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SOME SEPARATION AXIOMS IN SOFT IDEAL TOPOLOGICAL SPACES

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ABSTRACT. For dealing with uncertainties researchers introduced the concept of soft sets. In this paper, a new class of soft sets called soft delta pre ideal open sets in soft ideal topological space related to the notion of soft pre ideal regular pre ideal open sets is introduced. Also, some new soft separation axioms based on the soft delta pre ideal open sets are investigated.

1. INTRODUCTION AND PRELIMINARIES

In 1999, Molodtsov [22] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. Also, he applied this theory to several directions (see, for example, [23-26]). The soft set theory has been applied to many different fields (see, for example, [1-2], [4-5], [7-9], [16-21], [27], [31-32], [34]). Later, few researches (see, for example, [3], [6], [10-11], [15], [22], [28-30], [33]) introduced and studied the notion of soft topological spaces. In [13-14], the authors initiated the notion of soft ideal. They also introduced the concept of soft local function. These concepts were discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal $(X, \widetilde{\tau}, E, \widetilde{I})$. EL-Sheikh [8] introduced the notions of $\widetilde{I}$-open soft sets, pre-$\widetilde{I}$- open soft sets, α-$\widetilde{I}$-open soft sets, semi-$\widetilde{I}$-open soft sets and β-$\widetilde{I}$-open soft sets to soft topological spaces. He studied the relations between these different types of subsets of soft topological spaces with soft ideal. Also, he introduced the concepts of $\widetilde{I}$-continuous soft, pre-$\widetilde{I}$-continuous soft, α-$\widetilde{I}$-continuous soft, semi-$\widetilde{I}$- continuous soft and β-$\widetilde{I}$-continuous soft functions and discussed their properties. Jafari [11] introduced the concept of pre regular preopen set in general topological space. This paper extends this set to soft topological spaces with soft ideal, soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set, and study some of its properties. Also, the concept of a soft delta pre-$\widetilde{I}$-open set is given and some of its properties are investigated. Finally, a soft delta pre-$\widetilde{I}$-separation axioms are given.

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Definition 1.1. [23]. Let X be an initial universe set, $P(X)$ the power set of X, that is, the set of all subsets of X, and A a set of parameters. A pair (F, A), where F is a map from A to $P(X)$, is called a soft set over X.
In what follows by $SS(X, A)$ we denote the family of all soft sets (F, A) over X.

Definition 1.2. [23] Let $(F, A), (G, A) \in SS(X, A)$. We say that the pair (F, A) is a soft subset of (G, A) if $F(p) \subseteq G(p)$, for every $p \in A$. Symbolically, we write $(F, A) \sqsubseteq (G, A)$. Also, we say that the pairs (F, A) and (G, A) are soft equal if $(F, A) \sqsubseteq (G, A)$ and $(G, A) \sqsubseteq (F, A)$. Symbolically, we write $(F, A) = (G, A)$.

Definition 1.3. [23]. Let Λ be an arbitrary index set and $\{(F_i, A) : i \in \Lambda\} \subseteq SS(X, A)$. Then

(1) The soft union of these soft sets is the soft set $(F, A) \in SS(X, A)$, where the map $F : A \to P(X)$ is defined as follows: $F(p) = \bigcup\{F_i(p) : i \in \Lambda\}$, for every $p \in A$. Symbolically, we write $(F, A) = \sqcup\{(F_i, A) : i \in \Lambda\}$.

(2) The soft intersection of these soft sets is the soft set $(F, A) \in SS(X, A)$, where the map $F : A \to P(X)$ is defined as follows: $F(p) = \bigcap\{F_i(p) : i \in \Lambda\}$, for every $p \in A$. Symbolically, we write $(F, A) = \sqcap\{(F_i, A) : i \in \Lambda\}$.

Definition 1.4. [33]. Let $(F, A) \in SS(X, A)$. The soft complement of (F, A) is the soft set $(H, A) \in SS(X, A)$, where the map $H : A \to P(X)$ defined as follows: $H(p) = X \backslash F(p)$, for every $p \in A$. Symbolically, we write $(H, A) = (F, A)^{c}$. Obviously, $(F, A)^{c} = (F^{c}, A)$ [10]. For two given subsets $(M, A), (N, A) \in SS(X, A)$ [30], we have

(i) $((M, A) \sqcup (N, A))^c = (M, A)^c \sqcap (N, A)^c$;
(ii) $((M, A) \sqcap (N, A))^c = (M, A)^c \sqcup (N, A)^c$.

Definition 1.5. [23]. The soft set $(F, A) \in SS(X, A)$, where $F(p) = \phi$, for every $p \in A$ is called the A-null soft set of $SS(X, A)$ and denoted by $\mathbf{0}_A$. The soft set $(F, A) \in SS(X, A)$, where $F(p) = X$, for every $p \in A$ is called the A-absolute soft set of $SS(X, A)$ and denoted by $\mathbf{1}_A$.

Definition 1.6. [33]. The soft set $(F, A) \in SS(X, A)$ is called a soft point in X, denoted by x_e, if for the element $e \in A$, $F(e) = \{x\}$ and $F(e') = \phi$ for all $e' \in A \backslash \{e\}$. The set of all soft points of X is denoted by $SP(X, E)$. The soft point x_e is said to be in the soft set (G, A), denoted by $x_e \tilde{\in} (G, A)$, if for the element $e \in A$ and $x \in F(e)$.

Definition 1.7. [33]. Let $SS(X, A)$ and $SS(Y, B)$ be families of soft sets. Let $u : X \to Y$ and $p : A \to B$ be mappings. Then the mapping $f_{pu} : SS(X, A) \to SS(Y, B)$ is defined as:

(1) The image of $(F, A) \in SS(X, A)$ under f_{pu} is the soft set $f_{pu}(F, A) = (f_{pu}(F), B)$ in $SS(Y, B)$ such that

$$f_{pu}(F)(y) = \begin{cases} \bigcup\limits_{x \in p^{-1}(y)} u(F(x)), & p^{-1}(y) \neq \phi \\ \phi, & \text{otherwise} \end{cases}$$

for all $y \in B$.

(2) The inverse image of $(G, B) \in SS(Y, B)$ under f_{pu} is the soft set $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), A)$ in $SS(X, A)$ such that $f_{pu}^{-1}(G)(x) = u^{-1}(G(p(x)))$ for all $x \in A$.

Proposition 1.1. [10]. *Let* $(F, A), (F_1, A) \in SS(X, A)$ *and* $(G, B), (G_1, B) \in SS(Y, B)$. *The following statements are true:*

(1) If $(F,A) \sqsubseteq (F_1,A)$*, then* $f_{pu}(F,A) \sqsubseteq f_{pu}(F_1,A)$.
(2) If $(G,B) \sqsubseteq (G_1,B)$*, then* $f_{pu}^{-1}(G,B) \sqsubseteq f_{pu}^{-1}(G_1,B)$.
(3) $(F,A) \sqsubseteq f_{pu}^{-1}(f_{pu}(F,A))$.
(4) $f_{pu}(f_{pu}^{-1}(G,B)) \sqsubseteq (G,B)$.
(5) $f_{pu}^{-1}((G,B)^c) = (f_{pu}^{-1}(G,B))^c$.
(6) $f_{pu}((F,A) \sqcup (F_1,A)) = f_{pu}(F,A) \sqcup f_{pu}(F_1,A)$.
(7) $f_{pu}((F,A) \sqcap (F_1,A)) \sqsubseteq f_{pu}(F,A) \sqcap f_{pu}(F_1,A)$.
(8) $f_{pu}^{-1}((G,B) \sqcup (G_1,B)) = f_{pu}^{-1}(G,B) \sqcup f_{pu}^{-1}(G_1,B)$.
(9) $f_{pu}^{-1}((G,B) \sqcap (G_1,B)) = f_{pu}^{-1}(G,B) \sqcap f_{pu}^{-1}(G_1,B)$.

Definition 1.8. [33]. Let X be an initial universe set, A a set of parameters, and $\widetilde{\tau} \subseteq SS(X,A)$. We say that the family $\widetilde{\tau}$ defines a soft topology on X if the following axioms are true:
(1) $\mathbf{0}_A, \mathbf{1}_A \in \widetilde{\tau}$.
(2) If $(G,A),(H,A) \in \widetilde{\tau}$, then $(G,A) \sqcap (H,A) \in \widetilde{\tau}$.
(3) If $(G_i,A) \in \widetilde{\tau}$ for every $i \in I$, then $\sqcup\{(G_i,A) : i \in I\} \in \widetilde{\tau}$.
The triplet $(X,\widetilde{\tau},A)$ is called a soft topological space. The members of $\widetilde{\tau}$ are called soft open sets in X. Also, a soft set (F,A) is called soft closed if the complement $(F,A)^c$ belongs to $\widetilde{\tau}$. The family of all soft closed sets is denoted by $\widetilde{\tau}^c$.

Definition 1.9. Let $(X,\widetilde{\tau},A)$ be a soft topological space and $(F,A) \in SS(X,A)$.
(1) The soft closure of (F,A) [30] is the soft set
$\widetilde{s}Cl(F,A) = \sqcap\{(S,A) : (S,A) \in \widetilde{\tau}^c, (F,A) \sqsubseteq (S,A)\}$.
(2) The soft interior of (F,A) [33] is the soft set
$\widetilde{s}Int(F,A) = \sqcup\{(S,A) : (S,A) \in \widetilde{\tau}, (S,A) \sqsubseteq (F,A)\}$.

Definition 1.10. [13].

(1) Let $\widetilde{I}$ be a non-null collection of soft sets over a universe X with a fixed set of parameters E. Then $\widetilde{I} \subseteq SS(X,E)$ is called a soft ideal on X with the same set E if
  (a) $(F,E),(G,E) \in \widetilde{I}$, then $(F,E) \sqcup (G,E) \in \widetilde{I}$;
  (b) $(F,E) \in \widetilde{I}$ and $(G,E) \sqsubseteq (F,E)$, then $(G,E) \in \widetilde{I}$,
(2) Let $(X,\widetilde{\tau},E)$ be a soft topological space and $\widetilde{I}$ be a soft ideal over X. Then $(X,\widetilde{\tau},E,\widetilde{I})$ is called a soft ideal topological space. Let $(F,E) \widetilde{\in} SS(X,E)$, the soft operator $*\ : SS(X,E) \to SS(X,E)$, defined by
$(F,E)^*(\widetilde{I}, \widetilde{\tau})$ or $(F,E)^* = \sqcup\{x_e \in SP(X,E) : O_{x_e} \sqcap (F,E) \notin \widetilde{I}\ \forall\, O_{x_e} \in \widetilde{\tau}\}$
is called the soft local function of (F,E) with respect to $\widetilde{I}$ and $\widetilde{\tau}$, where O_{x_e} is a $\widetilde{\tau}$-soft open set containing x_e.

