^{\mu}(X)$, $GPRC^{\mu}(X)$ and $GPRO^{\mu}(X)$ respectively.

Definition 2. [11] Let A be a subset of (X,μ). Then:

(i) the supra pre-closure of a set A is defined as $pcl^{\mu}(A) = \bigcap$ (B: B is a supra pre-closed set and $A \subseteq B$).

(ii)the supra pre-interior of a set A is defined as $pint^{\mu}(A) = \bigcup$ (B: B is a supra pre-open set and $B \subseteq A$).

Definition 3. A subset A of a space (X,μ) is called:

(i) supra generalized closed (briefly g^{μ}-closed) [1] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X,μ).

(ii) supra generalized pre-closed (briefly gp^{μ}-closed) if $pcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ).

(iii) supra generalized pre-regular closed (briefly gpr^{μ}-closed) if $pcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra regular open in (X, μ).

Definition 4. Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ.A function f: (X, τ) $\rightarrow$ (Y, σ) is called:

(i) supra-continuous [8] if $f^{-1}(V)$ is supra closed in X for every closed set V of Y.

(ii) supra α-continuous [8] if $f^{-1}(V)$ is supra α-closed in X for every closed set V of Y.

(iii) supra pre-continuous [10] if $f^{-1}(V)$ is supra pre-closed in X for every closed set V of Y.

(iv) g^{μ}-continuous [8] if $f^{-1}(V)$ is g^{μ}-closed in X for every closed set V of Y.

(v) gp^{μ}-continuous if $f^{-1}(V)$ is gp^{μ}-closed in X for every closed set V of Y.

(vi) gpr^{μ}-continuous [11] if $f^{-1}(V)$ is gpr^{μ}-closed in X for every closed set V of Y.

3. gpr$^{\mu}$-Closed Sets

Theorem 5. *In a supra topological space (X, μ), let A be a subset of X. Then $x \in cl^{\mu}(A)$ iff every supra open set containing x intersects A.*

Proof. Let $x \notin cl^{\mu}(A)$, then the set $U = X - cl^{\mu}(A)$ is a supra open set containing x such that $U \cap A = \phi$.

Conversly if there exist a supra open set U containing x which does not intersect A, then $X - U$ is a supra closed set containing A. By definition of $cl^{\mu}(A)$, $X - U$ must contain $cl^{\mu}(A)$. Thus $cl^{\mu}(A) \subset X - U$ which implies $U \cap cl^{\mu}(A) = \phi$. Hence $x \notin cl^{\mu}(A)$. □

Definition 6. [11] A space (X, μ) is called supra pre-regular $T_{1/2}$ space if every gpr^{μ}-closed set is supra pre-closed.

Lemma 7. *(i) For an $x \in X$ in (X,μ), its complement $X - \{\text{x}\}$ is gpr^{μ}-closed or supra regular open.*

(ii) (X,μ) is supra pre-regular $T_{1/2}$ iff for each $\{x\}$ of (X,μ), $\{x\}$ is supra pre-open or X-$\{\text{x}\}$ is supra regular open.

Proof. (i) Let X-$\{x\}$ is not supra regular open. Then X is the only supra regular open set containing X-$\{x\}$. Thus pcl^{μ} $(X - \{\text{x}\}) \subseteq X$. Hence X-$\{x\}$ is gpr^{μ}-closed.

(ii) Suppose X-$\{x\}$ is not supra regular open. Then X is the only supra regular open set containing X-$\{x\}$. Thus $pcl^{\mu}(X - \{\text{x}\}) \subseteq X$. Hence X-$\{x\}$ is gpr^{μ}-closed. Therefore X-$\{x\}$ is supra pre-closed by definition 6. Hence $\{x\ \}$ is supra pre-open.

Conversely suppose that A is gpr^{μ}-closed such that $A = X - \{\text{x}\}$ and X-$\{x\}$ is supra regular open. Since A is gpr^{μ}-closed, $pcl^{\mu}(A) \subset X - \{\text{x}\} = A$. This implies $pcl^{\mu}(A) \subset A$ holds. Hence A is supra pre-closed. □

Theorem 8. *If $PO^{\mu}(X) = PC^{\mu}(X)$, then $GPRC^{\mu}(X)$ equals the power set of X.*

Proof. Suppose $A \subseteq O$, where O is supra regular open in (X,μ). Since O is supra pre-open, it is supra pre-closed by hypothesis. Hence $pcl^{\mu}(A) \subseteq O$ and so A is gpr^{μ}-closed. Thus $GPRC^{\mu}(X)$ equals the power set of (X,μ) □

Theorem 9. *Let $PO^{\mu}(X)$ be closed under finite intersections. If A is gpr^{μ}-open and B is gpr^{μ} open then $A \bigcap B$ is gpr^{μ}-open.*

Proof. Let

$$X - (A \bigcap B) = (X - A) \bigcup (X - B) \subseteq Q,$$

where Q is supra regular open. Then $X - A \subseteq Q$ and $X - B \subseteq Q$. Since A and B are gpr^{μ}-open, $pcl^{\mu}(X - A) \subseteq Q$ and $pcl^{\mu}(X - B) \subseteq Q$. By hypothesis $pcl^{\mu}((X - A) \bigcup (X - B)) = pcl^{\mu}(X - A) \bigcup pcl^{\mu}(X - B) \subseteq Q$. That is $pcl^{\mu}(X - (A \bigcap B)) \subseteq Q$. Hence $A \bigcap B$ is gpr^{μ}-open. □

Definition 10. Let (X,μ) be a supra topological space and $A \subset X$. A point $x \in A$ is called an supra interior point of A, iff there exist a supra open set G with $x \in G$ such that $G \subset A$.

Definition 11. [11] Let (X, μ) be a supra topological space, $A \subset X$ and $x \in X$. x is said to be a supra limit point of A iff every supra open set containing x contains a point of A different from x. The supra derived set of A denoted by $D^\mu[A]$ is the set of all supra limit points of A.

