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Generalized δ-$s\bigwedge_{ij}$-sets in bitopological spaces

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Abstract. The concepts of $ij-\delta-$semi closed and $ij-\delta-$semi open sets in bitopological spaces are introduced and studied. Also, the notions of $\delta-s\bigwedge_{ij}-$sets and $g\delta-s\bigwedge_{ij}-$sets are investigated. Furthermore , a new closure operator called $Cl_\delta^{s\bigwedge_{ij}}$ on the bitopological space (X,τ_1,τ_2) is defined and associated topology $\tau_\delta^{s\bigwedge_{ij}}$ is given.

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Contents

1. Introduction

In 1963, Kelly [4], initiated the definition of a bitopological space as a triple (X,τ_1,τ_2), where *X* is a nonempty set and τ_1 and τ_2 are topologies on *X*. In 1981, Bose [2], introduced the concept of ij-semi open sets in bitopological spaces. In 1987, Banerjee [1], gave the notion of $ij-\delta-$open sets in such spaces.Also, investigations of $ij-\delta-$open sets were found in [5, 6]. In this paper, we introduce and study $ij-\delta-$semi closed and $ij-\delta-$semi open sets in bitopological spaces. Also, we introduce and study the notions of $\delta-s\bigwedge_{ij}$-sets and $g\delta-s\bigwedge_{ij}-$sets in bitopological spaces by generalizing the results obtained in [3]. Furthermore, we define a closure operator $Cl_\delta^{s\bigwedge_{ij}}$ and associated topology $\tau_\delta^{s\bigwedge_{ij}}$ on the bitopological space (X,τ_1,τ_2).

Throughout this paper (X,τ_1,τ_2) (or briefly X) always mean a bitopological space on which no separation axioms are assumed unless explicitly stated. Let *A* be a subset of *X*, by $i-Cl(A)$ and $i-Int(A)$ we denote the closure and the interior of *A* in the topological space (X,τ_i). By *i*-open (or τ_i-open) and *i*-closed (or τ_i-closed) we mean open and closed in the topological space (X,τ_i). $X\backslash A=A^c$ will be denote the complement of *A* and *I* denote for an index set. Also $i,j=1,2$ and $i\neq j$. Let A be a subset of a bitopological space (X,τ_1,τ_2) . A point $x\in X$ is called an $ij-\delta-$cluster point [1] of *A* if $i-Int(j-Cl(U))\cap A\neq\phi$ for every τ_i-open set U containing *x*. The set of all $ij-\delta-$cluster points of *A* is called the $ij-\delta-$closure of *A* and is denoted by $ij-Cl_\delta(A)$. A subset *A* is said to be $ij-\delta-$closed if $ij-Cl_\delta(A)=A$. The complement of an $ij-\delta-$closed set is called $ij-\delta-$open. A subset *A* of *X* is called $ij-$semi open [2] if $A\subset j-Cl(i-Int(A))$.

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2. ij-δ-semi open sets

Definition 2.1. *A subset A of bitopological space (X, τ_1, τ_2) is called $ij-\delta-$semi open if there exists $ij-\delta$-open set U such that $U \subset A \subset j - Cl(U)$. The complement of an $ij - \delta-$semi open set is called $ij - \delta$-semi closed.*

A point $x \in X$ is called an $ij - \delta$-semi cluster point of *A* if $A \cap U \neq \phi$ for every $ij - \delta$-semi open set *U* of *X* containing *x*. The set of all $ij - \delta-$semi cluster points of *A* is called the $ij - \delta-$semi closure of *A* and is denoted by $ij - \delta sCl(A)$. The collection of all $ij - \delta$-semi open (resp. $ij - \delta$-semi closed) sets of *X* will be denoted by $ij - \delta SO(X)$ (resp. $ij - \delta SC(X)$).

A subset *U* of *X* is called $ij - \delta$-semi neighborhood (briefly, $ij - \delta$-semi nbd) of a point *x* if there exists an $ij - \delta$-semi open set *V* such that $x \in V \subseteq U$.