Theorem 1.2. [13]. *Let* $(X,\widetilde{\tau},E,\widetilde{I})$ *be a soft ideal topological space and* $(F,E),\ (U,E) \in SS(X,E)$. *Then we have*

(1) The soft closure operator $\widetilde{s}Cl^* : SS(X,E) \to SS(X,E)$*, defined by* $\widetilde{s}Cl^*(F,E) = (F,E) \sqcup (F,E)^*$*, satisfies Kuratwski's axioms.*
(2) If $(U,E) \in \widetilde{\tau}$*, then* $(U,E) \sqcap (F,E)^* \sqsubseteq [(U,E) \sqcap (F,E)]^*$.

Definition 1.11. [14]. A soft subset (F,E) of a soft ideal topological space $(X,\widetilde{\tau},E,\widetilde{I})$ is said to be τ^*-soft dense if $\widetilde{s}Cl^*(F,E) = \mathbf{1}_E$.

Definition 1.12. [8]. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X, E)$ and $x_e \in SP(X, E)$. Then:

(1) (F, E) is said to be a soft-$\widetilde{I}$-open set if $(F, E) \sqsubseteq \widetilde{s}Int(F, E)^*$. The complement of soft-$\widetilde{I}$-open set is called soft-$\widetilde{I}$-closed and we denote the set of all soft-$\widetilde{I}$-open (resp. soft-$\widetilde{I}$- closed) sets by $\widetilde{S}\widetilde{I}O(X, E)$(resp. $\widetilde{S}\widetilde{I}C(X, E)$).

(2) (F, E) is said to be a soft pre-$\widetilde{I}$-open set if $(F, E) \sqsubseteq \widetilde{s}Int(\widetilde{s}Cl^*(F, E))$. The complement of a soft pre-$\widetilde{I}$-open set is called soft pre-$\widetilde{I}$-closed and the family of all soft pre-$\widetilde{I}$-open (resp. soft pre-$\widetilde{I}$-closed) sets in $(X, \widetilde{\tau}, E, \widetilde{I})$ is denoted by $\widetilde{S}P\widetilde{I}O(X, E)$(resp. $\widetilde{S}P\widetilde{I}C(X, E)$).

(3) (F, E) is said to be a soft α-$\widetilde{I}$-open set if $(F, E) \sqsubseteq \widetilde{s}Int(\widetilde{s}Cl^*(\widetilde{s}Int(F, E)))$. The complement of a soft α-$\widetilde{I}$-open set is called soft α-$\widetilde{I}$-closed and the family of all soft α-$\widetilde{I}$-open(resp. soft α-$\widetilde{I}$-closed) sets in $(X, \widetilde{\tau}, E, \widetilde{I})$ is denoted by $\widetilde{S}\alpha\widetilde{I}O(X, E)$(resp. $\widetilde{S}\alpha\widetilde{I}C(X, E)$).

(4) x_e is called a soft pre-$\widetilde{I}$-Interior point of (F, E) if there exists $(G, E) \in \widetilde{S}P\widetilde{I}O(X, E)$ such that $x_e \widetilde{\in} (G, E) \sqsubseteq (F, E)$, the set of all soft pre-$\widetilde{I}$-interior points of (F, E) is called the soft pre-$\widetilde{I}$-interior of (F, E) and is denoted by $\widetilde{s}p\widetilde{I}Int(F, E)$. Consequently, $\widetilde{s}p\widetilde{I}Int(F, E) = \sqcup\{(G, E) : (G, E) \in \widetilde{S}P\widetilde{I}O(X, E), (G, E) \sqsubseteq (F, E)\}$

(5) x_e is called a soft pre-$\widetilde{I}$-closure point of (F, E) if $(F, E) \sqcap (H, E) \neq \mathbf{0}_E$ for every $(H, E) \in \widetilde{S}P\widetilde{I}O(X, E)$ and $x_e \widetilde{\in} (H, E)$. The set of all soft pre-$\widetilde{I}$-closure points of (F, E) is called the soft pre-$\widetilde{I}$-closure of (F, E) and is denoted by $\widetilde{s}p\widetilde{I}Cl(F, E)$. Consequently, $\widetilde{s}p\widetilde{I}Cl(F, E) = \sqcap\{(H, E) : (H, E) \in \widetilde{s}p\widetilde{I}C(X, E), (F, E) \sqsubseteq (H, E)\}$.

Theorem 1.3. [8]. *Every soft α-$\widetilde{I}$-open (resp. soft α-$\widetilde{I}$-closed) set is soft pre-$\widetilde{I}$-open (resp. soft pre-$\widetilde{I}$-closed).*

Theorem 1.4. [8]. *Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space, and $(F, E), (G, E) \in SS(X, E)$. Then the following hold:*

(1) $\widetilde{s}p\widetilde{I}Cl((F, E)^c) = (\widetilde{s}p\widetilde{I}Int(F, E))^c$.
(2) $\widetilde{s}p\widetilde{I}Int\ ((F, E)^c) = (\widetilde{s}p\widetilde{I}Cl(F, E))^c$.
(3) $\widetilde{s}p\widetilde{I}Int[(F, E) \sqcap (G, E)] \sqsubseteq \widetilde{s}p\widetilde{I}Int(F, E) \sqcap \widetilde{s}p\widetilde{I}Int(G, E)$.
(4) $(F, E) \in \widetilde{S}P\widetilde{I}C(X) \Longleftrightarrow (F, E) = \widetilde{s}p\widetilde{I}Cl(F, E)$.
(5) $(F, E) \in \widetilde{S}P\widetilde{I}O(X) \Longleftrightarrow (F, E) = \widetilde{s}p\widetilde{I}Int(F, E)$.
(6) $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Int(F, E)) = \widetilde{s}p\widetilde{I}Int(F, E)$.
(7) If $(F, E) \sqsubseteq (G, E)$*, then* $\widetilde{s}p\widetilde{I}Int(F, E) \sqsubseteq \widetilde{s}p\widetilde{I}Int(G, E)$.
(8) If $(F, E) \sqsubseteq (G, E)$*, then* $\widetilde{s}p\widetilde{I}Cl(F, E) \sqsubseteq \widetilde{s}p\widetilde{I}Cl(G, E)$.

2. SOFT PRE-$\widetilde{I}$-REGULAR PRE-$\widetilde{I}$-OPEN SETS

In this section, we introduce the concept of soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set in which the notion of soft pre-$\widetilde{I}$-open set is involved and study some of its properties. Also, we present other notions called extremely soft pre-$\widetilde{I}$-disconnected and soft pre-$\widetilde{I}$-regular sets.

Definition 2.1. Let $(X, \widetilde{\tau}, E, \widetilde{I}\ )$ be a soft ideal topological space. A soft pre-$\widetilde{I}$-open set (F, E) is said to be soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open if $(F, E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E))$. The complement of a soft pre- $\widetilde{I}$-regular pre-$\widetilde{I}$-open set is said to be soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed.

Example 2.2. Suppose that $(X, \widetilde{\tau}, E, \widetilde{I})$ is a soft ideal topological space, where $X = \{h_1, h_2, h_3\}$, $E = \{e\}$, $\widetilde{\tau} = \{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1\})\}, \{(e, \{h_2\})\}, \{(e, \{h_1, h_2\})\}$ and $\widetilde{I} = \{\mathbf{0}_E, \{(e, \{h_1\})\}$. Then one can deduce that $\widetilde{S}P\widetilde{I}O(X, E) = \{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1\})\}, \{(e, \{h_2\})\}, \{(e, \{h_1, h_2\})\}\}$. We have $\{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1\})\}, \{(e, \{h_2\})\}\}$ are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets.

Definition 2.3. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space and $(F, E) \in SS(X, E).(F, E)$ is said to be soft pre-$\widetilde{I}$-regular if it is soft pre-$\widetilde{I}$-open and soft pre-$\widetilde{I}$-closed.

Example 2.4. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space, where $X = \{h_1, h_2, h_3\}$, $E = \{e\}$, $\widetilde{\tau} = \{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_2, h_3\})\}\}$ and $\widetilde{I} = \{\mathbf{0}_E, \{(e, \{h_1\})\}\}$. Then one can deduce that $\widetilde{S}P\widetilde{I}O(X, E) = \{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_2, h_3\})\}, \{(e, \{h_2\})\}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_2\})\}, \{(e, \{h_1, h_3\})\}$, we have $\{\{(e, \{h_2\})\}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_2\})\}, \{(e, \{h_1, h_3\})\}, \mathbf{1}_E, \mathbf{0}_E\}$ are soft pre-$\widetilde{I}$-open and soft pre- $\widetilde{I}$-closed sets. So, they are soft pre-$\widetilde{I}$-regular.

Remark. (1) Soft pre-$\widetilde{I}$-regular set is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open;
(2) Soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set is soft pre-$\widetilde{I}$-open;
(3) Soft-$\widetilde{I}$-open set and soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set are independent.

Example 2.5. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be the soft ideal topological space as in Example 2.2. We have $(F, E) = \{(e, \{h_1\})\}$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open but not soft pre-$\widetilde{I}$-closed.

Example 2.6. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be the soft ideal topological space as in Example 2.4. We get $(F, E) = \{(e, \{h_2, h_3\})\}$ is a soft-$\widetilde{I}$-open set but it is not soft pre- $\widetilde{I}$-regular pre-$\widetilde{I}$-open and we have $(G, E) = \{(e, \{h_2, h_3\})\}$ is soft pre-$\widetilde{I}$-open set but it is not soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.

Example 2.7. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space, where $X = \{h_1, h_2, h_3\}, E = \{e\}$, $\widetilde{\tau} = \{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1\})\}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_2, h_3\})\}$ and $\widetilde{I} = \{\mathbf{0}_E, \{(e, \{h_1\})\}$. Then $\widetilde{S}P\widetilde{I}O(X, E) = \{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1\})\}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_2, h_3\})\}\}$.Therefore $\{(e, \{h_1\})\}$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set but it is not soft-$\widetilde{I}$-open.

Remark. The soft intersection of two soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets is not soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open, in general, as can be shown by the following example.

Example 2.8. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be the soft ideal topological space as in Example 2.4. Then, the soft sets $\{(e, \{h_1, h_2\})\}$ and $\{(e, \{h_1, h_3\})\}$ are both soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets, but their intersection $\{(e, \{h_1\})\}$ is not soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.