Theorem 12. *Let (X, μ) be a supra topological space and $A \subset X$. Then A is supra closed iff $D^\mu[A] \subset A$.*

Proof. Let (X, μ) be a supra topological space and $A \subset X$ be supra closed. By hypothesis $X - A$ is supra open. Let $x \in X - A$ be arbitrary. Then $X - A$ is a supra open set containing x such that $(X - A) \bigcap A = \phi$. This implies $x \notin D^\mu[A]$. Thus $(X - A) \in (X - D^\mu[A])$. Hence $D^\mu[A] \subset A$.

Conversely suppose that A is a subset of (X, μ) such that $D^\mu[A] \subset A$. Let $x \in (X - A)$ be arbitrary. Then $x \notin A$. This implies $x \notin D^\mu[A]$. Then there exist a supra open set G with $x \in G$ such that $(G - \{\text{x}\}) \bigcap A = \phi$. That is $G \bigcap A = \phi$. Thus $G \subset X - A$. Hence x is a supra interior point of $X - A$. This implies $X - A$ is a supra open set. Hence A is supra closed. □

Theorem 13. *In a supra topological space, $D^\mu[A]$ is supra closed for every supra closed set $A \subset X$.*

Proof. Let (X, μ) be a supra topological space and $A \subset X$. By theorem 12, A is supra closed iff $D^\mu[A] \subset A$. Hence $D^\mu[A]$ is supra closed iff $D^\mu[D^\mu[A]] \subset D^\mu[A]$. Let $x \in D^\mu[D^\mu[A]]$ be arbitrary. This implies for every supra open set G containing x such that $(G - \{\text{x}\}) \bigcap D^\mu[A] \neq \phi$. That is $(G - \{\text{x}\}) \bigcap A \neq \phi$. This shows that $x \in D^\mu[A]$. Thus $D^\mu[D^\mu[A]] \subset D^\mu[A]$. Hence $D^\mu[A]$ is supra closed. □

Definition 14. Let(X, μ) be a supra topological space with supra topology μ. If Y is a subset of X, the collection $\mu_Y = \{Y \bigcap U | \ U \in \mu\}$is a supra topology on Y called the supra subspace topology. With this supra topology, Y is called supra subspace of X ; its supra open sets consists of all intersection of supra open sets of X with Y.

Theorem 15. *Let Y be a supra subspace of X. Then the set A is supra closed in Y iff it equals intersection of a supra closed set of X with Y.*

Proof. Let $A = C \bigcap Y$, where C is supra closed in X. Then $X - C$ is supra open in X. Thus $(X - C) \bigcap Y$ is supra open in Y by the definition of supra subspace topology. But $(X - C) \bigcap Y = Y - A$. Hence $Y - A$ is supra open in Y, so that A is supra closed in Y.

Conversely, let A be supra closed in Y. Then $Y - A$ is supra open in Y, it equals the intersection of a supra open set U of X with Y. Thus $X - U$ is supra closed in X and $A = Y \bigcap (X - U)$, so that A equals the intersection of a supra closed set of X with Y. □

Theorem 16. *Let Y be a supra subspace of X; let A be subset of Y. Then $cl^{\mu}_{Y}(A) = cl^{\mu}(A) \bigcap Y$, where cl^{μ}_{Y} denotes the supra closure operator in the supra subspace Y.*

Proof. Let B denote $cl^{\mu}_{Y}(A)$. The set $cl^{\mu}(A)$ is supra closed in X, so $cl^{\mu}(A) \bigcap Y$ is supra closed in Y by theorem 15. Since$A \subset cl^{\mu}(A) \bigcap Y$ and by definition B equals intersection of all supra closed subsets of Y containing A, we have$B \subset (cl^{\mu}(A) \bigcap Y)$.

Conversely, we know that B is supra closed in Y, by the theorem 15,$B = C \bigcap Y$, for some supra closed set C in X. Then C is a supra closed set of X containing A. Thus $cl^{\mu}(A) \subset C$. Hence $cl^{\mu}(A) \bigcap Y \subset C \bigcap Y = B$. □

Lemma 17. *Let A be supra closed in (X, μ) and $B \subset A$. Then $cl^{\mu}(B) = cl^{\mu}_{A}(B)$ where cl^{μ}_{A} denotes the supra closure operator in the supra subspace A.*

Proof. As by hypothesis, A is a supra subspace of X and $B \subset A$. By theorem 16, $cl^{\mu}_{A}(B) = cl^{\mu}(B) \bigcap A = cl^{\mu}(B) \bigcap cl^{\mu}(A) = cl^{\mu}(B)$. □

Lemma 18. *Let A be supra open in (X, μ) and $B \subset A$. Then $int^{\mu}(B) = int^{\mu}_{A}(B)$ where int^{μ}_{A} denotes the supra interior operator in the supra subspace A.*

Proof.

$$int^{\mu}(B) = int^{\mu}(B \bigcap A) \subset int^{\mu}(B) \bigcap int^{\mu}(A) \subset int^{\mu}(B) \bigcap A = int^{\mu}_{A}(B).$$

$$\begin{aligned} int^{\mu}_{A}(B) = int^{\mu}(B) \bigcap A = int^{\mu}(B \bigcap A) \bigcap A \subset int^{\mu}(B) \bigcap int^{\mu}(A) \bigcap A \\ \subset int^{\mu}(B) \bigcap int^{\mu}(A) \subset int^{\mu}(B). \end{aligned}$$

Hence $int^{\mu}(B) = int^{\mu}_{A}(B)$. □

Lemma 19. *Let Y be a supra subspace of X. If U is supra closed in Y and Y is supra closed in X, then U is supra closed in X.*

Proof. Since U is supra closed in Y, $U = Y \bigcap V$ for some set V supra closed in X. Since Y and V are both supra closed in X, so is $V \bigcap Y$. This implies U is supra closed in X. □

Lemma 20. *Let Y be a supra subspace of X. If U is supra open in Y and Y is supra open in X, then U need not be supra open in X.*

Exambl e 21. Let X = {a,b,c,d},μ = {ϕ,X, {a}, {a,b}, {c,d}, {a,d}, {a,b,d}, {a,c,d}}, Y = {a, d}, μ_Y = { ϕ, Y, {a}, {d}}. U = {d} is supra open in Y and Y is supra open in X but U is not supra open in X.