Lemma 2.2. *The union of arbitrary collection of $ij - \delta-$semi open sets in (X, τ_1, τ_2) is $ij - \delta$-semi open.*

Proof. Since arbitrary union of $ij - \delta-$open sets is $ij - \delta$-open [4, Lemma 2.2], the result follows. ■

Lemma 2.3. *The intersection of arbitrary collection of $ij-\delta$-semi closed sets in (X, τ_1, τ_2) is $ij-\delta$-semi closed.*

Proof. Follows from Lemma 2.1. ■

Corollary 2.4. *Let $A \subset X$, $ij - \delta sCl(A) = \bigcap\{F : A \subseteq F,\ F \in ij - \delta SC(X)\}$.*

Corollary 2.5. *$ij - \delta sCl(A)$ is $ij - \delta$-semi closed, that is $ij - \delta sCl(ij - \delta sCl(A))$ =$ij - \delta sCl(A)$.*

Lemma 2.6. *Let (X, τ_1, τ_2) be a bitopological space. For subsets A, B and $A_k (k \in \Lambda)$ of* X, *we have*

(1) $A \subseteq ij - \delta sCl(A)$.
(2) $A \subseteq B \Rightarrow ij - \delta sCl(A) \subseteq ij - \delta sCl(B)$.
(3) $ij - \delta sCl(\bigcap_k A_k) \subseteq \bigcap_k ij - \delta sCl(A_k)$.
(4) $ij - \delta sCl(\bigcup_k A_k) = \cup_k \{ij - \delta sCl(A_k)\}$.
(5) A is $ij - \delta$-semi closed if and only if $A = ij - \delta sCl(A)$

3. $\delta - s\bigwedge_{ij}$ -sets and $g\delta - s\bigwedge_{ij}$-sets.

Definition 3.1. *For a subset B of a bitopological space (X, τ_1, τ_2), we define*

$B_\delta^{s\bigwedge_{ij}} = \bigcap\{O \in ij - \delta SO(X), B \subseteq O\}$
$B_\delta^{s\bigvee_{ij}} = \bigcup\{F \in ij - \delta SC(X), F \subseteq B\}$.

Definition 3.2. *A subset B of a bitopological space (X, τ_1, τ_2) is called $\delta - s\bigwedge_{ij}$-set (resp. $\delta - s\bigvee_{ij}$-set) if $B = B_\delta^{s\bigwedge_{ij}}$(resp. $B = B_\delta^{s\bigvee_{ij}}$).*

Definition 3.3. *A subset B of a bitopological space (X, τ_1, τ_2) is called*

(1) generalized $\delta - s\bigwedge_{ij}$-set (briefly, $g\delta - s\bigwedge_{ij}$-set) if $B_\delta^{s\bigwedge_{ij}} \subseteq F$ whenever $B \subseteq F$ and $F \in ji - \delta SC(X)$.

(2) generalized $\delta - s\bigvee_{ij}$-set (briefly, $g\delta - s\bigvee_{ij}$-set) if B^c is $g\delta - s\bigwedge_{ij}$.

By $G_\delta^{s\bigwedge_{ij}}$(resp. $G_\delta^{s\bigvee_{ij}}$) we will denote the family of all $g\delta - s\bigwedge_{ij}$-sets (resp. $g\delta - s\bigvee_{ij}$-sets).

Theorem 3.4. *Let A, B and B_k, $k \in I$ be subsets of a bitopological space (X, τ_1, τ_2). The following properties hold:*

(1) $B \subseteq B_\delta^{s\bigwedge_{ij}}$.

(2) If $A \subseteq B$, then $A_\delta^{s\bigwedge_{ij}} \subseteq B_\delta^{s\bigwedge_{ij}}$.

(3) $\left(\left(B_\delta^{s\bigwedge_{ij}}\right)_\delta\right)^{s\bigwedge_{ij}} = B_\delta^{s\bigwedge_{ij}}$.