Theorem 2.1. *Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space and $(F, E), (G, E) \widetilde{\in} SS(X, E)$. Then the following statement are true.*

(1) *If $(F, E) \sqsubseteq (G, E)$, then $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E)) \sqsubseteq \widetilde{s}p\widetilde{I}Int\widetilde{s}p\widetilde{I}Cl(G, E))$*
(2) *If $(F, E) \widetilde{\in} \widetilde{S}P\widetilde{I}O(X, E)$, then $(F, E) \sqsubseteq \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E))$.*
(3) *For every $(F, E) \widetilde{\in} \widetilde{S}P\widetilde{I}O(X, E)$, we have $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E)))) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E))$.*
(4) *If (F, E) and (G, E) are disjoint soft pre-$\widetilde{I}$-open sets, then $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E))$ and $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(G, E))$ are disjoint.*
(5) *If (F, E) is a soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set, then $\widetilde{s}p\widetilde{I}Cl((F, E)^c)$ is a soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed set.*

(6) If (F,E) is a soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set, then $\widetilde{s}p\widetilde{I}Int(F,E)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.

Proof. (1) Follows from Theorem 1.4 (7) and (8).

(2) $(F,E)\ \widetilde{\in}\ \widetilde{S}P\widetilde{I}O(X,E) \Longrightarrow (F,E)=\widetilde{s}p\widetilde{I}Int(F,E)\ \sqsubseteq \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))$.

(3) It is obvious that $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))\widetilde{\in}\widetilde{S}P\widetilde{I}O(X,E)$. So, by (2), we have $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))\ \sqsubseteq \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))))$. On the other hand, $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))\sqsubseteq \widetilde{s}p\widetilde{I}Cl\,(F,E)$, which implies that $\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E)))\ \sqsubseteq \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Cl(F,E))=\widetilde{s}p\widetilde{I}Cl(F,E)$. Hence $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))))\sqsubseteq \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))$. Therefore, we obtain $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))))$ = $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))$.

(4) Since (F,E) and (G,E) are disjoint soft pre-$\widetilde{I}$-open sets, $(F,E)\sqcap(G,E)=\mathbf{0}_E$ which implies that $(F,E)\sqcap\widetilde{s}p\widetilde{I}Cl(G,E)=\mathbf{0}_E$ and so $(F,E)\sqcap\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(G,E))=\mathbf{0}_E$. Since $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(G,E)$ is soft pre-$\widetilde{I}$-open, $\widetilde{s}p\widetilde{I}Cl(F,E)\sqcap\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(G,E)=\mathbf{0}_E$. Hence $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))\ \sqcap\ \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(G,E))=\mathbf{0}_E$.

(5) Given that (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set. So (F,E)= $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl\,(F,E))$ which implies that $(F,E)^c=\left(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))\right)^c=\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int((F,E)^c))$. Therefore, $\widetilde{s}p\widetilde{I}C((F,E)^c)=\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl((F,E)^c)))$. Hence $\widetilde{s}p\widetilde{I}Cl((F,E)^c)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed set.

(6) By (5), if (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open, then $\widetilde{s}p\widetilde{I}Cl((F,E)^c)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed. Hence $\left(\widetilde{s}p\widetilde{I}Cl((F,E)^c)\right)^c$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open which implies that $\widetilde{s}p\widetilde{I}Int(F,E)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set. □

Definition 2.9. A soft ideal topological space $(X,\widetilde{\tau},E,\widetilde{I})$ is said to be soft-$\widetilde{I}$-submaximal if each τ^*-soft dense subset is soft open.

Lemma 2.2. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space, the following are equivalent:*

(i) Every soft pre-$\widetilde{I}$-open set is soft open.

(ii) $(X,\widetilde{\tau},E,\widetilde{I})$ is soft-$\widetilde{I}$-submaximal.

Proof. (i)$\Longrightarrow$(ii). Suppose that (F,E) is a τ^*-soft dense set, then $\widetilde{s}Cl^*(F,E)=\mathbf{1}_E$ which implies that $\widetilde{s}Int(\widetilde{s}Cl^*(F,E))=\mathbf{1}_E$. Hence $(F,E)\ \sqsubseteq \widetilde{s}Int(\widetilde{s}Cl^*(F,E))=\mathbf{1}_E$. Therefore, by (i), we have (F,E) is soft open.

(ii)$\Longrightarrow$(i). Let (G,E) be a soft pre-$\widetilde{I}$-open subset of X. Then $(G,E)\sqsubseteq\widetilde{s}Int(\widetilde{s}Cl^*(G,E))=(U,E)$, say. Then $\widetilde{s}Cl^*(G,E)=\widetilde{s}Cl^*(U,E)$, so that $\widetilde{s}Cl^*[(U,E)^c\ \sqcup\ (G,E)]=\widetilde{s}Cl^*((U,E)^c)\sqcup\widetilde{s}Cl^*(G,E)=(U,E)^c\sqcup\widetilde{s}Cl^*(G,E)=\mathbf{1}_E$ and thus $(U,E)^c\sqcup(G,E)$ is τ^*-soft dense set in X. Thus $(U,E)^c\sqcup(G,E)$ is soft open. Now, we have $(G,E)=((U,E)^c\sqcup(G,E))\sqcap(U,E)$, is the intersection of two soft open sets, so that (G,E) is soft open. □

Theorem 2.3. *In a soft-$\widetilde{I}$-submaximal soft ideal topological space, the intersection of any finite number of soft pre-$\widetilde{I}$-open sets is soft pre-$\widetilde{I}$-open.*

Proof. It's clear from Lemma 2.2. □

Theorem 2.4. *In a soft-$\widetilde{I}$-submaximal soft ideal topological space, the intersection of any finite number of soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.*

Proof. Let $\{(F_i, E) : i = 1, 2, ..., n\}$ be a finite family of soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets. Since the space $(X, \widetilde{\tau}, E, \widetilde{I})$ is soft-$\widetilde{I}$-submaximal, then, by Theorem 2.2, $\sqcap\{(F_i, E) : i = 1, 2, ..., n\}$ is soft pre-$\widetilde{I}$-open. Therefore, $\sqcap\{(F_i, E) : i = 1, 2, ..., n\} \sqsubseteq \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\sqcap(F_i\ ,\ E))$. Also, for each $i = 1, 2, ..., n, \sqcap(F_i, E) \sqsubseteq (F_i, E)$ which implies that $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\sqcap(F_i\ ,\ E)\ )) \sqsubseteq \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F_i\ ,\ E))$. Also, each (F_i, E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open implies that $(F_i, E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F_i, E))$ which implies that $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\sqcap(F_i\ ,\ E))) \sqsubseteq \sqcap(F_i\ ,\ E)$ and so $\sqcap(F_i\ ,\ E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\sqcap(F_i\ ,\ E))$. Hence $\sqcap(F_i, E)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open. □

Remark. It should be noted that an arbitrary union of soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open. But the Intersection of two soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed sets fails to be soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed as can be shown by the following example.

Example 2.10. Consider the soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ as in Example 2.2. We have the two soft sets $(F, E) = \{(e, \{h_1,\ h_3\})\}$ and $(G, E) = \{(e, \{h_2,\ h_3\})\}$ are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed sets but their intersection $(F, E) \sqcap (G, E) = \{(e, \{h_3\})\}$ is not soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed set.

Theorem 2.5. *Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space and let $(F, E), (G, E) \in SS(X, E)$, then the following hold.*

(1) If (F, E) is soft pre-$\widetilde{I}$-closed, then $\widetilde{s}p\widetilde{I}Int(F, E)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.
(2) If (F, E) is soft pre-$\widetilde{I}$-open, then $\widetilde{s}p\widetilde{I}Cl(F, E)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed.
(3) If (F, E) and (G, E) are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed sets, then $(F, E) \sqsubseteq (G, E)$ if and only if $\widetilde{s}p\widetilde{I}Int(F, E) \sqsubseteq \widetilde{s}p\widetilde{I}Int(G, E)$.
(4) If (F, E) and (G, E) are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets, then $(F, E) \sqsubseteq (G, E)$ if and only if $\widetilde{s}p\widetilde{I}Cl(F, E) \sqsubseteq \widetilde{s}p\widetilde{I}Cl(G, E)$.

Proof. (1) Since (F, E) is soft pre-$\widetilde{I}$-closed, we have $(F, E) = \widetilde{s}p\widetilde{I}Cl(F, E)$. Now, we obtain
$\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F, E)) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E)))) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E)) = \widetilde{s}p\widetilde{I}Int(F, E)$. Hence $\widetilde{s}p\widetilde{I}Int(F, E)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.

(2) Now, we have $\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E))) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F, E))$ =
$\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F, E) = \widetilde{s}p\widetilde{I}Cl(F, E)$. Hence $\widetilde{s}p\widetilde{I}Cl(F, E)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed.

(3) Given (F, E) and (G, E) are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed sets. Therefore, we have $(F, E) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F, E))$ and $(G, E) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int((F, E))$. Clearly, we have $\widetilde{s}p\widetilde{I}Int(F, E) \sqsubseteq \widetilde{s}p\widetilde{I}Int(G, E)$ whenever $(F, E) \sqsubseteq (G, E)$. Conversely, suppose that $\widetilde{s}p\widetilde{I}Int(F, E) \sqsubseteq \widetilde{s}p\widetilde{I}Int(G, E)$. Now, we obtain $(F, E) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F, E)) \sqsubseteq$
$\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(G, E))) = (G, E)$. Hence $(F, E) \sqsubseteq (G, E)$.

(4) Given (F, E) and (G, E) are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets. Therefore, we obtain $(F, E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E))$ and $(G, E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(G, E))$. Suppose $(F, E) \sqsubseteq (G, E), \widetilde{s}p\widetilde{I}Cl(F, E) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F, E))) \sqsubseteq \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(G, E))) = \widetilde{s}p\widetilde{I}Cl(G, E)$. Therefore, $\widetilde{s}p\widetilde{I}Cl(F, E) \sqsubseteq \widetilde{s}p\widetilde{I}Cl(G, E)$.

Conversely, $\widetilde{s}p\widetilde{I}Cl(F,E) \sqsubseteq \widetilde{s}p\widetilde{I}Cl(G,E)$. Now, we obtain that $(F,E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E)) \sqsubseteq \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(G,E)) = (G,E)$. Therefore, $(F,E) \sqsubseteq (G,E)$.

□

Definition 2.11. Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space and $(F,E)\in SS(X,E)$. A soft subset (F,E) is said to be soft-$\widetilde{I}$-rare if $\widetilde{s}Int^*(F,E) = \mathbf{0}_E$.