Lemma 22. *Let $A \subset Y \subset X$, Y be supra open and supra closed in (X, μ). Then $A \in PC^\mu(X)$ iff $A \in PC^\mu(Y)$.*

Proof. Let $A \in PC^\mu(X, \mu)$. Then $cl^\mu(int^\mu(A)) \subseteq A$. Since Y is supra open, $cl^\mu\ (int^\mu_Y(A)) \subseteq A$ by lemma 18. Then $cl^\mu(int^\mu_Y(A)) \bigcap Y \subseteq A \bigcap Y = A$. That is $cl^\mu_Y\ (int^\mu_Y(A)) \subseteq A$. Thus $A \in PC^\mu(Y)$.

Conversely, let $A \in PC^\mu(Y)$. Then $cl^\mu_Y\ (int^\mu_Y(A)) \subseteq A$. Since Y is supra closed and supra open, $cl^\mu\ (int^\mu(A)) = cl^\mu_Y\ (int^\mu_Y(A)) \subseteq A$ by lemmas 17 & 18. Thus $A \in PC^\mu(X, \mu)$. □

Definition 23. In a supra topological space (X, μ), let A be a subset of X. Then $x \in pcl^\mu(A)$ iff every supra pre-open set containing x intersects A.

Theorem 24. *In a supra topological space $(X, \mu), A \cup cl^\mu(int^\mu(A))$ is supra pre-closed.*

Proof. $A \bigcup cl^\mu(int^\mu(A))$ is supra pre-closed iff

$$pcl^\mu(A \bigcup cl^\mu(int^\mu(A))) = A \bigcup cl^\mu(int^\mu(A)). A \bigcup cl^\mu(int^\mu(A)) \subset pcl^\mu(A \bigcup cl^\mu(int^\mu(A))).$$

Now to prove that

$$pcl^\mu(A \bigcup cl^\mu(int^\mu(A))) \subset A \bigcup cl^\mu(int^\mu(A)).$$

Let $x \notin (A \bigcup cl^\mu(A))$. Then $x \notin A$, $x \notin cl^\mu(A)$. This implies by definition 23, there exist a supra open set U with $x \in U$ such that $U \bigcap A = \phi$. That is, there exist a supra pre-open set U with $x \in U$ such that $U \bigcap A = \phi$.

By hypothesis $x \notin (A \bigcup cl^\mu(A))$, implies that $x \notin (A \bigcup cl^\mu(int^\mu(A))$. Thus $x \notin A$, $x \notin cl^\mu(int^\mu(A))$. As $cl^\mu(cl^\mu(A)) = cl^\mu(A)$, $x \notin cl^\mu(cl^\mu(int^\mu\ (A)))$. Hence $x \notin pcl^\mu(cl^\mu(int^\mu(A)))$. That is there exist a supra pre-open set U with

$x \in U$ such that $U \bigcap cl^{\mu}(int^{\mu}(A)) = \phi$. Also $U \bigcap A = \phi$. Thus there exist a supra pre-open set U containing x such that $U \bigcap (A \bigcup cl^{\mu}(int^{\mu}(A))) = \phi$. This implies $x \notin pcl^{\mu}(A \bigcup cl^{\mu}(int^{\mu}(A)))$. Thus

$$pcl^{\mu}(A \bigcup cl^{\mu}(int^{\mu}(A))) \subset A \bigcup cl^{\mu}(int^{\mu}(A)).$$

Hence $A \bigcup cl^{\mu}(int^{\mu}(A))$ is supra pre-closed. □

Proposition 25. *Let A be a subset of a supra topological space (X, μ). Then:*

(i) $pcl^{\mu}(A)$ is supra preclosed.

(ii) $pcl^{\mu}(A) = A \bigcup cl^{\mu}(int^{\mu}(A))$.

Proof. (i) By definition 2 (i)$pcl^{\mu}(A) = \bigcap(B : B$ is a supra pre-closed set and $A \subseteq B)$. Also by theorem 2.3(i)[10], arbitrary intersection of supra pre-closed sets is always supra pre-closed, $pcl^{\mu}(A)$ is supra preclosed.

(ii)

$$cl^{\mu}(int^{\mu}(A \bigcup cl^{\mu}(int^{\mu}(A)))) \subset A \bigcup cl^{\mu}(int^{\mu}(A))$$

as $A \bigcup cl^{\mu}(int^{\mu}(A))$ is supra pre-closed by theorem 24. Thus

$$pcl^{\mu}(A \bigcup cl^{\mu}(int^{\mu}(A))) = A \bigcup cl^{\mu}(int^{\mu}(A)).$$

Hence $pcl^{\mu}(A) \subset pcl^{\mu}(A \bigcup cl^{\mu}(int^{\mu}(A))) = A \bigcup cl^{\mu} (int^{\mu}(A))$. On the other hand $pcl^{\mu}(A)$ is supra pre-closed. Therefore

$$cl^{\mu}(int^{\mu}(A)) \subset cl^{\mu}(int^{\mu}(pcl^{\mu}(A))) \subset pcl^{\mu}(A).$$

Hence $A \bigcup cl^{\mu}(int^{\mu} (A)) \subset pcl^{\mu}(A)$. That is

$$pcl^{\mu}(A) = A \bigcup cl^{\mu}(int^{\mu}(A)).$$

□

Lemma 26. *If $A \subset Y \subset X$ and Y be supra open in (X, μ), then $pcl^{\mu}_{Y}(A) = pcl^{\mu}_{X}(A) \bigcap Y$*