(4) $(\bigcup_{k\in I} B_\lambda)_\delta^{s\bigwedge_{ij}} = \bigcup_{k\in I}(B_k)_\delta^{s\bigwedge_{ij}}$.

(5) If $A \in ij-\delta SO(X)$, then $A = A_\delta^{s\bigwedge_{ij}}$.

(6) $(B^c)_\delta^{s\bigwedge_{ij}} = \left(B_\delta^{s\bigvee_{ij}}\right)^c$.

(7) $B_\delta^{s\bigvee_{ij}} \subseteq B$.

(8) If $B \in ij-\delta SC(X)$, then $B = B_\delta^{s\bigvee_{ij}}$.

(9) $(\bigcap_{k\in I} B_k)_\delta^{s\bigwedge_{ij}} \subseteq \bigcap_{k\in I}(B_k)_\delta^{s\bigwedge_{ij}}$.

(10) $(\bigcup_{k\in I} B_k)_\delta^{s\bigvee_{ij}} \supseteq \bigcup_{k\in I}(B_k)_\delta^{s\bigvee_{ij}}$.

Proof. (1) Clear.

(2) Suppose $x \notin B_\delta^{s\bigwedge_{ij}}$. Then there exists an $ij-\delta$-semi open set U such that $B \subseteq U$ and $x \notin U$. Since $A \subseteq B$, then $x \notin A_\delta^{s\bigwedge_{ij}}$ and therefore $A_\delta^{s\bigwedge_{ij}} \subseteq B_\delta^{s\bigwedge_{ij}}$.

(3) Follows from (2).

(4) Let $x \notin \left(\bigcup_{k\in I} B_k\right)_\delta^{s\bigwedge_{ij}}$. Then there exists an $ij-\delta-$semi open set U such that $\bigcup_{k\in I} B_k \subseteq U$ and $x \notin U$. Thus for each $k \in I$ we have $x \notin (B_k)_\delta^{s\bigwedge_{ij}}$. So, $x \notin \bigcup_{k\in I}(B_k)_\delta^{s\bigwedge_{ij}}$.

Conversely, suppose that $x \notin \bigcup_{k\in I}(B_k)_\delta^{s\bigwedge_{ij}}$. Then there exists an $ij-\delta$-semi open set U_k (for each $k \in I$) such that $x \notin U_k$, $B_k \subseteq U_k$. Let $U = \bigcup_{k\in I} U_k$. Then, $x \notin U = \bigcup_{k\in I} U_k$, $\bigcup_{k\in I} B_k \subseteq U$ and U is $ij-\delta$-semi open. So, $x \notin \left(\bigcup_{k\in I} B_k\right)_\delta^{s\bigwedge_{ij}}$. This completes the proof of (4).

(5) Since A is an $ij-\delta-$semi open set, then $A_\delta^{s\bigwedge_{ij}} \subseteq A$. By (1) , we have $A_\delta^{s\bigwedge_{ij}} = A$.

(6) $(B_\delta^{s\bigvee_{ij}})^c = \bigcap F^c : F^c \supseteq B^c, F^c \in ij-\delta SO(X) = (B^c)_\delta^{s\bigwedge_{ij}}$.

(7) Clear.

(8) If $B \in ij-\delta SC(X)$, $B^c \in ij-\delta SO(X)$. By (5) and (6) $B^c = (B^c)_\delta^{s\bigwedge_{ij}} = \left(B_\delta^{s\bigvee_{ij}}\right)^c$. Hence $B = B_\delta^{s\bigvee_{ij}}$.

(9) Let $x \notin \bigcap_{k\in I}(B_k)_\delta^{s\bigwedge_{ij}}$. Then there exists $k \in I$ such that $x \notin (B_k)_\delta^{s\bigwedge_{ij}}$. Hence there exists $U \in ij-\delta SO(X)$ such that $B_k \subseteq U$ and $x \notin U$. Therefore $x \notin \left(\bigcap_{k\in I} B_k\right)_\delta^{s\bigwedge_{ij}}$.