Example 2.12. Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space where $X = \{h_1,h_2,h_3\}$, $E = \{e\}, \widetilde{\tau} = \{\mathbf{1}_E,\mathbf{0}_E,\{(e,\{h_2\})\}\}$ and $\widetilde{I} = \{\mathbf{0}_E,\{(e,\{h_2\})\}\}$, then $\tau^* = \{\mathbf{1}_E,\mathbf{0}_E,\{(e,\{h_2\})\},\{(e,\{h_1,\ h_3\})\}\}$. Take $(F,E) = \{(e,\{h_1\})\}$, so $\widetilde{s}Int^*(F,E) = \mathbf{0}_E$. Hence we get $(F,E) = \{(e,\{h_1\})\}$ is a soft-$\widetilde{I}$-rare set.

Definition 2.13. A subset (F,E) of soft ideal topological space $(X,\widetilde{\tau},E,\widetilde{I})$ is said to be soft nowhere dense set if $\widetilde{s}Int(\widetilde{s}Cl(F,E)) = \mathbf{0}_E$.

Example 2.14. Let $(X,\widetilde{\tau},E,\widetilde{I})$ be the soft ideal topological space as in Example 2.12. Take $(F,E) = \{(e,\{h_1\})\}$, $\widetilde{s}Int(\widetilde{s}Cl(F,E)) = \mathbf{0}_E$, so (F,E) is soft nowhere dense.

Lemma 2.6. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space and $(F,E),(U,E)\in SS(X,E)$. If (U,E) is soft open set, then $(U,E)\sqcap \widetilde{s}Cl^*(F,E) \sqsubseteq \widetilde{s}Cl^*((U,E)\sqcap(F,E))$.*

Proof. Since $(U,E)\in\widetilde{\tau}$, by Theorem 1.2 we obtain $(U,E)\sqcap\widetilde{s}Cl^*(F,E) = (U,E)\sqcap[(F,E)\sqcup(F,E)^*] = [(U,E)\sqcap(F,E)]\sqcup[(U,E)\sqcap(F,E)^*] \sqsubseteq [(U,E)\sqcap(F,E)]\sqcup[(U,E)\sqcap(F,E)]^*$ (Theorem 2.1) = $\widetilde{s}Cl^*((U,E)\sqcap(F,E))$. Hence $(U,E)\sqcap\widetilde{s}Cl^*(F,E) \sqsubseteq \widetilde{s}Cl^*((U,E)\sqcap(F,E))$. □

Lemma 2.7. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space and $(F,E)\in SS(X,E)$. Then*

(1) $\widetilde{s}p\widetilde{I}Int(F,E) = (F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E))$.
(2) $\widetilde{s}p\widetilde{I}Cl(F,E) = (F,E)\sqcup\widetilde{s}Cl(\widetilde{s}Int^*(F,E))$.

Proof. (1) Since $(F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E)) \sqsubseteq \widetilde{s}Int(\widetilde{s}Cl^*(F,E)) = \widetilde{s}Int(\widetilde{s}Int(\widetilde{s}Cl^*(F,E))) = \widetilde{s}Int(\widetilde{s}Cl^*(F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E))) \sqsubseteq \widetilde{s}Int(\widetilde{s}Cl^*((F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E))))$, then we have $(F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E))$ is soft pre-$\widetilde{I}$-open set contained in (F,E) and so $(F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E)) \sqsubseteq \widetilde{s}p\widetilde{I}Int(F,E)$. On other hand, $\widetilde{s}p\widetilde{I}Int(F,E)$ is soft pre-$\widetilde{I}$- open, $\widetilde{s}p\widetilde{I}Int(F,E) \sqsubseteq \widetilde{s}Int(\widetilde{s}Cl^*(\widetilde{s}p\widetilde{I}Int(F,E))) \sqsubseteq \widetilde{s}Int(\widetilde{s}Cl^*(F,E))$ and so $\widetilde{s}p\widetilde{I}Int(F,E) \sqsubseteq (F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E))$. Hence $\widetilde{s}p\widetilde{I}Int(F,E) = (F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E))$.

(2) By (1), $\widetilde{s}p\widetilde{I}Int(F,E) = (F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E))$. So, $(\widetilde{s}p\widetilde{I}Int(F,E))^c = [(F,E)\sqcap\widetilde{s}Int(\widetilde{s}Cl^*(F,E))]^c$, $\widetilde{s}p\widetilde{I}Cl((F,E)^c) = (F,E)^c\sqcup[\widetilde{s}Int(\widetilde{s}Cl*(F,E))]^c$ and $\widetilde{s}p\widetilde{I}Cl((F,E)^c) = (F,E)^c\sqcup\widetilde{s}Cl(\widetilde{s}Int^*((F,E)^c))$. Assume, $(F,E)^c = (G,E)$. Hence $\widetilde{s}p\widetilde{I}Cl(G,E) = (G,E)\sqcup\widetilde{s}Cl(\widetilde{s}Int^*(G,E))$.

□

Theorem 2.8. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space and $(F,E)\in SS(X)_E$. Then the following hold.*

(1) The empty set is the only soft subset which is nowhere dense and soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open;
(2) If (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed, then every soft-$\widetilde{I}$-rare set is soft pre-$\widetilde{I}$-open.

Proof. (1) Suppose that (F,E) is soft nowhere dense and soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open. Then, by Lemma 2.7(1) we have $(F,E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E)) = \widetilde{s}p\widetilde{I}Cl(F,E) \sqcap \widetilde{s}Int(\widetilde{s}Cl^*(\widetilde{s}p\widetilde{I}Cl((F,E)))$. Therefore $(F,E) \sqsubseteq \widetilde{s}p\widetilde{I}Cl(F,E) \sqcap \widetilde{s}Int(\widetilde{s}Cl^*(\widetilde{s}Cl(F,E))) \sqsubseteq \widetilde{s}p\widetilde{I}Cl(F,E) \sqcap \widetilde{s}Int(\widetilde{s}Cl(F,E)) = \widetilde{s}p\widetilde{I}Cl(F,E) \sqcap \mathbf{0}_E = \mathbf{0}_E$.

(2) Suppose that (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed. Then $(F,E) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F,E)) = \widetilde{s}p\widetilde{I}Int(F,E) \sqcup \widetilde{s}Cl(\widetilde{s}Int^*(\widetilde{s}p\widetilde{I}Int(F,E))) \sqsubseteq \widetilde{s}p\widetilde{I}Int(F,E) \sqcup \widetilde{s}Cl(\widetilde{s}Int^*(F,E)) = \widetilde{s}p\widetilde{I}Int(F,E) \sqcup \mathbf{0}_E = \widetilde{s}p\widetilde{I}Int(F,E)$. Therefore, $(F,E) = \widetilde{s}p\widetilde{I}Int(F,E)$. Hence (F,E) is soft pre- $\widetilde{I}$-open.

□

Definition 2.15. A soft ideal topological space $(X,\widetilde{\tau},E,\widetilde{I})$ is called soft extremely pre-$\widetilde{I}$-disconnected if the soft pre- $\widetilde{I}$-closure of every soft pre-$\widetilde{I}$-open set is soft pre-$\widetilde{I}$-open.

Example 2.16. Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space where $X = \{h_1,h_2,h_3\}$, $E = \{e\}$,$\widetilde{\tau} = \{\mathbf{1}_E, \mathbf{0}_E, \{(e,\{h_1\})\}, \{(e,\{h_2,h_3\})\}\}$ and $\widetilde{I} = \{\mathbf{0}_E, \{(e,\{h_1\})\}, \{(e,\{h_3\})\}, \{(e,\{h_1,h_3\})\}\}$. Then $\widetilde{S}P\widetilde{I}O(X,E) = \{\mathbf{1}_E, \mathbf{0}_E, \{(e,\{h_1\})\}, \{(e,\{h_2,h_3\})\}, \{(e,\{h_2\})\}, \{(e,\{h_1,h_2\})\}\}$.
So, $(X,\widetilde{\tau},E,\widetilde{I})$ is extremely soft pre-$\widetilde{I}$-disconnected.

Theorem 2.9. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space. Then the following are equivalent:*

(1) $(X,\widetilde{\tau},E,\widetilde{I})$ is extremely soft pre-$\widetilde{I}$-disconnected;
(2) Every soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set is soft pre-$\widetilde{I}$-regular.

Proof. $(1) \Rightarrow (2)$: Assume that $(X,\widetilde{\tau},E,\widetilde{I})$ is extremely soft pre-$\widetilde{I}$-disconnected and (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open. Then (F,E) is soft pre-$\widetilde{I}$-open and so, $\widetilde{s}p\widetilde{I}Cl(F,E)$ is a soft pre-$\widetilde{I}$-open set. Hence $(F,E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E)) = \widetilde{s}p\widetilde{I}Cl(F,E)$. Hence (F,E) is soft pre-$\widetilde{I}$-closed. Therefore (F,E) is soft pre-$\widetilde{I}$-regular.
$(2) \Rightarrow (1)$: Suppose that $(F,\ E)$ is soft pre-$\widetilde{I}$- open. Then, by Theorem 2.5(2), $\widetilde{s}p\widetilde{I}Cl(F,E)$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed which implies that $(\widetilde{s}p\widetilde{I}Cl(F,\ E))^c$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$- open. Hence $(\widetilde{s}p\widetilde{I}Cl(F,E))^c$ is soft pre-$\widetilde{I}$-regular. Therefore, $(\widetilde{s}p\widetilde{I}Cl(F,\ E))^c$ is soft pre-$\widetilde{I}$-closed and so $\widetilde{s}p\widetilde{I}Cl(F,E)$ is soft pre-$\widetilde{I}$-open. Hence $(X,\widetilde{\tau},E,\widetilde{I})$ is soft extremely pre-$\widetilde{I}$-disconnected. □

Theorem 2.10. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ is a soft extremely pre-$\widetilde{I}$-disconnected space and $(F,E) \widetilde{\in} SS(X,E)$. Then the following are equivalent:*

(1) (F,E) is soft pre-$\widetilde{I}$-regular.
(2) $(F,E) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F,E))$;
(3) $(F,E)^c$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open;
(4) (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.

Proof. $(1) \Rightarrow (2)$: Suppose that (F,E) is soft pre-$\widetilde{I}$-regular. Then (F,E) is soft pre-$\widetilde{I}$-open and soft pre-$\widetilde{I}$-closed and so $(F,E) = \widetilde{s}p\widetilde{I}Int(F,E)$ and $(F,E) = \widetilde{s}p\widetilde{I}Cl(F,E)$. Hence $(F,E) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F,E))$.
$(2) \Rightarrow (3)$: Suppose that $(F,E) = \widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F,E))$. Then $(F,E)^c = (\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(F,E)))^c = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl((F,E)^c))$. So $(F,E)^c$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.
$(3) \Rightarrow (4)$: Since $(F,E)^c$ is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open, then $(F,E)^c$ is soft pre- $\widetilde{I}$-regular (Theorem 2.9). So, (F,E) is soft pre-$\widetilde{I}$-open and soft pre-$\widetilde{I}$-closed, thus $(F,E) =$

$\widetilde{s}p\widetilde{I}Int(F,E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))$. Hence (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.
$(4) \Rightarrow (1)$: The proof follows from Theorem 2.9.

