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Corrigendum to "On (L, M)-Fuzzy Convex Structures"

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Abstract. In this paper, we point out that the proof of Theorem 2.4(5), Proposition 2.6(1) and Proposition 2.8(1) in the paper titled "On (L, M)-fuzzy convex structures" (Filomat 33(13): 4151-4163, 2019) are not true in general. Then, we give three correct proofs of these results.

1. Introduction

Sayed et al.[4] defined a new class of L-fuzzy sets called r-L-fuzzy biconvex sets in (L, M)-fuzzy convex structures. The transformation method between L-fuzzy hull operators and (L, M)-fuzzy convex structures were introduced, and a characterization of the product of the L-fuzzy hull operator was obtained. The aim of this article is to correct some errors in the proof of Theorem 2.4(5),Proposition 2.6(1) and Proposition 2.8(1) proposed by Sayed et al. ([4]).

2. Preliminaries

Throughout this paper, let X be a non-empty set, both L and M be two completely distributive lattices with order reversing involution $'$ where $\perp_M$ $(\perp_L)$ and $\top_M(\top_L)$ denote the least and the greatest elements in $M(L)$ respectively, and $M_{\perp_M} = M - \{\perp_M\}(L_{\perp_L} = L - \{\perp_L\})$. Recall that an order-reversing involution $'$ on L is a map $(-)' : L \longrightarrow L$ such that for any $a, b \in L$, the following conditions hold: (1) $a \leq b$ implies $b' \leq a'$. (2) $a'' = a$. The following properties hold for any subset $\{b_i : i \in I\} \in L$: (1) $(\bigvee_{i\in I} b_i)' = \bigwedge_{i\in I} b_i'$; (2) $(\bigwedge_{i\in I} b_i)' = \bigvee_{i\in I} b_i'$. An L-fuzzy subset of X is a mapping $\mu : X \longrightarrow L$ and the family L^X denoted the set of all fuzzy subsets of a given X ([1]). The least and the greatest elements in L^X are denoted by $\chi_\emptyset$ and χ_X, respectively. For each $\alpha \in L$, let $\underline{\alpha}$ denote the constant L-fuzzy subset of X with the value α. The complementation of a fuzzy subset are defined as $\mu'(x) = (\mu(x))'$ for all $x \in X$,(e.g. $\mu'(x) = 1 - \mu(x)$in the case of$L = [0, 1]$). We say $\{\mu_i : i \in \Gamma\}$ is a directed (resp. co-directed) subset of L^X, in symbols $\{\mu_i : i \in \Gamma\} \overset{dir}{\subseteq} L^X$ (resp. $\{\mu_i : i \in \Gamma\} \overset{cdir}{\subseteq} L^X$) if

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for each $\mu_1, \mu_2 \in \{\mu_i : i \in \Gamma\}$, there exists $\mu_3 \in \{\mu_i : i \in \Gamma\}$ such that $\mu_1, \mu_2 \leq \mu_3$ (resp. $\mu_1, \mu_2 \geq \mu_3$). An element $a \neq \perp_M$ in a lattice is called coprime if $a \leq b \vee c$ implies $a \leq b$ or $a \leq c$ for all $b, c \in M$. Further, a is said to be join-irreducible if $a = b \vee c$ implies $a = b$ or $a = c$ for all $b, c \in M$. The set of all coprime elements (resp. join-irreducible elements) of M is denoted Copr(M) (resp. $J(M)$). It can be verified that if M is distributive, then $a \in M$ is coprime iff it is join-irreducible, which means Copr(M) = $J(M)$. So, for convenience, we usually use $J(M)$ to stand for the set of all coprime elements of M if M is distributive. If M is a completely distributive lattice and $x \triangleleft \bigvee_{t\in T} y_t$, then there must be $t^\star \in T$ such that $x \triangleleft y_{t^\star}$ (here $x \triangleleft a$ means: $K \subset M, a \leq \bigvee K \Rightarrow \exists y \in K$ such that $x \leq y$), and for each $b \in M$, $b = \bigvee\{a \in M : a \triangleleft b\} = \bigvee\{a \in J(M) : a \triangleleft b\}$. Some more properties of $\triangleleft$ can be found in [2] and [6].