(10)$(\bigcup_{k\in I} B_k)_\delta^{s\bigvee_{ij}} = \left(((\bigcup_{k\in I} B_k)^c)_\delta^{s\bigvee_{ij}}\right)^c = \left((\bigcap_{k\in I} B_k^c)_\delta^{s\bigvee_{ij}}\right)^c \supseteq \left(\bigcap_{k\in I}\left((B_k)_\delta^{s\bigvee_{ij}}\right)^c\right)^c = \bigcup_{k\in I}(B_k)_\delta^{s\bigvee_{ij}}$. ■

Theorem 3.5. *Let B be a subset of a bitopological space (X, τ_1, τ_2). Then*

(1) ϕ and X are $\delta-s\bigwedge_{ij}$-sets and $\delta-s\bigvee_{ij}$-sets.

(2) Every union of $\delta-s\bigwedge_{ij}$-sets (resp. $\delta-s\bigvee_{ij}$-sets) is $\delta-s\bigwedge_{ij}$-sets (resp. $\delta-s\bigvee_{ij}$-sets).

(3) Every intersection of $\delta-s\bigwedge_{ij}$-sets (resp. $\delta-s\bigvee_{ij}$-sets) is $\delta-s\bigwedge_{ij}$-sets (resp. $\delta-s\bigvee_{ij}$-sets).

(4) B is a $\delta-s\bigwedge_{ij}$ $-$set if and only if B^c is a $\delta-s\bigvee_{ij}$-set.

Proof. (1) and (4) are obvious.

(2) Let $\{B_k : k \in I\}$ be a family of $\delta - s\bigwedge_{ij}$-sets in (X, τ_1, τ_2). Then by Theorem 3.1(4) we have $\bigcup_{k\in I} B_k = \bigcup_{k\in I} (B_k)_\delta^{s\bigwedge_{ij}} = \left(\bigcup_{k\in I} B_k\right)_\delta^{s\bigwedge_{ij}}$.

(3) Let $\{B_k : k \in I\}$ be a family of $\delta - s\bigwedge_{ij}$ −sets in (X, τ_1, τ_2). Then, by Theorem 3.1(9), we have $\left(\bigcap_{k\in I} B_k\right)_\delta^{s\bigwedge_{ij}} \subseteq \bigcap_{k\in I} (B_k)_\delta^{s\bigwedge_{ij}} = \bigcap_{k\in I} B_k$. Hence, by Theorem 3.1, $\bigcap_{k\in I} B_k = \left(\bigcap_{k\in I} B_k\right)_\delta^{s\bigwedge_{ij}}$. ■

Remark 3.6. *By Theorem 3.2, the family of all $\delta - s\bigwedge_{ij}$-sets (resp. $\delta - s\bigvee_{ij}$ −sets), denoted by $\lambda_\delta^{s\bigwedge_{ij}}$ (resp. $\lambda_\delta^{s\bigvee_{ij}}$) in (X, τ_1, τ_2) is a topology on X containing all $ij - \delta$-semi open (resp. $ij - \delta$-semi closed) sets. Clearly $(X, \lambda_\delta^{s\bigwedge_{ij}})$ and $(X, \lambda_\delta^{s\bigvee_{ij}})$ are Alexandroff spaces.*

Theorem 3.7. *Let (X, τ_1, τ_2) be a bitopological space. Then*

(1) Every $\delta - s\bigwedge_{ij}$-set is a $g\delta - s\bigwedge_{ij}$-set.

(2) Every $\delta - s\bigvee_{ij}$-set is a $g\delta - s\bigvee_{ij}$-set.

(3) If B_k is a $g\delta - s\bigwedge_{ij}$ −set for all $k \in I$ then $\bigcup_{k\in I} B_k$ is a $g\delta - s\bigwedge_{ij}$-set.