□

Definition 2.17. A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is said to be $\widetilde{s}p\widetilde{I}R$-door space if every soft subset of $\widetilde{\tau}$ is either soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open or soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed.

Example 2.18. The soft ideal topological space of Example 2.1 is a $\widetilde{s}p\widetilde{I}R$-door space.

Theorem 2.11. *Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a $\widetilde{s}p\widetilde{I}R$-door space. Then every soft pre-$\widetilde{I}$-open set in the space is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open.*

Proof. Let (F,E) be a soft pre-$\widetilde{I}$-open subset of X. Since $(X, \widetilde{\tau}, E, \widetilde{I})$ is a $\widetilde{s}p\widetilde{I}R$-door space, then (F,E) is either soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$- open or soft pre- $\widetilde{I}$-regular pre-$\widetilde{I}$-closed. If (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open, then the proof is complete. If (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed, so $(F,E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))$. □

Theorem 2.12. *Let $(X,\ \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space and $(F,E) \widetilde{\in} SS(X)_E$. If (F,E) is both soft $\alpha - \widetilde{I}$-open and soft $\alpha - \widetilde{I}$-closed, then (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$- open set.*

Proof. Suppose (F,E) is a soft $\alpha - \widetilde{I}$-open and soft $\alpha - \widetilde{I}$-closed set. Then (F,E) is soft pre-$\widetilde{I}$-open and soft pre-$\widetilde{I}$-closed set (Theorem 1.3). Hence (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set. □

Theorem 2.13. *Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space and $(F,E) \widetilde{\in} SS(X)_E$. If (F,E) is soft $\alpha{-}\widetilde{I}$- open and soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open, then $(F,E) = \widetilde{s}Int(\widetilde{s}Cl^*(\widetilde{s}Int(F,E)))$.*

Proof. Suppose that (F,E) is a soft $\alpha{-}\widetilde{I}$-open set. Then $(F,E) \sqsubseteq \widetilde{s}Int(\widetilde{s}Cl^*(\widetilde{s}Int(F,E)))$ and (F,E) is soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open. Hence, we have $(F,E) = \widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E))$ =
$\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl(F,E)))) \sqsupseteq \widetilde{s}p\widetilde{I}Int(\widetilde{s}Cl^*(\widetilde{s}Int(\widetilde{s}p\widetilde{I}Cl(F,E)))) \sqsupseteq$
$\widetilde{s}p\widetilde{I}Int(\widetilde{s}Cl^*(\widetilde{s}Int(F,E))) \sqsupseteq \widetilde{s}Int(\widetilde{s}Cl^*(\widetilde{s}Int(F,E)))$. Therefore $(F,E) = \widetilde{s}Int(\widetilde{s}Cl^*(\widetilde{s}Int(F,E)))$.

□

3. $\widetilde{s}\delta p\widetilde{I}$-OPEN SETS

In this section, we define the soft delta pre-$\widetilde{I}$-open set by using the notion of soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets and study some of their properties.

Definition 3.1. A soft point $x_e \in SP(X,E)$ is called a $\widetilde{s}\delta p\widetilde{I}$-cluster point of (F,E) if $(F,E) \sqcap (U,E) \neq \mathbf{0}_E$ for every soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set (U,E) containing x_e. The set of all $\widetilde{s}\delta p\widetilde{I}$-cluster points of (F,E) is called the $\widetilde{s}\delta p\widetilde{I}$-closure of (F,E) and is denoted by $\widetilde{s}\delta p\widetilde{I}Cl(F,E)$. The complement of an $\widetilde{s}\delta p\widetilde{I}$-closed set is called an $\widetilde{s}\delta p\widetilde{I}$-open set. We denote the collection of all $\widetilde{s}\delta p\widetilde{I}$-open set (resp. $\widetilde{s}\delta p\widetilde{I}$-closed) sets by $\widetilde{S}\delta P\widetilde{I}O(X,E)$ (resp. $\widetilde{S}\delta P\widetilde{I}C(X,E)$).

Example 3.2. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be as in Example 2.4. Then, we have $\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1,\ h_2\})\}$, $\{(e, \{h_1,\ h_3)\}$, $\{(e, \{h_2\})\}$, $\{(e, \{h_3\})\}$ are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets. Therefore $\widetilde{S}\delta P\widetilde{I}C(X,E) = \{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1,\ h_2\})\}, \{(e, \{h_1,\ h_3\})\}, \{(e, \{h_2\})\}, \{(e, \{h_3\})\}, \{(e, \{h_1\})\}\}$

and
$\widetilde{S}\delta P\widetilde{I}O(X,E) = \{\mathbf{1}_E, \mathbf{0}_E, \{(e,\{h_1,\ h_2\})\}, \{(e,\{h_1,\ h_3\})\}, \{(e,\{h_2\})\}, \{(e,\{h_3\})\}, \{(e,\{h_2,\ h_3\})\}\}$.

Definition 3.3. A soft set (F,E) in a soft ideal topological space $(X,\widetilde{\tau},E,\widetilde{I})$ is called $\widetilde{s}\delta p\widetilde{I}$-neighborhood of a soft point x_e if there exists an $\widetilde{s}\delta p\widetilde{I}$-open set (U,E) such that $x_e \widetilde{\in} (U,E) \sqsubseteq (F,E)$.

Example 3.4. Let $(X,\widetilde{\tau},E,\widetilde{I})$ be the soft ideal topological space as in Example 2.4. Then we have $\{(e,\{h_2,\ h_3\})\}$ is an $\widetilde{s}\delta p\widetilde{I}$-neighborhood of a soft point $x_e = \{(e,\{h_2\})\}$. Indeed, $\{(e,\{h_2\})\} \widetilde{\in} \{(e,\{h_2,\ h_3\})\} \sqsubseteq \{(e,\{h_2,\ h_3\})\}$ and $\{(e,\{h_2,\ h_3\})\} \in \widetilde{S}\delta P\widetilde{I}O(X,E)$.

Lemma 3.1. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space and $(F,E),(G,E) \in SS(X,E)$ and $\{(U_i,E) : i \in \Lambda\} \subseteq SS(X,E)$. Then the following hold.*

(1) $(F,E) \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl(F,E)$;
(2) *If* $(F,E) \sqsubseteq (G,E)$, *then* $\widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl(G,E)$;
(3) $\widetilde{s}\delta p\widetilde{I}Cl\{\sqcap\{(U_i,E) : i \in \Lambda\}\} \sqsubseteq \sqcap\{\widetilde{s}\delta p\widetilde{I}Cl(U_i,E) : i \in \Lambda\}$;
(4) $\sqcup\{\widetilde{s}\delta p\widetilde{I}Cl(U_i,E) : i \in \Lambda\} \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl\{\sqcup(U_i,E) : i \in \Lambda\}$;
(5) $\widetilde{s}\delta p\widetilde{I}Cl\{(F,E) \sqcup (G,E)\} = \widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqcup \widetilde{s}\delta p\widetilde{I}Cl(G,E)$.

Proof. (1) Let $x_e \widetilde{\in} (F,E)$ and (G,E) be a soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set containing x_e. Therefore $(F,E) \sqcap (G,E) \neq \mathbf{0}_E$ implies $x_e \widetilde{\in} \widetilde{s}\delta p\widetilde{I}Cl(F,E)$ which implies that $(F,E) \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl(F,E)$;

(2) Suppose $(F,E) \sqsubseteq (G,E)$ and $x_e \widetilde{\in} \widetilde{s}\delta p\widetilde{I}Cl(F,E)$. Then $(F,E) \widetilde{\cap} (H,E) \neq \widetilde{\emptyset}$ for every soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set (H,E) containing x_e. Since $(F,E) \sqsubseteq (G,E)$, then $(G,E) \sqcap (H,E) \neq \mathbf{0}_E$. Therefore $x_e \widetilde{\in} \widetilde{s}\delta p\widetilde{I}Cl(G,E)$. So, $\widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl(G,E)$;

(3) Since $\sqcap(U_i,E) \sqsubseteq (U_i,E)$ for each $i \in \Lambda$, by (2) $\widetilde{s}\delta p\widetilde{I}Cl\{\sqcap\{(U_i,E) : i \in \Lambda\}\} \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl(U_i,E)$ for each $i \in \Lambda$. So, $\widetilde{s}\delta p\widetilde{I}Cl\{\sqcap\{(U_i,E) : i \in \Lambda\}\} \sqsubseteq \sqcap\{\widetilde{s}\delta p\widetilde{I}Cl(U_i,E) : i \in \Lambda\}$. Therefore $\widetilde{s}\delta p\widetilde{I}Cl\{\sqcap\{(U_i,E) : i \in \Lambda\}\} \sqsubseteq \sqcap\{\widetilde{s}\delta p\widetilde{I}Cl(U_i,E) : i \in \Lambda\}$;

(4) Since $(U_{i_\circ},E) \sqsubseteq \sqcup_{i\in\Lambda}(U_i,E)$ for each $i_\circ \in \Lambda$, $\widetilde{s}\delta p\widetilde{I}Cl(U_{i_\circ},E) \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl\{\sqcup\{(U_i,E) : i \in \Lambda\}\}$. Hence $\sqcup\{\widetilde{s}\delta p\widetilde{I}Cl(U_i,E) : i \in \Lambda\} \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl\{\sqcup\{(U_i,E) : i \in \Lambda\}\}$;

(5) Let $x_e \widetilde{\in} \widetilde{s}\delta p\widetilde{I}Cl\{(F,E) \sqcup (G,E)\}$. Then $((F,E) \sqcup (G,E)) \sqcap (H,E) \neq \mathbf{0}_E$, for every soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set (H,E) containing x_e. Hence $(F,E) \sqcap (H,E) \neq \mathbf{0}_E$ or $(G,E) \sqcap (H,E) \neq \mathbf{0}_E$ which implies that $x_e \widetilde{\in} \widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqcup \widetilde{s}\delta p\widetilde{I}Cl(G,E)$. Therefore $\widetilde{s}\delta p\widetilde{I}Cl\{(F,E) \sqcup (G,E)\} \sqsubseteq \widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqcup \widetilde{s}\delta p\widetilde{I}Cl(G,E)$. Also, $\widetilde{s}\delta p\widetilde{I}Cl\{(F,E) \sqcup (G,E)\} \sqsupseteq \widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqcup \widetilde{s}\delta p\widetilde{I}Cl(G,E)$. So, $\widetilde{s}\delta p\widetilde{I}Cl\{(F,E) \sqcup (G,E)\} = \widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqcup \widetilde{s}\delta p\widetilde{I}Cl(G,E)$. □