Proof.

$$pcl^{\mu}_{Y}(A) = A \bigcup (cl^{\mu}_{Y}(int^{\mu}_{Y}(A))) = A \bigcup (cl^{\mu}_{Y}(int^{\mu}(A)))$$
$$= A \bigcup (cl^{\mu}(int^{\mu}(A)) \bigcap Y) = (A \bigcup cl^{\mu}(int^{\mu}(A))) \bigcap (A \bigcup Y) = pcl^{\mu}_{X}(A) \bigcap Y.$$

□

Lemma 27. *If Y is supra open and supra pre-closed in (X, μ), then $pcl^{\mu}_{Y}(A) = pcl^{\mu}_{X}(A)$.*

Proof. By lemma 26, $pcl^{\mu}_{Y}(A) = pcl^{\mu}_{X}(A) \bigcap Y$. Since Y is supra pre-closed, $pcl^{\mu}_{X}(A) \subseteq Y$. Therefore $pcl^{\mu}_{Y}(A) = pcl^{\mu}_{X}(A)$. □

Lemma 28. *In a supra topological space (X, μ), let $A \subset X$. If A is supra open then $RO^{\mu}(A, \mu|A) = \{ V \bigcap A : V \in RO^{\mu}(X, \mu)\}$.*

Proof. Let A be a supra open set in X and let $W \in RO^{\mu}(A, \mu|A)$. Then $int^{\mu}_{A}\ (cl^{\mu}_{A}(W)) = int^{\mu}\ (cl^{\mu}(W) \bigcap A) \bigcap A \subset int^{\mu}(cl^{\mu}(W)) \bigcap int^{\mu}(A) \bigcap A = int^{\mu}(cl^{\mu}(W)) \bigcap A = V \bigcap A$ where $V \in RO^{\mu}\ (X, \mu)$.

Conversely let $V \in RO^{\mu}(X, \mu)$ and $W = V \bigcap A$. Then

$$\begin{aligned} int^{\mu}_{A}(cl^{\mu}_{A}(W)) &= int^{\mu}(cl^{\mu}(W) \bigcap A) \bigcap A = int^{\mu}(cl^{\mu}(V \bigcap A) \bigcap A) \bigcap A \\ &\subset int^{\mu}(cl^{\mu}(V) \bigcap A) \bigcap A \subset int^{\mu}(cl^{\mu}(V)) \bigcap int^{\mu}(A) \bigcap A \\ &= int^{\mu}(cl^{\mu}(V)) \bigcap A = V \bigcap A = W. \end{aligned}$$

Therefore $W \in RO^{\mu}\ (A, \mu|A)$. □

Lemma 29. *In a supra topological space (X, μ), let $A \subset Y \subset X$. Then:*

(i) if Y is supra open in (X, μ) then $A \in GPRC^{\mu}(X)$ implies $A \in GPRC^{\mu}(Y)$; and

(ii) if Y is supra open & supra pre-closed in (X, μ) then $A \in GPRC^{\mu}\ (Y)$ implies $A \in GPRC^{\mu}(X)$.

Proof. (i) Let A be gpr^{μ}-closed in (X, μ). Let $A \subseteq O$ where O is supra regular open in Y. Then $O = O^{*} \bigcap Y$ where O^{*} is supra regular open in (X, μ) by lemma 28. That is $A \subseteq O^{*}$. Since $A \in GPRC^{\mu}(X), pcl^{\mu}_{X}\ (A) \subseteq O^{*}$. Then $pcl^{\mu}_{X}(A) \bigcap Y \subseteq O^{*} \bigcap Y$. That is $pcl^{\mu}_{Y}(A) \subseteq O$. Hence $A \in GPRC^{\mu}(Y)$.

(ii) Let $A \in GPRC^{\mu}(Y)$. Then $A \subseteq U$ where U is supra regular open in X. Now $A = A \bigcap Y \subseteq U \bigcap Y$ where $U \bigcap Y$ is supra regular open in Y by lemma 28. By hypothesis $pcl^{\mu}_{Y}(A) \subseteq U \bigcap Y$. Then by lemma 27, $pcl^{\mu}_{X}(A) \subseteq U \bigcap Y \subseteq U$. Hence $A \in GPRC^{\mu}(X)$. □

The following example shows that the assumption Y is supra open and supra pre-closed in lemma 29 (ii) cannot be removed.

Examble 30. Let X = {a,b,c,d}, Y = {a, b, d} and μ ={ ϕ, X, {a}, {c}, {a,b}, {a, c}, {b, c},{a, b, c}}. Then $GPRC^{\mu}$ (X, μ) = { ϕ, X, {b}, {d}, {c,d}, {a,d}, {b,d}, {a,c}, {a,b,c}, {a,b,d}, {b,c,d}, {a,c,d}} and $GPRC^{\mu}$ (Y, $\mu|Y$) = { ϕ, Y, {a}, {b}, {d}, {a,b}, {a,d}, {b,d}}. {a} $\in$ $GPRC^{\mu}$ (Y, $\mu|Y$) but {a} does belongs to $GPRC^{\mu}$ (X, μ). Here, Y is supra pre-closed but not supra open in (X, μ).

Corollary 31. *If Y is supra open and supra pre-closed in (X, μ) then $A \in GPRC^{\mu}(X)$ iff $A \in GPRC^{\mu}(Y)$.*

Proof. It follows from lemma 29 (i) and (ii). □

4. gpr^{μ}-Closure and gpr^{μ}-Interior

Definition 32. Let (X, μ) be a supra topological space and $A \subset X$. Then:

(i) $gpr^{\mu} - cl(A) = \bigcap$ [$F : A \subset F$, F is gpr^{μ}-closed set in (X, μ)].

(ii)$gpr^{\mu} - int(A) = \bigcup$ [$M : M \subset A, M$ is gpr^{μ}-open set in (X, μ)].