(4) If B_k is a $g\delta - s\bigvee_{ij}$-set for all $k \in I$ then $\bigcap_{k\in I} B_k$ is a $g\delta - s\bigvee_{ij}$-set.

Proof. (1) Obvious.

(2)Let *B* be a $\delta - s\bigvee_{ij}$-subset of *X*. Then $B = B_\delta^{s\bigvee_{ij}}$. By Theorem 3.1(6), $(B^c)_\delta^{s\bigwedge_{ij}} = \left(B_\delta^{s\bigvee_{ij}}\right)^c = B^c$. Therefore, by (1), *B* is a $g\delta - s\bigvee_{ij}$-set.

(3)Let B_k is a $g\delta - s\bigwedge_{ij}$-subset of *X* for all $k \in I$. Then by Theorem 3.1 (4), $\left(\bigcup_{k\in I} B_k\right)_\delta^{s\bigwedge_{ij}} = \bigcup_{k\in I} (B_k)_\delta^{s\bigwedge_{ij}}$. Hence, by hypothesis, $\bigcup_{k\in I} B_k$ is a $g\delta - s\bigwedge_{ij}$-set.

(4)Follows from (3). ■

Theorem 3.8. *A subset B of a bitopological space $(X, \tau_1, \tau_2)$is a $g\delta - s\bigvee_{ij}$-set if and only if $U \subseteq B_\delta^{s\bigvee_{ij}}$, whenever $U \subseteq B$ and U is an $ij - \delta$-semi open subset of X.*

Proof. Let *U* be an $ij - \delta$-semi open subset of *X* such that $U \subseteq B$. Then, since U^c is $ij - \delta$-semi closed and $B^c \subseteq U^c$, we have $(B^c)_\delta^{s\bigwedge_{ij}} \subseteq U^c$. Hence, by Theorem 3.1(6), $\left(B_\delta^{s\bigvee_{ij}}\right)^c \subseteq U^c$. Thus $U \subseteq B_\delta^{s\bigvee_{ij}}$. On the other hand, let *F* be an $ij - \delta$-semi closed subset of X such that $B^c \subseteq F$. Since F^c is $ij - \delta$-semi open and $F^c \subseteq B$, by assumption we have $F^c \subseteq B_\delta^{s\bigvee_{ij}}$. Then $F \supseteq \left(B_\delta^{s\bigvee_{ij}}\right)^c = (B^c)_\delta^{s\bigvee_{ij}}$. Thus B^c is a $g\delta - s\bigwedge_{ij}$-set, i.e., *B* is a $g\delta - s\bigvee_{ij}$-set. ■

4. $Cl_\delta^{s\bigwedge_{ij}}$ closure operator and associated $\tau_\delta^{s\bigwedge_{ij}}$

In this section, we define a closure operator $Cl_\delta^{s\bigwedge_{ij}}$ and the associated topology $\tau_\delta^{s\bigwedge_{ij}}$ on the bitopological space (X, τ_1, τ_2) using the family of $g\delta - s\bigwedge_{ij}$-sets.

Definition 4.1. *For any subset B of a bitopological space (X, τ_1, τ_2), define $Cl_\delta^{s\bigwedge_{ij}}(B) = \bigcap\{U : B \subseteq U, U \in G_\delta^{s\bigwedge_{ij}}\}$ and $Int_\delta^{s\bigwedge_{ij}}(B) = \bigcup\{F : B \supseteq F, F^c \in G_\delta^{s\bigwedge_{ij}}\}$.*

Theorem 4.2. *Let A, B and $B_k : k\epsilon I$ be subsets of a bitopological space (X, τ_1, τ_2). Then the following statements are true:*

(1) $B \subseteq Cl_\delta^{s\bigwedge_{ij}}(B)$.

(2) $Cl_\delta^{s\bigwedge_{ij}}(B^c) = \left(Int_\delta^{s\bigwedge_{ij}}(B)\right)^c$.