First, we recall two definitions which will be used in this paper.

Definition 2.1. ([5]) The pair $(X, \mathcal{C})$ is called an (L, M)-fuzzy convex structure ((L, M)-fcs, for short), where $\mathcal{C} : L^X \longrightarrow M$ satisfying the following axioms:

(LMC1) $\mathcal{C}(\underline{0}) = \mathcal{C}(\underline{1}) = \top_M$.

(LMC2) If $\{\mu_i : i \in \Gamma\} \subseteq L^X$ is nonempty, then

$$\mathcal{C}(\bigwedge_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \mathcal{C}(\mu_i).$$

(LMC3) If $\{\mu_i : i \in \Gamma\} \subseteq L^X$ is nonempty and totally ordered by inclusion, then

$$\mathcal{C}(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \mathcal{C}(\mu_i).$$

The mapping $\mathcal{C}$ is called an (L, M)-fuzzy convexity on X and $\mathcal{C}(\mu)$ can be regarded as the degree to which μ is an L-convex fuzzy set.

Definition 2.2. ([3]) Let $f : X \longrightarrow Y$. Then the image $f^{\rightarrow}(\mu)$ of $\mu \in L^X$ and the preimage $f^{\leftarrow}(\nu)$ of $\nu \in L^Y$ are defined by:

$$f^{\rightarrow}(\mu)(y) = \bigvee\{\mu(x) : x \in X, f(x) = y\}$$

and $f^{\leftarrow}(\nu) = \nu \circ f$, respectively. It can be verified that the pair $(f^{\rightarrow}, f^{\leftarrow})$ is a Galois connection on $(L^X, \leq)$ and $(L^Y, \leq)$.

Next, we recall Theorem 2.4, Proposition 2.6 and Proposition 2.8 of [4] as follows.

Theorem 2.3. ([4, Theorem 2.4]) *Let $(X, \mathcal{C})$ be an (L, M)-fuzzy convex structure. For each $\mu \in L^X$ and $r \in M_{\perp_M}$, we define a mapping $CO_{\mathcal{C}} : L^X \times M_{\perp_M} \longrightarrow L^X$ as follows:*

$$CO_{\mathcal{C}}(\mu, r) = \bigwedge\{\nu \in L^X : \mu \leq \nu,\ \mathcal{C}(\nu) \geq r\}.$$

For $\mu, \nu \in L^X$ and $r, s \in M_{\perp_M}$ the operator $CO_{\mathcal{C}}$ satisfies the following conditions:

(1) $CO_{\mathcal{C}}(\underline{0}, r) = \underline{0}$.

(2) $\mu \leq CO_{\mathcal{C}}(\mu, r)$.

(3) If $\mu \leq \nu$, then $CO_{\mathcal{C}}(\mu, r) \leq CO_{\mathcal{C}}(\nu, r)$.

(4) If $r \leq s$, then $CO_{\mathcal{C}}(\mu, r) \leq CO_{\mathcal{C}}(\mu, s)$.

(5) $CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) = CO_{\mathcal{C}}(\mu, r)$.

(6) For $\{\mu_i : i \in \Gamma\} \subseteq L^X$ is nonempty and totally ordered by inclusion,

$$CO_{\mathcal{C}}(\bigvee_{i\in\Gamma} \mu_i, r) = \bigvee_{i\in\Gamma} CO_{\mathcal{C}}(\mu_i, r).$$

A mapping $CO_{\mathcal{C}}$ is called L-fuzzy hull operator.