Lemma 3.2. *Arbitrary soft intersection of $\widetilde{s}\delta p\widetilde{I}$-closed sets in a soft ideal topological space $(X,\widetilde{\tau},E,\widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I}$-closed.*

Proof. Suppose that $(F,E) = \sqcap\{(F_i,E) : (F_i,E) \in \widetilde{S}\delta P\widetilde{I}C(X), i \in \Lambda\}$. Then we obtain $\widetilde{s}\delta p\widetilde{I}Cl(F_i,E) = (F_i,E),\ i \in \Lambda$. Thus $\widetilde{s}\delta p\widetilde{I}Cl[\sqcap\{(F_i,E) : i \in \Lambda\}] \sqsubseteq \sqcap\{(F_i,E), i \in \Lambda\}$. Therefore, $\widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqsubseteq (F,E)$.Therefore $(F,E) = \widetilde{s}\delta p\widetilde{I}Cl(F,E)$ and (F,E) is $\widetilde{s}\delta p\widetilde{I}$-closed. □

Lemma 3.3. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space and $(F,E) \in SS(X,E)$. Then*

(1) $\widetilde{s}\delta p\widetilde{I}Cl(F,E) = \sqcap\{(G_i,E) : (G_i,E) \in \widetilde{S}\delta P\widetilde{I}C(X), (F,E) \sqsubseteq (G_i,E), i \in \Lambda\}$,
(2) $\widetilde{s}\delta p\widetilde{I}Cl(F,E) = \sqcap\{(G_i,E) : (G_i,E) \in \widetilde{S}P\widetilde{I}RC(X), (F,E) \sqsubseteq (G_i,E), i \in \Lambda\}$.

Proof. (1) Let $x_e \widetilde{\notin} \sqcap\{(G_i,E) : (G_i,E) \in \widetilde{S}\delta P\widetilde{I}C(X), (F,E) \sqsubseteq (G_i,E), , i \in \Lambda\}$. Then there exists $(G_{i_\circ},E) \in \widetilde{S}\delta P\widetilde{I}C(X)$ such that $x_e \widetilde{\notin} (G_{i_\circ},E)$ and $x_e \widetilde{\notin} (F,E)$ as $(F,E) \sqsubseteq (G_{i_\circ},E)$. Since $(G_{i_\circ},E)^c \in \widetilde{S}\delta P\widetilde{I}O(X)$ and $x_e \widetilde{\in} (G_{i_\circ},E)^c$, then we have $(G_{i_\circ},E)^c \sqcap (F,E) = \mathbf{0}_E$. Therefore $x_e \widetilde{\notin} \widetilde{s}\delta p\widetilde{I}Cl(F,E)$. Conversely, suppose $x_e \widetilde{\notin} \widetilde{s}\delta p\widetilde{I}Cl(F,E)$. Then there exists $(U,E) \in \widetilde{S}\delta P\widetilde{I}O(X)$ such that $x_e \widetilde{\in} (U,E)$ and $(U,E) \sqcap (F,E) = \mathbf{0}_E$. Thus $x_e \widetilde{\notin} (U,E)^c$ and $(U,E)^c \in S\delta P\widetilde{I}C(X)$. We can replace $(U,E)^c$ with (G_i,E) for some $i \in \Lambda$ and obtain $(F,E) \sqsubseteq (G_i,E)$. So $x_e \widetilde{\notin} \sqcap\{(G_i,E) : (G_i,E) \in \widetilde{S}P\widetilde{I}RC(X), (F,E) \sqsubseteq (G_i,E), i \in \Lambda\}$.
(2) The proof is similar to the proof of (1).

□

Lemma 3.4. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space, $(F,E) \in SS(X,E)$ and $x_e \in SP(X,E)$. Then $x_e \widetilde{\in} \widetilde{s}\delta p\widetilde{I}Cl(F,E)$ if and only if $(U, E) \sqcap (F,E) \neq \mathbf{0}_E$ for every $\widetilde{s}\delta p\widetilde{I}$-open (soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open) set (U,E) containing x_e.*

Proof. Suppose that $x_e \widetilde{\notin} \widetilde{s}\delta p\widetilde{I}Cl(F,E)$. Then, $\{\widetilde{s}\delta p\widetilde{I}Cl(F,E)\}^c$ is an $\widetilde{s}\delta p\widetilde{I}$-open set containing x_e that doesn't intersect (F,E). That is, $(U,E) \sqcap (F,E) = \mathbf{0}_E$, where $(U,E) = \{\widetilde{s}\delta p\widetilde{I}Cl(F,E)\}^c$. The converse is obvious. □

Corollary 3.5. *(1) $\widetilde{s}\delta p\widetilde{I}Cl(F,E)$ is $\widetilde{s}\delta p\widetilde{I}$-closed in $(X,\widetilde{\tau},E,\widetilde{I})$ for any $(F,E) \in SS(X,E)$;*
(2) $(F,E) \in SS(X,E)$ is $\widetilde{s}\delta p\widetilde{I}$-closed (resp. $\widetilde{s}\delta p\widetilde{I}$- open) if and only if it is the soft intersection (resp. soft union) of soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed (soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open) sets.

Lemma 3.6. *Let $(X,\widetilde{\tau},E,\widetilde{I})$ be a soft ideal topological space and $(F,E) \in SS(X,E)$. Then $\widetilde{s}\delta p\widetilde{I}Cl(F,E)$ is the smallest $\widetilde{s}\delta p\widetilde{I}$-closed set in $(X,\widetilde{\tau},E,\widetilde{I})$ containing (F,E).*

Proof. Let $\{(F_i,E) : i \in \Lambda\}$ be the collection of all soft $\widetilde{s}\delta p\widetilde{I}$-closed subsets of $(X,\widetilde{\tau},E,\widetilde{I})$ containing (F,E). So by Lemma 3.2, $\widetilde{s}\delta p\widetilde{I}Cl(F,E) = \sqcap\{(F_i,E) : i \in \Lambda\}$ is $\widetilde{s}\delta p\widetilde{I}$-closed. Since $(F,E) \sqsubseteq (F_i,E)$ for each $i \in \Lambda$, we have $(F,E) \sqsubseteq \sqcap\{(F_i,E) : i \in \Lambda\} = \widetilde{s}\delta p\widetilde{I}Cl(F,E)$. Thus $\widetilde{s}\delta p\widetilde{I}Cl(F,E)$ is a soft $\widetilde{s}\delta p\widetilde{I}$-closed set containing (F,E). Also, since $\widetilde{s}\delta p\widetilde{I}Cl(F,E) = \sqcap\{(F_i,E) : i \in \Lambda\}$, then $\widetilde{s}\delta p\widetilde{I}Cl(F,E) \sqsubseteq (F_i,E)$ for each $i \in \Lambda$. Consequently, $\widetilde{s}\delta p\widetilde{I}Cl(F,E)$ is the smallest $\widetilde{s}\delta p\widetilde{I}$-closed set in $(X,\widetilde{\tau},E,\widetilde{I})$ containing (F,E). □

Remark. It is clear that by Corollary 3.1, every soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set is $\widetilde{s}\delta p\widetilde{I}$-open. However, the converse is not true as shown by the following example.

Example 3.5. Let$(X,\widetilde{\tau},E,\widetilde{I})$ be the soft ideal topological space as in Example 2.4, then we have $\{(e,\{h_1,\ h_2\})\}, \{(e,\{\ h_1,\ h_3\})\}$ are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed sets. Thus by Corollary 3.5, $\{(e,\{h_1,\ h_2\})\} \sqcap \{(e,\{\ h_1,\ h_3\})\} = \{(e,\{h_1\})\}$ is $\widetilde{s}\delta p\widetilde{I}$-closed, and so $\{(e,\{\ h_2,\ h_3\})\}$ is $\widetilde{s}\delta p\widetilde{I}$-open. But $\widetilde{s}p\widetilde{I}Int(\widetilde{s}p\widetilde{I}Cl\{(e,\{\ h_2,\ h_3\})\}) = \mathbf{1}_E$, and so $\{(e,\{\ h_2,\ h_3\})\}$ is not soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$- open.

Remark. The soft union of even two $\widetilde{s}\delta p\widetilde{I}$-closed sets need not be a $\widetilde{s}\delta p\widetilde{I}$-closed set as shown by the following example.

Example 3.6. Let$(X, \widetilde{\tau}, E, \widetilde{I})$ be the soft ideal topological space as in Example 3.4. $\{(e, \{h_2\})\}$, $\{(e, \{h_3\})\}$ are soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-closed sets. Thus, by Corollary 3.5, $\{(e, \{h_2\})\}$, $\{(e, \{h_3\})\}$ are $\widetilde{s}\delta p\widetilde{I}$-closed. However, $\{(e, \{ h_2,\ h_3\})\}$ is not $\widetilde{s}\delta p\widetilde{I}$-closed.

4. SOFT DELTA PRE-$\widetilde{I}$-SEPARATION AXIOMS

In this section, we introduce the concept of soft separation axioms using soft delta pre-$\widetilde{I}$-open sets. We define a $\widetilde{s}\delta p\widetilde{I} - T_0$ space, a $\widetilde{s}\delta p\widetilde{I} - T_1$ space and a $\widetilde{s}\delta p\widetilde{I} - T_2$ space and study some of their properties.

Definition 4.1. A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is said to be a $\widetilde{s}\delta p\widetilde{I} - T_0$ space if for every pair of soft points $x_e,\ y_e \in SP(X, E)$ such that $x_e \neq\ y_e$, there exists $(F, E) \in \widetilde{S}\delta P\widetilde{I}O(X, E)$, containing one of them but not the other.

Example 4.2. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space, where $X = \{h_1, h_2\}, E = \{e_1, e_2\}, \widetilde{\tau} = \{\mathbf{1}_E, \mathbf{0}_E, \{(e_1, \{h_1\})\}, \{(e_1, \{h_2\}), (e_2, \{h_1\})\}, \{(e_1, \{h_1\}),\ (e_2, \{h_2\})\}, \{(e_1, X), (e_2, \{h_1\})\}\}$ and $\widetilde{I} = \{\mathbf{0}_E, \{(e_1, \{h_1\})\}\}$. The space $(X, \widetilde{\tau}, E, \widetilde{I})$ is a $\widetilde{s}\delta p\widetilde{I} - T_0$ space.

Definition 4.3. A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is said to be a $\widetilde{s}\delta p\widetilde{I} - T_1$ space if for every pair of soft points $x_e,\ y_e \in SP(X, E)$ such that $x_e \neq\ y_e$, there exist $(F, E),\ (G, E) \in\widetilde{S}\delta P\widetilde{I}O(X)$, such that $x_e \widetilde{\in} (F, E), y_e \widetilde{\notin} (F, E)$ and $y_e \widetilde{\in} (G, E)$, $x_e \widetilde{\notin} (G, E)$.