(3) $Cl_\delta^{s\bigwedge_{ij}}(\phi) = \phi$.

(4) $\bigcup_{k\in I} Cl_\delta^{s\bigwedge_{ij}}(B_k) = Cl_\delta^{s\bigwedge_{ij}}(\bigcup_{k\in I} B_k)$.

(5) $Cl_\delta^{s\bigwedge_{ij}}\left(Cl_\delta^{s\bigwedge_{ij}}(B)\right) = Cl_\delta^{s\bigwedge_{ij}}(B)$.

(6) If $A \subseteq B$*, then* $Cl_\delta^{s\bigwedge_{ij}}(A) \subseteq Cl_\delta^{s\bigwedge_{ij}}(B)$.

(7) If B is $g\delta - s\bigwedge_{ij}-$*set, then* $Cl_\delta^{s\bigwedge_{ij}}(B) = B$.

(8) If B is $g\delta - s\bigvee_{ij}$*-set, then* $Int_\delta^{s\bigwedge_{ij}}(B) = B$.

Proof. (1), (2) and (3) are clear.

(4) Let $x \notin Cl_\delta^{s\bigwedge_{ij}}(\bigcup_{k\in I} B_k)$. Then, there exists $U \in G_\delta^{s\bigwedge_{ij}}$ such that $\bigcup_{k\in I} B_k \subseteq U$ and $x \notin U$. Thus for each $k \in I$ we have $x \notin Cl_\delta^{s\bigwedge_{ij}}(B_k)$. This implies that $x \notin \bigcup_{k\in I} Cl_\delta^{s\bigwedge_{ij}}(B_k)$. Conversely, suppose $x \notin \bigcup_{k\in I} Cl_\delta^{s\bigwedge_{ij}}(B_k)$. Then there exist subsets $U_k \in G_\delta^{s\bigwedge_{ij}}$ for all $k \in I$ such that $x \notin U_k$ and $B_k \subseteq U_k$. Let $U = \bigcup_{k\in I} U_k$. Then $x \notin U$, $\bigcup_{k\in I} B_k \subseteq U$ and $U \in G_\delta^{s\bigwedge_{ij}}$. Thus, $x \notin Cl_\delta^{s\bigwedge_{ij}}(\bigcup_{k\in I} B_k)$.

(5) Suppose that $x \notin Cl_\delta^{s\bigwedge_{ij}}(B)$. Then there exists a subset $U \in G_\delta^{s\bigwedge_{ij}}$ such that $x \notin U$ and $B \subseteq U$. Since $U \in G_\delta^{s\bigwedge_{ij}}$ we have $Cl_\delta^{s\bigwedge_{ij}}(B) \subseteq U$. Thus we have $x \notin Cl_\delta^{s\bigwedge_{ij}}(Cl_\delta^{s\bigwedge_{ij}}(B))$. Therefore $Cl_\delta^{s\bigwedge_{ij}}\left(Cl_\delta^{s\bigwedge_{ij}}(B)\right) \subseteq Cl_\delta^{s\bigwedge_{ij}}(B)$. But by (6) $Cl_\delta^{s\bigwedge_{ij}}(B) \subseteq Cl_\delta^{s\bigwedge_{ij}}\left(Cl_\delta^{s\bigwedge_{ij}}(B)\right)$. Then the result follows.

(6) It is clear.

(7) Follows from (1).

(8) Follows from (7) and (2). ■

Theorem 4.3. $Cl_\delta^{s\bigwedge_{ij}}$ *is a Kuratowski closure operator on X.*

Definition 4.4. *Let* $\tau_\delta^{s\bigwedge_{ij}}$ *be the topology on X generated by* $Cl_\delta^{s\bigwedge_{ij}}$ *in the usual manner, i.e.,* $\tau_\delta^{s\bigwedge_{ij}} = \{B \subseteq X,\ Cl_\delta^{s\bigwedge_{ij}}(B^c) = B^c\}$.
We define a family $\rho_\delta^{s\bigwedge_{ij}}$ *by* $\rho_\delta^{s\bigwedge_{ij}} = \{B \subseteq X,\ Cl_\delta^{s\bigwedge_{ij}}(B) = B\}$*, equivalently* $\rho_\delta^{s\bigwedge_{ij}} = \{B \subseteq X,\ B^c \in \tau_\delta^{s\bigwedge_{ij}}\}$.