Lemma 33. *Let A and B be subsets of the supra topological space (X, μ). Then:*

(i) $gpr^{\mu} - cl(\phi) = \phi$ and $gpr^{\mu} - cl(X) = X$.

(ii) $A \subset gpr^{\mu} - cl(A)$.

(iii) If B is any gpr^{μ}-closed set containing A, then $gpr^{\mu} - cl(A) \subset B$.

(iv) If $A \subset B$, then gpr^{μ}-$cl(A) \subset gpr^{\mu} - cl(B)$.

(v) $gpr^{\mu} - cl(A) = gpr^{\mu} - cl(gpr^{\mu} - cl(A))$.

(vi) $gpr^{\mu} - cl(A \bigcup B) \supset gpr^{\mu} - cl(A) \bigcup gpr^{\mu} - cl(B)$.

(vii)$gpr^{\mu} - cl(A \bigcap B) \subset gpr^{\mu} - cl(A) \bigcap gpr^{\mu} - cl(B)$.

Proof. Obvious. □

Remark 34. The equality does not hold in lemma 33(vi & vii) as per the following examples.

Examble 35. i) Let X = {a,b,c,d,e}. Consider μ = {X, ϕ, {a,b}, {b,c,d}, {a,b,c,d}, {a}}. Consider A = {b}, B = {c,d}. $A\bigcup B$= {b,c,d}. $gpr^{\mu}-cl(A\bigcup B)$ = {b,c,d,e}. $gpr^{\mu}-cl(A)=\{b\}, gpr^{\mu}-cl(B)=\{c,d\}.gpr^{\mu}-cl(A)\bigcup gpr^{\mu}-cl(B)$ = {b,c,d}.

ii) Let X= {a,b,c,d,e}. Consider $\mu=\{X,\ \phi,$ {a,b}, {b,c,d}, {a,b,c,d}, {a}}. Consider A = {b,c,d}, B = {e,b,d}. $A\bigcap B$ = {d,b}. $gpr^{\mu}-cl(A)$ = {b,c,d,e}, $gpr^{\mu}-cl(B)$ = {e,d,b}. $gpr^{\mu}-cl(A\bigcap B)$ = {b,d}. $gpr^{\mu}-cl(A)\bigcap gpr^{\mu}-cl(B)$ = {e,d,b}.

Lemma 36. *For an $x\in X$, $x\in gpr^{\mu}-cl(A)$ iff $V\bigcap A\neq\phi$ for every gpr^{μ}-open set V containing x.*

Proof. Necessity: Let $x\in gpr^{\mu}-cl(A)$. Suppose that there exist a gpr^{μ}-open set V containing x such that $V\bigcap A=\phi$. Since $A\subset X-V$, $gpr^{\mu}-cl(A)\subset X-V$, implies $x\notin gpr^{\mu}-cl(A)$, a contradiction. Therefore $V\bigcap A\neq\phi$ for every gpr^{μ}-open set V containing x.

Sufficiency: Let $x\notin gpr^{\mu}-cl(A)$. Then there exist a gpr^{μ}-closed subset F containing A such that $x\notin F$. Then $x\in X-F$ and $X-F$ is gpr^{μ}-open. Also $(X-F)\bigcap A=\phi$ which is a contradiction. Hence the lemma. □

Remark 37. In a supra topological space (X,μ), if $A\subset X$ is gpr^{μ}-closed then $gpr^{\mu}-cl(A)=A$.

Proof. Obvious. □

Remark 38. If $gpr^{\mu}-cl(A)=A$, then A need not be gpr^{μ}-closed in (X,μ).

Examble 39. Let X = {a,b,c,d}.Consider $\mu=\{X,\ \phi,$ {a}, {a,c}, {b,c}, {a,b,c}}. $GPRC^{\mu}(X)$ = { X, ϕ, {b}, {c}, {d}, {a,b}, {c,d}, {a,c}, {b,d}, {a,d}, {a,b,c}, {a,b,d}, {b,c,d}, {c,d,a}}. $gpr^{\mu}-cl\{a\}$= {a,b} $\bigcap$ {a,c} $\bigcap$ {a,b,c} $\bigcap$ {a,b,d} $\bigcap$ {a,c,d} $\bigcap X$ = {a}. But {a} is not gpr^{μ}-closed.

Lemma 40. *Let A and B be the subsets of the supra topological space (X,μ). Then:*

(i) $gpr^{\mu}-int(\phi)=\phi$ and $gpr^{\mu}-int(X)=X$.

(ii)$gpr^{\mu}-int(A)\subset A$.
(iii) If B is a gpr^{μ}-open set contained in A, then $B\subset gpr^{\mu}-int(A)$.

(iv) If $A\subset B$, then $gpr^{\mu}-int(A)\subset gpr^{\mu}-int(B)$.

(v) $gpr^{\mu}-int(gpr^{\mu}-int(A))=gpr^{\mu}-int(A)$.

Proof. Follows from definitions. □

Lemma 41. $(X - gpr^{\mu} - int(A)) = gpr^{\mu} - cl(X - A)$.

Proof. Let $x \in X - (gpr^{\mu} - int(A))$. Then $x \notin gpr^{\mu} - int(A))$. That is every gpr^{μ}-open set B containing x is such that $B \not\subset A$. This implies every gpr^{μ}-open set B containing x intersects $X - A$. So $x \in gpr^{\mu} - cl(X - A)$. Hence $(X - gpr^{\mu} - int(A)) \subset gpr^{\mu} - cl(X - A)$.