Example 4.4. The soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ in Example 4.2 is a $\widetilde{s}\delta p\widetilde{I} - T_1$ space.

Definition 4.5. A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is said to be a $\widetilde{s}\delta p\widetilde{I} - T_2$ space if for every a pair soft points $x_e,\ y_e \in SP(X, E)$ such that $x_e \neq y_e$, there exist $(F, E),\ (G, E) \in \widetilde{S}\delta P\widetilde{I}O(X, E)$, such that $x_e \widetilde{\in} (F, E), y_e \widetilde{\in} (G, E)$ and $(F, E)\ \sqcap (G, E) = \mathbf{0}_E$.

Example 4.6. The soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I}\ )$ in Example 4.2 is a $\widetilde{s}\delta p\widetilde{I} - T_2$ space.

Remark. From Corollary 3.5(2) we have:

(1) A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I} - T_0$ if and only if for every pair of distinct soft points $x_e,\ y_e$ of $SP(X, E)$, there exists a soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set containing one of the soft point but not the other.

(2) A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I} - T_1$ if and only if for every pair of distinct soft points $x_e,\ y_e$ of $SP(X, E)$, there exists a soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set (U, E) in $SS(X, E)$ containing x_e but not y_e and a soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set (V, E) in $SS(X, E)$ containing y_e but not x_e.

(3) A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I} - T_2$ if and only if for every pair of distinct soft points $x_e,\ y_e$ of $SP(X, E)$, there exists a soft pre-$\widetilde{I}$-regular pre- $\widetilde{I}$-open set (U, E) and (V, E) in $SS(X, E)$ containing x_e and y_e, respectively, such that $(U, E)\ \sqcap\ (V, E) = \mathbf{0}_E$.

Remark. If $(X, \widetilde{\tau}, E, \widetilde{I})$ is an $\widetilde{s}\delta p\widetilde{I} - T_i$ space, then it is $\widetilde{s}\delta p\widetilde{I} - T_{i-1}$, $i = 1,\ 2$. The converse need not be true as shown in the following example.

Example 4.7. Suppose that $(X, \widetilde{\tau}, E, \widetilde{I})$ is the soft ideal topological space in Example 2.4. The soft pre-$\widetilde{I}$- regular pre-$\widetilde{I}$-open sets are $\{\mathbf{1}_E,\ \mathbf{0}_E, \{(e, \{h_1\})\}, \{(e, \{h_2\})\}\}$. It is clear from Remark 4.1, that $(X, \widetilde{\tau}, E, \widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I} - T_0$ space but it is not $\widetilde{s}\delta p\widetilde{I} - T_1$.

Remark. It is easy to see from Remark 4.1 and the fact that every soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open set is soft pre-$\widetilde{I}$-open, that if a space $(X, \widetilde{\tau}, E, \widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I} - T_i$, then it is soft pre-$\widetilde{I} - T_i,\ i = 0, 1,\ 2$. The converse need not be true as seen from the following example.

Example 4.8. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space where $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$, $\widetilde{\tau} = \{\mathbf{1}_E, \mathbf{0}_E, (e, \{h_1\}), (e, \{h_3\}), (e, \{h_1, h_3\})\}$ and $\widetilde{I} = \{\mathbf{0}_E, (e, \{h_1\})\}$. Then we have that $\widetilde{SPIO}(X, E) = \{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1\})\}, \{(e, \{h_1, h_3\})\}, \{(e, \{h_3\})\}, \{(e, \{h_1, h_2, h_3\})\}, \{(e, h_1, h_3, h_4$ and the soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets are $\{\mathbf{1}_E, \mathbf{0}_E, \{(e, \{h_1\})\}, \{(e, \{h_3\})\}\}$. Then we have $(X, \widetilde{\tau}, E, \widetilde{I})$ is soft pre-$\widetilde{I} - T_0$ but not $\widetilde{s}\delta p\widetilde{I} - T_0$.

Theorem 4.1. *Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a soft ideal topological space. A space $(X, \widetilde{\tau}, E, \widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I} - T_2$ if and only if it is soft pre-$\widetilde{I}$- T_2.*

Proof. Assume that $(X, \widetilde{\tau}, E, \widetilde{I})$ is a soft pre-$\widetilde{I} - T_2$ space. Let $x_e,\ y_e\ \in SP(X, E)$ such that $x_e\ \neq\ y_e$, then by the assumption, there exist disjoint soft pre-$\widetilde{I}$-open sets (U, E) and (V, E) containing x_e and y_e, respectively. Since $(U, E) \sqcap (V, E) = \mathbf{0}_E$ and (V, E) is soft pre-$\widetilde{I}$-open, then $\widetilde{sp}\widetilde{I}Cl(U, E)\ \sqcap\ (V, E) = \mathbf{0}_E$ and thus, $\widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(U, E))\ \sqcap\ (V, E) = \mathbf{0}_E$. Similarly, since $\widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(U, E))$ is soft pre-$\widetilde{I}$-open, then $\widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(U, E)) \sqcap \widetilde{sp}\widetilde{I}Cl(V, E) = \mathbf{0}_E$ which implies that $\widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(U, E)) \sqcap \widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(V, E)) = \mathbf{0}_E$. Now, $(U, E)\ \sqsubseteq\ \widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(U, E))$ and $(V, E)\ \sqsubseteq\ \widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(V, E))$ as (U, E) and (V, E) are soft pre-$\widetilde{I}$-open sets. Thus, $\widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(U, E))$ and $\widetilde{sp}\widetilde{I}Int(\widetilde{sp}\widetilde{I}Cl(V, E))$ are disjoint soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$- open sets containing x_e and y_e, respectively. Hence by Remark 4.1 (3), $(X, \widetilde{\tau}, E, \widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I} - T_2$. The converse of proof follows from Remark 4.3. □

Theorem 4.2. *A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is a $\widetilde{s}\delta p\widetilde{I} - T_0$ space if and only if for each pair of distinct soft points x_e and y_e of $SP(X, E)$, $\widetilde{s}\delta p\widetilde{I}Cl(\{x_e\})\ \neq\ \widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$.*

Proof. Let $(X, \widetilde{\tau}, E, \widetilde{I})$ be a $\widetilde{s}\delta p\widetilde{I} - T_0$ space and $x_e,\ y_e \in SP(X, E)$, such that $x_e\ \neq\ y_e$. Then, there exists a $\widetilde{s}\delta p\widetilde{I}$-open set (G, E) containing x_e but not y_e and therefore $(G, E)^c$ is a $\widetilde{s}\delta p\widetilde{I}$-closed set which contains y_e but not x_e. By Lemma 3.6, we have $\widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})\ \sqsubseteq\ (G, E)^c$ and $x_e\ \widetilde{\notin}\widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$. Hence $\widetilde{s}\delta p\widetilde{I}Cl(\{x_e\})\ \neq\ \widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$. Conversely, suppose that $x_e,\ y_e\ \in SP(X, E), x_e\ \neq\ y_e$. Then by the assumption, $\widetilde{s}\delta p\widetilde{I}Cl(\{x_e\})\ \neq\ \widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$. Hence there exists at least one soft point $z_e\ \in\ SP(X, E)$ such that $z_e\ \widetilde{\in}\ \widetilde{s}\delta p\widetilde{I}Cl(\{x_e\})$ and $z_e\ \widetilde{\notin}\ \widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$, say. We claim that $x_e\ \widetilde{\notin}\widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$. If $x_e\ \widetilde{\in}\widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$, by Lemma 3.1(2), $\widetilde{s}\delta p\widetilde{I}Cl(\{x_e\})\ \sqsubseteq\ \widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$ which is a contradiction to the fact that $z_e\ \widetilde{\notin}\ \widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$. Thus $x_e\ \widetilde{\in}\ \{\widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})\}^c$. But by Lemma 3.1(1) and Corollary 3.5(1) we have $\{\widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})\}^c$ is a $\widetilde{s}\delta p\widetilde{I}$-open set that doesn't contain y_e. Therefore $(X, \widetilde{\tau}, E, \widetilde{I})$ is a $\widetilde{s}\delta p\widetilde{I} - T_0$ space. □

Theorem 4.3. *A soft ideal topological space $(X, \widetilde{\tau}, E, \widetilde{I})$ is a $\widetilde{s}\delta p\widetilde{I} - T_1$ space if and only if the soft singleton sets of $SS(X, E)$ are $\widetilde{s}\delta p\widetilde{I}$-closed.*

Proof. Suppose that $(X, \widetilde{\tau}, E, \widetilde{I})$ is a $\widetilde{s}\delta p\widetilde{I}-T_1$ space and $x_e \in SP(X, E)$. Let $y_e \in SP(X, E)\setminus\{x_e\}$. Then $x_e \neq y_e$ and so there exists a $\widetilde{s}\delta p\widetilde{I}$-open set (U, E) such that $y_e \widetilde{\in} (U, E)$ but $x_e \widetilde{\notin} (U, E)$. Consequently, $y_e \widetilde{\in} (U, E) \sqsubseteq x_e^c$. Now, $x_e^c = \sqcup\{(U, E) : y_e \widetilde{\in} x_e^c \widetilde{\in} S\delta P\widetilde{I}O(X)\}$. Therefore, x_e is $\widetilde{s}\delta p\widetilde{I}$-closed. Conversely, Let $x_e,\ y_e \in SP(X, E)$ such that $x_e \neq y_e$ and $x_e,\ y_e$ are $\widetilde{s}\delta p\widetilde{I}$-closed sets. Then by the assumption, x_e^c is a $\widetilde{s}\delta p\widetilde{I}$-open set containing y_e but not x_e. Similarly, y_e^c is a $\widetilde{s}\delta p\widetilde{I}$-open set containing x_e but not y_e. Therefore $(X, \widetilde{\tau}, E, \widetilde{I})$ is a $\widetilde{s}\delta p\widetilde{I} - T_1$ space. □

Definition 4.9. A soft function $f_{pu} : (X, \widetilde{\tau}, E, \widetilde{I}_1) \to (Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is called $\widetilde{s}\delta p\widetilde{I}$-continuous if $f_{pu}^{-1}(G, E) \in \widetilde{S}\delta P\widetilde{I}O(X)$ for every $(G, E) \in \widetilde{S}\delta P\widetilde{I}O(Y)$.