Theorem 4.5. *Let* (X, τ_1, τ_2) *be a bitopological space. Then*

(1) $\tau_\delta^{s\bigwedge_{ij}} = \{B \subseteq X,\ Int_\delta^{s\bigwedge_{ij}}(B) = B\}$.

(2) $ij - \delta SO(X) \subseteq G_\delta^{s\bigwedge_{ij}} \subseteq \rho_\delta^{s\bigwedge_{ij}}$.

(3) $ij - \delta SC(X) \subseteq G_\delta^{s\bigwedge_{ij}} \subseteq \tau_\delta^{s\bigwedge_{ij}}$.

(4) If $ij - \delta SC(X) = \tau_\delta^{s\bigwedge_{ij}}$*, then every* $g\delta - s\bigwedge_{ij}$*-set of X is* $ij - \delta$*-semi open.*

(5) If every $g\delta - s\bigwedge_{ij}$*-set of X is* $ij - \delta$*-semi open (i.e., if* $G_\delta^{s\bigwedge_{ij}} \subseteq ij - \delta SO(X)$*), then* $\tau_\delta^{s\bigwedge_{ij}} = \{B \subseteq X,\ B = B_\delta^{s\bigwedge_{ij}}\}$.

(6) If every $g\delta - s\bigwedge_{ij}$*-set of X is* $ij - \delta$*-semi closed (i.e., if* $G_\delta^{s\bigwedge_{ij}} \subseteq ij - \delta SC(X)$*), then* $ij - \delta SO(X) = \tau_\delta^{s\bigwedge_{ij}}$.

Proof. (1) By Theorem 4.1 (2), we have: If $A \subseteq X$, then $A \in \tau_\delta^{s \bigwedge_{ij}}$ if and only if $Cl_\delta^{s \bigwedge_{ij}}(A^c) = A^c$ if and only if $\left(Int_\delta^{s \bigvee_{ij}}(A)\right)^c = A^c$ if and only if $Int_\delta^{s \bigvee_{ij}}(A) = A$. Thus, $\tau_\delta^{s \bigwedge_{ij}} = \{B \subseteq X, Int_\delta^{s \bigvee_{ij}}(B) = B\}$.

(2) Let $B \in ij - \delta SO(X)$. By Theorem 3.1(5) *B* is a $\delta - s \bigwedge_{ij}$-set. By Theorem 3.3, *B* is a $g\delta - s \bigwedge_{ij}$ −set, i.e., $B \in G_\delta^{s \bigwedge_{ij}}$. Suppose *B* any element of $G_\delta^{s \bigwedge_{ij}}$. By Theorem 3.1, $B = Cl_\delta^{s \bigwedge_{ij}}(B)$, i.e., $B \in \rho_\delta^{s \bigwedge_{ij}}$. Therefore $ij - \delta SO(X) \subseteq G_\delta^{s \bigwedge_{ij}} \subseteq \rho_\delta^{s \bigwedge_{ij}}$.

(3) Let $B \in ij - \delta SC(X)$. By Theorem 3.3, $B = B_\delta^{s \bigvee_{ij}}$. Thus *B* is a $\delta - s \bigvee_{ij}$ −set. By Theorem 3.1, *B* is a $g\delta - s \bigvee_{ij}$ −set. Hence $B \in G_\delta^{s \bigvee_{ij}}$. Now, if $B \in G_\delta^{s \bigvee_{ij}}$, then by (1) and Theorem 3.4(8), $B \in \tau_\delta^{s \bigwedge_{ij}}$.