Conversely, let $x \in gpr^{\mu} - cl(X - A)$. Then every gpr^{μ}-open set D containing x intersect $X - A$. That is, every gpr^{μ}-open set D containing x is such that $D \not\subset A$. This implies $x \notin gpr^{\mu} - int(A)$. Thus $gpr^{\mu} - cl(X - A) \subset X - gpr^{\mu} - int(A)$. Hence $(X - gpr^{\mu} - int(A)) = gpr^{\mu} - cl(X - A)$. □

Proposition 42. *If $GPRC^{\mu}$ (X, μ) is closed under finite unions, then $gpr^{\mu} - cl(A \bigcup B) = gpr^{\mu} - cl(A) \bigcup gpr^{\mu} - cl(B)$.*

Proof. Let A and B be gpr^{μ}-closed in (X, μ). Then by remark 37, $gpr^{\mu} - cl(A) = A$ and $gpr^{\mu} - cl(B) = B$.Thus $gpr^{\mu} - cl(A) \bigcup gpr^{\mu} - cl(B) = A \bigcup B$. Also by hypothesis $A \bigcup B$ is gpr^{μ}-closed. That is $gpr^{\mu} - cl(A \bigcup B) = A \bigcup B = gpr^{\mu} - cl(A) \bigcup gpr^{\mu} - cl(B)$. □

Theorem 43. *If $PC^{\mu}(X, \mu)$ is closed under finite unions, then $GPRC^{\mu}(X, \mu)$ is closed under finite unions.*

Proof. Let $PC^{\mu}(X, \mu)$ be closed under finite unions. Let $A, B \in GPRC^{\mu}(X, \mu)$ and let $A \bigcup B \subseteq U$, where U is supra regular open in (X, μ). Then $A \subseteq U$ and $B \subseteq U$. Hence, $pcl^{\mu}(A) \subseteq U$ and $pcl^{\mu}(B) \subseteq U$. This implies $pcl^{\mu}(A) \bigcup pcl^{\mu}(B) \subseteq U$. By hypothesis, $pcl^{\mu}(A \bigcup B) = A \bigcup B \subseteq U$. That is $pcl^{\mu}(A \bigcup B) \subseteq U$. Hence $A \bigcup B \in GPRC^{\mu}(X, \mu)$. □

Corollary 44. *If $PO^{\mu}(X, \mu)$ is closed under finite intersections, then $GPRO^{\mu}(X, \mu)$ is closed under finite intersections.*

Proof. Let $PO^{\mu}(X, \mu)$ be closed under finite intersections. Let $A, B \in GPRO^{\mu}(X, \mu)$ and let $U \subseteq A \bigcap B$, where U is supra regular closed in (X, μ). Then $U \subseteq A$ and $U \subseteq B$. Hence, $U \subseteq pint^{\mu}(A)$ and $U \subseteq pint^{\mu}(B)$. This implies $U \subseteq pint^{\mu}(A) \bigcap pint^{\mu}(B)$. By hypothesis, $pint^{\mu}(A \bigcap B) = A \bigcap B \supseteq U$. That is $U \subseteq pint^{\mu}(A \bigcap B)$. Hence $A \bigcap B \in GPRO^{\mu}(X, \mu)$. □

Lemma 45. *Let (X, μ) be a supra topological space.*

(i) If $U \in X$ is supra closed and $V \in PC^{\mu}(X)$ then $U \bigcap V \in PC^{\mu}(X)$.

(ii) If $V \in PC^{\mu}(X)$ and $U \in SC^{\mu}(X)$ then $U \bigcap V \in PC^{\mu}(U)$.

Proof. (i)

$$cl^\mu(int^\mu(U\bigcap V)) \subset cl^\mu(int^\mu(U)\bigcap int^\mu(V)) \subset cl^\mu(int^\mu(U)) \bigcap cl^\mu(int^\mu(V)) \subset cl^\mu(U)\bigcap V = U\bigcap V.$$

Therefore $cl^\mu int^\mu(U\bigcap V)) \subset U\bigcap V$.

(ii)

$$\begin{aligned} cl^\mu_U(int^\mu_U(U\bigcap V)) &= cl^\mu_U(int^\mu(U\bigcap V)\bigcap U) \subset cl^\mu_U(int^\mu(U)\bigcap int^\mu(V)\bigcap U) \\ &\subset cl^\mu_U(int^\mu(U)\bigcap int^\mu(V)) = cl^\mu(int^\mu(U)\bigcap int^\mu(V))\bigcap U \\ &\subset cl^\mu(int^\mu(U))\bigcap cl^\mu(int^\mu(V))\bigcap U \subset cl^\mu(int^\mu(cl^\mu(U)))\bigcap cl^\mu(int^\mu(V))\bigcap U \\ &\subset cl^\mu(U)\bigcap cl^\mu(int^\mu(V))\bigcap U \subset U\bigcap cl^\mu(int^\mu(V)) \subset U\bigcap V. \end{aligned}$$

Therefore $U\bigcap V \in PC^\mu(U)$. □

Lemma 46. *If $U \in PC^\mu(X)$ and $V \in PC^\mu(U)$ then $V \in PC^\mu(X)$ in (X,μ).*

Proof. Since $V \in PC^\mu(\mathrm{U})$, $cl^\mu_U(int^\mu_U(V)) \subseteq V$ and $cl^\mu_U(int^\mu_U(V))$ is supra closed in U, there exist an supra closed set $W \subset X$ such that $U\bigcap W = cl^\mu_U(int^\mu_U(V))$. This implies

$$\begin{aligned} V \supset U\bigcap W &\supseteq cl^\mu(int^\mu(U))\bigcap W \supseteq cl^\mu(int^\mu(U))\bigcap cl^\mu(W) \\ &\supseteq cl^\mu(int^\mu(U))\bigcap cl^\mu(int^\mu(W)) \supseteq cl^\mu(int^\mu(U)\bigcap int^\mu(W)) \\ &\supseteq cl^\mu(int^\mu(U\bigcap W)) = cl^\mu(int^\mu(cl^\mu_U(int^\mu_U(V)))) \\ &\supset cl^\mu(int^\mu(int^\mu_U(V))) \supset cl^\mu(int^\mu(int^\mu(V))) = cl^\mu(int^\mu(V)). \end{aligned}$$

Therefore $cl^\mu(int^\mu(V)) \in V$. So $V \in PC^\mu(X)$. □

5. gpr^μ-Continuous Functions

Definition 47 (11)**.** Let (X,τ), (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function $f:(X,\tau)\to(Y,\sigma)$ is called gpr^μ-continuous if $f^{-1}(V)$ is gpr^μ-closed in X for every closed set V of Y.