Proposition 2.4. ([4, Proposition 2.6(1)]) *Let $(X, \mathcal{C}_1, \mathcal{C}_2)$ be an (L, M)-fbcs. For each $r \in M_{\perp_M}$ and $\mu \in L^X$, a mapping $\mathcal{C}_{CO_{12}} : L^X \longrightarrow M$ is defined as follows*

$$\mathcal{C}_{CO_{12}}(\mu) = \bigvee \{r \in M_{\perp_M} : \mu = CO_{12}(\mu, r)\},$$

where $CO_{12}(\mu, r) = CO_{\mathcal{C}1}(\mu, r) \wedge CO_{\mathcal{C}2}(\mu, r)$ satisfies the conditions (1)-(6) of Theorem 2.3 (see [4]). Then $\mathcal{C}_{CO_{12}}$ is an (L, M)-fuzzy convexity on X.

Proposition 2.5. ([4, Proposition 2.8]) *Let $(X, \mathcal{C})$ and $(Y, \mathcal{D})$ be (L, M)-fuzzy convex structures. Then $f : X \longrightarrow Y$ is*

(1) An (L, M)-fuzzy convexity preserving function if and only if $f^{\rightarrow}(CO_{\mathcal{C}}(\mu, r)) \leq CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r)$ for all $\mu \in L^X$ and $r \in M_{\perp_M}$.

(2) An (L, M)-fuzzy convex-to-convex function if and only if $CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r) \leq f^{\rightarrow}(CO_{\mathcal{C}}(\mu, r))$ for all $\mu \in L^X$ and $r \in M_{\perp_M}$.

3. Main Results

First, we point out that the proof of Theorem 2.4(5), Proposition 2.6(1) and Proposition 2.8(1) are not true in general (see [4]). Here is why:

Notice that L (M) is a completely distributive lattice, not a unit interval [0,1]. So, if $a \nleq b$, it doesn't imply $a > b$. Because there exists another case that a and b may are not compparable, i.e., $a \parallel b$.

Now, we provide three correct proofs of these results as follows.

Proposition 3.1. ([4, Theorem 2.4(5)]) *Let $(X, \mathcal{C})$ be an (L, M)-fuzzy convex structure. For each $\mu \in L^X$ and $r \in M_{\perp_M}$, we define a mapping $CO_{\mathcal{C}} : L^X \times M_{\perp_M} \longrightarrow L^X$ as follows:*

$$CO_{\mathcal{C}}(\mu, r) = \bigwedge \{\nu \in L^X : \mu \leq \nu,\ \mathcal{C}(\nu) \geq r\}.$$

Then

$$CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) = CO_{\mathcal{C}}(\mu, r).$$

Proof. For all $\mu \in L^X$ and $r \in M_{\perp_M}$. By the definition of $CO_{\mathcal{C}}(\mu, r)$, we have $\mu \leq CO_{\mathcal{C}}(\mu, r)$. Hence, $CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) \geq CO_{\mathcal{C}}(\mu, r)$.

On the other hand,

$$\begin{aligned} CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) &= CO_{\mathcal{C}}\left(\bigwedge \left\{\nu \in L^X : \mu \leq \nu,\ \mathcal{C}(\nu) \geq r\right\}, r\right) \\ &\leq \bigwedge_{\mu \leq \nu,\ \mathcal{C}(\nu) \geq r} CO_{\mathcal{C}}(\nu, r) \\ &= \bigwedge_{\mu \leq \nu,\ \mathcal{C}(\nu) \geq r} \ \bigwedge_{\nu \leq \varpi,\ \mathcal{C}(\varpi) \geq r} \varpi \\ &= \bigwedge_{\mu \leq \varpi,\ \mathcal{C}(\varpi) \geq r} \varpi \\ &= CO_{\mathcal{C}}(\mu, r). \end{aligned}$$

Hence $CO_{\mathcal{C}}(CO_{\mathcal{C}}(\mu, r), r) = CO_{\mathcal{C}}(\mu, r)$. □

Proposition 3.2. ([4, Proposition 2.6(1)]) *Let $(X, \mathcal{C}_1, \mathcal{C}_2)$ be an (L, M)-fbcs. For each $r \in M_{\perp_M}$ and $\mu \in L^X$, a mapping $\mathcal{C}_{CO_{12}} : L^X \longrightarrow M$ is defined as follows*

$$\mathcal{C}_{CO_{12}}(\mu) = \bigvee \{r \in M_{\perp_M} : \mu = CO_{12}(\mu, r)\}.$$

Then $\mathcal{C}_{CO_{12}}$ is an (L, M)-fuzzy convexity on X.