(4) Let *B* be any $g\delta - s \bigwedge_{ij}$-set, i.e., $B \in G_\delta^{s \bigwedge_{ij}}$. By (2), $B \in \rho_\delta^{s \bigwedge_{ij}}$. Thus, $B^c \in \tau_\delta^{s \bigwedge_{ij}}$. From assumption, we have $B^c \in ij - \delta SC(X)$. Hence $B \in ij - \delta SO(X)$.

(5) Let $A \subseteq X$ and $A \in \tau_\delta^{s \bigwedge_{ij}}$. Then, $A^c = Cl_\delta^{s \bigwedge_{ij}}(A^c) = \bigcap\{U : U \supseteq A, U \in G_\delta^{s \bigwedge_{ij}}\} = \bigcap\{U : U \supseteq A^c, U \in ij - \delta SO(X)\} = (A^c)_\delta^{s \bigwedge_{ij}}$. Using Theorem 3.1, we have $A = A_\delta^{s \bigvee_{ij}}$, i.e., $A \in \{B \subseteq X : B = B_\delta^{s \bigvee_{ij}}\}$.

Conversely, if $A \in \{B \subseteq X : B = B_\delta^{s \bigvee_{ij}}\}$, then by Theorem 3.3, A is a $g\delta - s \bigvee_{ij}$ −set. Thus $A \in G_\delta^{s \bigvee_{ij}}$. By using (3), $A \in \tau_\delta^{s \bigwedge_{ij}}$.

(6) Let $A \subseteq X$ and $A \in \tau_\delta^{s \bigwedge_{ij}}$. Then $A = \left(Cl_\delta^{s \bigwedge_{ij}}(A^c)\right)^c = \left(\bigcap\{U : A^c \subseteq U, U \in G_\delta^{s \bigwedge_{ij}}\}\right)^c = \bigcup\{U^c : U^c \in ij - \delta SO(X)\} \in ij - \delta SO(X)$.

Conversely, if $A \in ij - \delta SO(X)$, then by Theorems 3.1 and 3.3, $A \in G_\delta^{s \bigwedge_{ij}}$. By assumption $A \in ij - \delta SC(X)$. Using (3), $A \in \tau_\delta^{s \bigwedge_{ij}}$. ■

Lemma 4.6. *Let (X, τ_1, τ_2) be a bitopological space.*
(1)For each $x \in X$, $\{x\}$ is an $ij - \delta-$semi open set or $\{x\}^c$ is a $g\delta - s \bigwedge_{ij}$ −set of X.
(2)For each $x \in X$, $\{x\}$ is an $ij - \delta-$semi open set or $\{x\}$ is a $g\delta - s \bigvee_{ij}$ −set of X.

Proof. (1) Suppose that $\{x\}$ is not $ij - \delta-$semi open. Then the only $ij - \delta$-semi closed set *F* containing $\{x\}^c$ is *X*. Thus $(\{x\}^c)_\delta^{s \bigwedge_{ij}} \subseteq F = X$ and $\{x\}^c$ is a $g\delta - s \bigwedge_{ij}$ −set of *X*.
(2) Follows from (1). ■

Theorem 4.7. *If $ij - \delta SO(X) = \tau_\delta^{s \bigwedge_{ij}}$, then every singleton $\{x\}$ is $\tau_\delta^{s \bigwedge_{ij}}$-open.*

Proof. Suppose that $\{x\}$ is not $ij - \delta$-semi open. Then by Lemma 4.1, $\{x\}^c$ is a $g\delta - s \bigwedge_{ij}$-set. Thus $\{x\} \in \tau_\delta^{s \bigwedge_{ij}}$. Suppose that $\{x\}$ is $ij - \delta$-semi open. Then $\{x\} \in ij - \delta SO(X) = \tau_\delta^{s \bigwedge_{ij}}$. Therefore, every singleton $\{x\}$ is $\tau_\delta^{s \bigwedge_{ij}}$-open. ■

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