Theorem 48. *Let (X, τ), (Y, σ) be two topological spaces and μ be an associated supra topology with τ. Let f be a map from X into Y. Then the following are equivalent:*

(i) f is gpr^{μ}-continuous;

(ii) The inverse image of every open set in Y is gpr^{μ}-open in X;

(iii)gpr^{μ}-$cl(f^{-1}(V)) \subset f^{-1}(cl(V))$, for every $V \subset Y$;

(iv) $f(gpr^{\mu} - cl(A)) \subset cl(f(A))$ for every $A \subset X$.

Proof. (i) $\Longleftrightarrow$ (ii) Obviously:

(i) $\Longrightarrow$ (iii) Since $cl(V)$ is a closed set for every V of Y, then $f^{-1}(cl(V))$ is gpr^{μ}-closed. $f^{-1}(cl(V)) = gpr^{\mu} - cl(f^{-1}(cl(V))) \supset \ gpr^{\mu} - cl(f^{-1}(V))$.

(iii) $\Longrightarrow$ (iv) Let $A \subset X$ and $f(A) = V$. Then $gpr^{\mu} - cl(f^{-1}(V)) \subset f^{-1}(cl(V))$. Thus $gpr^{\mu} - cl(f^{-1}(f(A))) \subset f^{-1}(cl(f(A)))$. This implies $gpr^{\mu} - cl(A) \subset f^{-1}(cl(f(A)))$. Hence $f(gpr^{\mu} - cl(A)) \subset cl(f(A))$.

(iv) $\Longrightarrow$ (i) Let $V \subset Y$ be a closed set and $U = f^{-1}(V)$. Then $f(gpr^{\mu} - cl(U)) \subset cl(f(U)) = cl(f(f^{-1}(V))) \subset cl(V) = V$. $gpr^{\mu} - cl(U) \subset f^{-1}(f(gpr^{\mu} - cl(U)) \subset f^{-1}(V) = U$. Thus U is gpr^{μ}-closed. □

Theorem 49. *Let (X, τ), (Y, σ) be two topological spaces and μ be an associated supra topology with τ. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. For $A \subset X$, if $gpr^{\mu} - int(A) = A$ implies that A is gpr^{μ}-open then, the following are equivalent:*

(i) f is gpr^{μ}-continuous;

(ii) $f^{-1}(int(B)) \subset gpr^{\mu} - int(f^{-1}(B))$ for every $B \subset Y$.

Proof. (i)$\Longrightarrow$ (ii). Let $B \subset Y$. This implies $f^{-1}(int(B))$ is gpr^{μ}-open in X. Therefore $f^{-1}(int(B)) = gpr^{\mu} - int(f^{-1}(int(B))) \subset gpr^{\mu} - int(f^{-1}(B))$.

(ii) $\Longrightarrow$ (i). Let $V \subset Y$ be an open set, then $f^{-1}(V) = f^{-1}(int(V)) \subset gpr^{\mu} - int(f^{-1}(V))$. Hence $f^{-1}(V)$ is gpr^{μ}-open. Thus f is gpr^{μ}-continuous. □

Theorem 50. *(i) Every supra continuous function is gpr^{μ}-continuous function.*

(ii) Every supra α-continuous function is gpr^{μ}-continuous function.

(iii) Every supra pre-continuous function is gpr^{μ}-continuous function.

(iv) Every g^{μ}-continuous function is gpr^{μ}-continuous function.

(v) Every gp^{μ}-continuous function is gpr^{μ}-continuous function.

Proof. Obvious. □

However, the converse of the above theorems are not true as seen in the following examples.

Exmable 51. (i) Let X = {a,b,c,d}, $\tau = \{\ \phi,\ X,\ \{a\},\{a,b\}\}$, $\mu = \{\ \phi,\ X,$ $\{a\},\{a,b\},\{b,c,d\}\}$ and σ={ ϕ, X, {a},{a,b,c}}. Define $f : (X, \tau) \to (X, \sigma)$ by $f(a) = b, f(b) = c, f(c) = d, f(d) = a$. $f^{-1}(b, c, d)$ = {a, b, c} is not supra closed in X. Therefore the function is not supra continuous but gpr^{μ}-continuous.

(ii) Let X = {a,b,c}, τ = { ϕ, X, {a,b}}, μ = { ϕ, X, {a,b},{a,c}} and σ = { ϕ, X,{a}}. Define $f : (X, \tau) \to (X, \sigma)$ by f(a) = b, f (b) = c, f (c) = a. $f^{-1}(b, c)$ = {a, b} is not supra pre-closed. Therefore the function is not supra pre-continuous but gpr^{μ}-continuous.

(iii) Let X = {a, b, c}, τ={ ϕ, X, {b, c}}, μ = { ϕ,X,{a, c},{b, c}} and σ={ ϕ, X, {a}}. Define $g : (X, \tau) \to (X, \sigma)$ by g(a) = b, g (b) = a, g (c) = c, g^{-1} (b,c) = {a, c} is not supra α-closed, therefore the given function is not supra α-continuous but gpr^{μ}-continuous.

(iv)Let X = {a, b, c}, τ = { ϕ, X, {b, c}}, μ = { ϕ, X, {a},{a,b}, {a,c}, {b,c}} and σ = { ϕ, X, {a}}. Define $f : (X, \tau) \to (X, \sigma)$ by f(a) = b, f (b) = c, f (c) = a. f^{-1} (b,c) =(a,b) is not gp^{μ}-closed, therefore the function is not gp^{μ}-continuous but gpr^{μ}-continuous.