Proof. (LMC1) Since for all $r \in M_{\perp_M}$, $CO_{12}(\underline{1}, r) \geq \underline{1}$ and $CO_{12}(\underline{0}, r) = \underline{0}$, we have

$$\mathcal{C}_{CO_{12}}(\underline{0}) = \mathcal{C}_{CO_{12}}(\underline{1}) = \top_M.$$

(LMC2) Suppose that $b \in M$ and $b \triangleleft \bigwedge_{i\in\Gamma} \mathcal{C}_{CO_{12}}(\mu_i)$. Then $b \triangleleft \mathcal{C}_{CO_{12}}(\mu_i)$ for all $i \in \Gamma$. There exists $r_0^i \in M_{\perp_M}$ such that $\mu_i = CO_{12}(\mu_i, r_0^i)$ and $b \triangleleft r_0^i$ (thus $b \leq r_0^i$). Put $r_0 = \bigwedge_{i\in\Gamma} r_0^i$, then $b \leq r_0$. Since CO_{12} satisfies the conditions (1)-(6) of Theorem 2.3, we have $CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0^i) \leq CO_{12}(\mu_i, r_0^i)$ for all $i \in \Gamma$. Then it follows that

$$CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0) \leq CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0^i) \leq \bigwedge_{i\in\Gamma} CO_{12}(\mu_i, r_0^i) = \bigwedge_{i\in\Gamma} \mu_i.$$

On the other hand, by Theorem 2.3 (2), we have

$$CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0) \geq \bigwedge_{i\in\Gamma} \mu_i.$$

So, we obtain

$$CO_{12}(\bigwedge_{i\in\Gamma} \mu_i, r_0) = \bigwedge_{i\in\Gamma} \mu_i.$$

By the definition of $\mathcal{C}_{CO_{12}}(\bigwedge_{i\in\Gamma} \mu_i)$, we obtain $\mathcal{C}_{CO_{12}}(\bigwedge_{i\in\Gamma} \mu_i) \geq r_0 \geq b$. Hence

$$\mathcal{C}_{CO_{12}}(\bigwedge_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \mathcal{C}_{CO_{12}}(\mu_i).$$

(LMC3) Let $\{\mu_i : i \in \Gamma\} \overset{cdir}{\subseteq} L^X$. Suppose that $b \in M$ and $b \triangleleft \bigwedge_{i\in\Gamma} \mathcal{C}_{CO_{12}}(\mu_i)$. Then $b \triangleleft \mathcal{C}_{CO_{12}}(\mu_i)$ for all $i \in \Gamma$. There exists $r_0^i \in M_{\perp_M}$ such that $\mu_i = CO_{12}(\mu_i, r_0^i)$ and $b \triangleleft r_0^i$ (thus $b \leq r_0^i$). Put $r_0 = \bigwedge_{i\in\Gamma} r_0^i$, then $b \leq r_0$. By Theorem 2.3 (6), we have

$$\bigvee_{i\in\Gamma} \mu_i \leq CO_{12}(\bigvee_{i\in\Gamma} \mu_i, r_0) \leq CO_{12}(\bigvee_{i\in\Gamma} \mu_i, r_0^i) = \bigvee_{i\in\Gamma} CO_{12}(\mu_i, r_0^i) = \bigvee_{i\in\Gamma} \mu_i.$$

So, we obtain

$$CO_{12}(\bigvee_{i\in\Gamma} \mu_i, r_0) = \bigvee_{i\in\Gamma} \mu_i.$$

By the definition of $\mathcal{C}_{CO_{12}}(\bigvee_{i\in\Gamma} \mu_i)$, we obtain $\mathcal{C}_{CO_{12}}(\bigvee_{i\in\Gamma} \mu_i) \geq r_0 \geq b$. Hence $\mathcal{C}_{CO_{12}}(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \mathcal{C}_{CO_{12}}(\mu_i)$. □