(v) Let X = {a, b, c}, τ = { ϕ, X, {a}}, μ = { ϕ, X, {a},{a,b},{a,c}, {b,c}}, σ = { ϕ, X, {a}}. Define $f : (X, \tau) \to (X, \sigma)$ by f (a) = b, f (b) = c, f (c) = a. f^{-1} (b,c) =(a,b) is not g^{μ}-closed, therefore the function is not g$^{\mu}$-continuous but gpr^{μ}-continuous.

Theorem 52. *(i) Every continuous function is gpr^{μ}-continuous.*

(ii) If a function $f : (X, \tau) \to (Y, \sigma)$ is gpr^{μ}-continuous and $g : (Y, \sigma) \to (Z, \eta)$ is continuous then g o f is gpr^{μ}-continuous.

(iii) Every gpr^{μ}-continuous function defined on a supra pre-regular $T_{1/2}$ space is supra pre-continuous.

Proof. (i) Let $f : (X, \tau) \to (Y, \sigma)$ be a continuous function and A be an open set in Y. Then $f^{-1}(A)$ is an open set in X. Since μ is an associated supra topology with τ, then $\tau \subset \mu$. Therefore $f^{-1}(A)$ is a supra open set in X which is a gpr^{μ}-open set in X. Hence f is gpr^{μ}-continuous function.

(ii) Let V be closed set in (Z, η). Since $g : (Y, \sigma) \to (Z, \eta)$ is a continuous function, $g^{-1}(V)$ is closed in (Y, σ). Also gpr^{μ}-continuity of f implies that

$f^{-1}(g^{-1}(V))$ is gpr^μ-closed in X. That is $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is gpr^μ-closed in X. Hence gof is gpr^μ-continuous.

(iii) Let $f : (X, \tau) \to (Y, \sigma)$ be gpr^μ-continuous function. Then $f^{-1}(V)$ is gpr^μ-closed in X for every closed set V of Y. Since X is supra pre-regular $T_{1/2}$ space, every gpr^μ-closed set is supra pre-closed. Hence $f^{-1}(V)$ is supra pre-closed in X for every closed set V in Y. Hence f is supra pre-continuous. □

Theorem 53. *Consider the map $f : (X, \tau) \to (Y, \sigma)$. If for each $x \in X$ and each open set V containing $f(x)$ there exist a gpr^μ-open set U containing x such that $f(U) \subset V$, then $f(gpr^\mu - cl(A)) \subset cl(f(A))$ for every subset A of X and hence f is gpr^μ-continuous by theorem 48.*

Proof. Let $y \in f(gpr^\mu - cl(A))$. Let V be an open set containing y. Then by hypothesis, there exist an $x \in X$ such that $f(x) = y$ and a gpr^μ-open set U containing x such that $f(U) \subset V$ and $x \in gpr^\mu - cl(A)$. Therefore, by lemma 36, $U \bigcap A \neq \phi$. Then $f(U \bigcap A) \neq \phi$. Thus $V \bigcap f(A) \neq \phi$. Hence, $y \in cl(f(A))$. □

Theorem 54. *(i) If $f : X \to Y$ is supra pre-continuous and $U \subset X$ is supra closed, then the restriction $f|U : U \to Y$ is gpr^μ-continuous.*

(ii) If $f : X \to Y$ is supra pre-continuous and $U \in SC^\mu(X)$, then the restriction $f|U : U \to Y$ is gpr^μ-continuous.

Proof. (i) Let $V \subset Y$ be a closed set.Then $f^{-1}(V) \in PC^\mu(X)$. Since $U \subset X$ is supra closed, by lemma 45 (i), $f^{-1}(V) \bigcap U = (f|U)^{-1}(V) \in PC^\mu(X)$. Hence $f|U$ is supra pre-continuous. That is $f|U$ is gpr^μ-continuous.

(ii) Let $V \subset Y$ be a closed set. Then $f^{-1}(V) \in PC^\mu(X)$. Since $U \in SC^\mu(X)$, by lemma 45 (ii), $f^{-1}(V) \bigcap U = (f|U)^{-1}(V) \in PC^\mu(U)$. Hence $f|U$ is supra pre-continuous. That is $f|U$ is gpr^μ-continuous. □

References

[1] I. Arockiarani and M. Trinita Pricilla, On generalized b-regular closed sets in supra topological spaces, *Asian Journal of Current Engineering and Maths,* **1(1)** (2012), 1-4.

[2] K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological spaces, *Mem Fac. Sci. Kochi Univ. Ser A. Math*, **12** (1991), 5-13.

[3] Y. Gnanambal and K. Balachandran, On gpr-continuous functions in topological spaces, *Indian J. Pure Appl Math,* **30(6)** (1999), 581-593.

[4] Kangweon-Kyungki, The generalized open sets on supra topology, *Math. Jour,* **10(1)** (2002), 25-28.

[5] N. Levine, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo,* **19** (1970), 89-96.

[6] H. Maki, J. Umehara and T. Noiri, Every topological space is pre-$T_{1/2}$, *Mem. Fac. Sci. Kochi Univ. Ser. A. Math,* **17** (1996), 33-42.

[7] A. S. Mashour, A. A. Allam, F. S. Mahmoud and F. H. Khedr, On supra topological spaces, *Indian J. Pure and Appl. Math,* **14(4)** (1983), 502-510.

[8] O. Ravi, G. Ramkumar and M. Kamaraj, On supra g-closed sets, *International Journal of Advances in Pure and Applied Mathematics,* **1** (2011), 52-66.

[9] O. Ravi, G. Ramkumar and M. Kamaraj, Two classes of supra generalized closed sets, *Global Journal of Advances in Pure and Applied Mathematics,* **1** (2012), 24-33.

[10] O. R. Sayed, Supra pre-open sets and supra pre continuity on topological spaces, *Scientific Studies and Research, Series Mathematics and Informatics*, **20(2)** (2010), 79-88.

[11] Vidhya Menon, On generalized preregular closed sets in supra topological spaces, *International Journal of Scientific & Engineering Research*, **3(11)** (2012), 2229-5518.