Proposition 3.3. ([4, Proposition 2.8(1)]) *Let $(X, \mathcal{C})$ and $(Y, \mathcal{D})$ be (L, M)-fuzzy convex structures. Then $f : X \longrightarrow Y$ is an (L, M)-fuzzy convexity preserving function if and only if $f^{\rightarrow}(CO_{\mathcal{C}}(\mu, r)) \leq CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r)$ for all $\mu \in L^X$ and $r \in M_{\perp_M}$.*

Proof. ($\Longrightarrow$) Since $f : X \longrightarrow Y$ is an (L, M)-fuzzy convexity preserving function, we obtain $\mathcal{C}(f^{\leftarrow}(\varpi)) \geq \mathcal{D}(\varpi)$ for all $\varpi \in L^Y$. So, for each $r \in M_{\perp_M}$ and $\mu \in L^X$, we obtain

$$\begin{aligned} f^{\leftarrow}[CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r)] &= f^{\leftarrow}\left[\bigwedge \left\{\varpi \in L^Y : f^{\rightarrow}(\mu) \leq \varpi,\ \mathcal{D}(\varpi) \geq r\right\}\right] \\ &= \bigwedge \left\{f^{\leftarrow}(\varpi) \in L^X : f^{\rightarrow}(\mu) \leq \varpi,\ \mathcal{D}(\varpi) \geq r\right\} \\ &\geq \bigwedge \left\{f^{\leftarrow}(\varpi) \in L^X : \mu \leq f^{\leftarrow}(\varpi),\ \mathcal{C}(f^{\leftarrow}(\varpi)) \geq r\right\} \\ &\geq \bigwedge \left\{\nu \in L^X : \mu \leq \nu,\ \mathcal{C}(\nu) \geq r\right\} = CO_{\mathcal{C}}(\mu, r). \end{aligned}$$

Hence

$$f^{\longrightarrow}(CO_{\mathcal{C}}(\mu, r)) \leq f^{\longrightarrow} f^{\leftarrow}[CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r)] \leq CO_{\mathcal{D}}(f^{\rightarrow}(\mu), r).$$

($\Longleftarrow$) Suppose that $b \in J(M)$ and $b \lhd \mathcal{D}(\varpi)$ for all $\varpi \in L^Y$, then $b \leq \mathcal{D}(\varpi)$. So,

$$f^{\rightarrow}(CO_{\mathcal{C}}(f^{\leftarrow}(\varpi), b)) \leq CO_{\mathcal{D}}(f^{\rightarrow}(f^{\leftarrow}(\varpi)), b) \leq CO_{\mathcal{D}}(\varpi, b) = \varpi.$$

It follows that

$$f^{\leftarrow}(\varpi) \leq CO_{\mathcal{C}}(f^{\leftarrow}(\varpi), b) \leq f^{\leftarrow}(\varpi).$$

Therefore, $CO_{\mathcal{C}}(f^{\leftarrow}(\varpi), b) = f^{\leftarrow}(\varpi)$. Furthermore,

$$\mathcal{C}(f^{\leftarrow}(\varpi)) = \mathcal{C}(CO_{\mathcal{C}}(f^{\leftarrow}(\varpi), b)) = \mathcal{C}\left(\bigwedge \left\{\nu \in L^X : f^{\leftarrow}(\varpi) \leq \nu,\ \mathcal{C}(\nu) \geq b\right\}\right) \geq \bigwedge_{f^{\leftarrow}(\varpi) \leq \nu,\ \mathcal{C}(\nu) \geq b} \mathcal{C}(\nu) \geq b.$$

Hence $\mathcal{C}(f^{\leftarrow}(\varpi)) \geq \mathcal{D}(\varpi)$ and $f : X \longrightarrow Y$ is an (L, M)-fuzzy convexity preserving function. □

4. Conclusion

In this paper, we point out that the proof of Theorem 2.4(5),Proposition 2.6(1) and Proposition 2.8(1) in [4] are incorrect, and then, we present the modified versions.

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