Theorem 4.4. *If $(Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is $\widetilde{s}\delta p\widetilde{I} - T_0$ space and $f_{pu} : (X, \widetilde{\tau}, E, \widetilde{I}_1) \to (Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is $\widetilde{s}\delta p\widetilde{I}$-continuous and soft injective, then $(X, \widetilde{\tau}, E, \widetilde{I}_1)$ is $\widetilde{s}\delta p\widetilde{I} - T_0$ space.*

Proof. Let $x_e,\ y_e \in SP(X, E)$ such that $x_e \neq y_e$. Since f_{pu} is soft injective and $(Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is $\widetilde{s}\delta p\widetilde{I}-T_0$, there exists $(G, E) \in \widetilde{S}\delta P\widetilde{I}O(Y)$ containing x_e and $f_{pu}(x_e) \widetilde{\in} (G, E),\ f_{pu}(y_e) \widetilde{\notin} (G, E)$ with $f_{pu}(x_e) \neq f_{pu}(y_e)$. Since f_{pu} is $\widetilde{s}\delta p\widetilde{I}$-continuous, then we have $f_{pu}^{-1}(G, E) \in \widetilde{S}\delta P\widetilde{I}O(X)$ such that $x_e \widetilde{\in} f_{pu}^{-1}(G, E)$ and $y_e \widetilde{\notin} f_{pu}^{-1}(G, E)$. Therefore $(X, \widetilde{\tau}, E, \widetilde{I}_1)$ is $\widetilde{s}\delta p\widetilde{I} - T_0$ space. □

Definition 4.10. A soft function $f_{pu} : (X, \widetilde{\tau}, E, \widetilde{I}_1) \to (Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is said to be soft point $\delta p\widetilde{I}$-closure one-to-one if $x_e,\ y_e \in SP(X, E)$ such that $\widetilde{s}\delta p\widetilde{I}Cl(\{x_e\}) \neq \widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$, implies $\widetilde{s}\delta p\widetilde{I}Cl(\{f_{pu}(x_e)\}) \neq \widetilde{s}\delta p\widetilde{I}Cl(\{f_{pu}(y_e)\})$.

Theorem 4.5. *If $f_{pu} : (X, \widetilde{\tau}, E, \widetilde{I}_1) \to (Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is soft point $\delta p\widetilde{I}$-closure one-to-one and $(X, \widetilde{\tau}, E, \widetilde{I}_1)$ is $\widetilde{s}\delta p\widetilde{I} - T_0$ space, then f_{pu} is one-to-one.*

Proof. Let $x_e,\ y_e \in SP(X, E)$ such that $x_e \neq y_e$. Since $(X, \widetilde{\tau}, E, \widetilde{I}_1)$ is $\widetilde{s}\delta p\widetilde{I} - T_0$, then by Theorem 4.2 we have $\widetilde{s}\delta p\widetilde{I}Cl(\{x_e\}) \neq \widetilde{s}\delta p\widetilde{I}Cl(\{y_e\})$. But f_{pu} is soft point $\delta p\widetilde{I}$-closure one-to-one implies that $\widetilde{s}\delta p\widetilde{I}Cl(\{f_{pu}(x_e)\}) \neq \widetilde{s}\delta p\widetilde{I}Cl(\{f_{pu}(y_e)\})$. Hence $f_{pu}(x_e) \neq f_{pu}(y_e)$. Therefore f_{pu} is one-to-one. □

Definition 4.11. A soft function $f_{pu} : (X, \widetilde{\tau}, E, \widetilde{I}_1) \to (Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is said to be $\widetilde{s}\delta p\widetilde{I}$-closed if $f_{pu}(G, E) \in \widetilde{S}\delta P\widetilde{I}C(Y)$ for every $(G, E) \in \widetilde{S}\delta P\widetilde{I}C(X)$.

Theorem 4.6. *Let $(X, \widetilde{\tau}, E, \widetilde{I}_1)$ be $\widetilde{s}\delta p\widetilde{I} - T_1$ and $f_{pu} : (X, \widetilde{\tau}, E, \widetilde{I}_1) \to (Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is $\widetilde{s}\delta p\widetilde{I}$-closed surjective function. Then $(Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is $\widetilde{s}\delta p\widetilde{I} - T_1$.*

Proof. Suppose that $y_e \in SP(Y)$. Since f_{pu} is soft surjective, then there exists $x_e \in SP(X, E)$ such that $f_{pu}(x_e) = y_e$. Since $(X, \widetilde{\tau}, E, \widetilde{I}_1)$ is $\widetilde{s}\delta p\widetilde{I} - T_1$, then from Theorem 4.3, we obtain $x_e \in \widetilde{S}\delta P\widetilde{I}C(X)$. Again by the hypothesis, we have $f_{pu}(x_e) = y_e \in \widetilde{S}\delta P\widetilde{I}C(Y)$. Hence, $(Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is $\widetilde{s}\delta p\widetilde{I} - T_1$. □

Theorem 4.7. *Let $f_{pu} : (X, \widetilde{\tau}, E, \widetilde{I}_1) \to (Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ be a soft injective and $\widetilde{s}\delta p\widetilde{I}$-continuous function. If $(Y, \widetilde{\sigma}, E, \widetilde{I}_2)$ is $\widetilde{s}\delta p\widetilde{I} - T_1$, then $(X, \widetilde{\tau}, E, \widetilde{I}_1)$ is $\widetilde{s}\delta p\widetilde{I} - T_1$.*

Proof. It's similar to the proof of Theorem 4.4. □

Theorem 4.8. *If $(X, \widetilde{\tau}, E, \widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I} - T_2$ space, then for all $x_e,\ y_e \in SP(X, E)$ with $x_e \neq y_e$ there exists $(F, E) \in \widetilde{S}\delta P\widetilde{I}O(X)$ such that $x_e \widetilde{\in} (F, E)$ and $y_e \widetilde{\notin} \widetilde{s}\delta p\widetilde{I}Cl(F, E)$.*

Proof. Let $x_e,\ y_e\ \in\ SP(X,E)$ such that $x_e\ \neq\ y_e$. Since $(X,\widetilde{\tau},E,\widetilde{I})$ is $\widetilde{s}\delta p\widetilde{I}-T_2$, then there exist two disjoint soft sets $(F,E),(G,E)\in\widetilde{S}\delta P\widetilde{I}O(X)$ such that $x_e\,\widetilde{\in}\,(F,E)$ and $y_e\,\widetilde{\in}\,(G,E)$. Clearly we have, $(G,E)^c\in\widetilde{S}\delta P\widetilde{I}C(X)$, $\widetilde{s}\delta p\widetilde{I}Cl(F,E)\ \sqsubseteq\ (G,E)^c$ and therefore, $y_e\,\widetilde{\notin}\,\widetilde{s}\delta p\widetilde{I}Cl(F,E)$. □

Theorem 4.9. *Let $f_{pu}\ :\ (X,\widetilde{\tau},E,\widetilde{I}_1)\ \to\ (Y,\widetilde{\sigma},E,\widetilde{I}_2)$ be a soft injective and $\widetilde{s}\delta p\widetilde{I}$-continuous function. If $(Y,\widetilde{\sigma},E,\widetilde{I}_2)$ is $\widetilde{s}\delta p\widetilde{I}-T_2$, then $(X,\widetilde{\tau},E,\widetilde{I}_1)$ is $\widetilde{s}\delta p\widetilde{I}-T_2$.*

Proof. Since f_{pu} is soft injective, so $f_{pu}(x_e)\ \neq\ f_{pu}(y_e)$ for each $x_e,\ y_e\in SP(X,E)$ and $x_e\neq y_e$. Now, $(Y,\ \widetilde{\sigma},E,\ \widetilde{I}_2)$ being $\widetilde{s}\delta p\widetilde{I}-T_2$, there exists $(F,E),(G,E)\in\widetilde{S}\delta P\widetilde{I}O(Y)$ such that $f_{pu}(x_e)\ \widetilde{\in}\ (F,E),\ f_{pu}(y_e)\ \widetilde{\in}\ (G,E)$ and $(F,E)\ \sqcap\ (G,E)\ =\ \mathbf{0}_E$. Suppose that $(U,E)\ =\ f_{pu}^{-1}(F,E)$ and $(V,E)\ =\ f_{pu}^{-1}(G,E)$. Then by $\widetilde{s}\delta p\widetilde{I}$-continuity, $(U,E),(V,E)\in\widetilde{S}\delta P\widetilde{I}O(X)$. Also, $x_e\,\widetilde{\in}\,f_{pu}^{-1}(F,E)=(U,E), y_e\,\widetilde{\in}\,f_{pu}^{-1}(G,E)=(V,E)$ and $(U,E)\ \sqcap\ (V,E)\ =\ f_{pu}^{-1}(F,E)\sqcap\ f_{pu}^{-1}(G,E)\ =\ \mathbf{0}_E$. Hence $(X,\widetilde{\tau},E,\widetilde{I}_1)$ is $\widetilde{s}\delta p\widetilde{I}-T_2$. □

5. Conclusion

The present work is devoted to define and study new classes of soft sets, namely soft delta pre-$\widetilde{I}$-open sets and soft pre-$\widetilde{I}$-regular pre-$\widetilde{I}$-open sets in soft ideal topological space. Also, a new class of soft separation axioms, namely soft delta pre ideal- T_i spaces, $i=0,1,2$ is introduced. We believe that it would be interesting to extend this approach to other structures such as Fuzzy soft topology, fuzzifying soft topology etc. We intend to investigate all these issues in future research works.

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References

[1] H. Aktas and N. Cağman, Soft sets and soft groups, Inf. Sci., **177** (2007), 2726-2735.
[2] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl., **57** (2009), 1547-1553.
[3] A. Aygünoğlu and H. Aygün, Some notes on soft topological spaces, Neural. Comput. Appl., **21** (1) (2012), 113-119.
[4] N. Cağman and S. Enginoglu, Soft set theory and uni-int decision making, Eur. J. Oper. Res., **207** (2010), 848-855.
[5] N. Cağman and S. Enginoglu, Soft matrix theory and its decision making, Comput. Math. Appl., **59** (2010), 3308-3314.
[6] N. Cağman, S. Karatas and S. Enginoglu, Soft topology, Comput. Math. Appl., **62** (2011), 351-358.
[7] D. Chen, E. C. C. Tsang, D. S. Yeung and X. Wang, The parameterization reduction of soft sets and its applications, Comput. Math. Appl., **49** (2005), 757-763.
[8] S.A. El-Sheikh, Decomposition of some types of soft sets and soft continuity via soft ideals, Gen. Math. Notes, **24** (2014), 103-124.
[9] F. Feng, Y. B. Jun and X. Z. Zhao, Soft semirings, Comput. Math. Appl., **56** (2008), 2621-2628.
[10] D. N. Georgiou, A. C. Megaritis and V. I. Petropoulos, On soft topological spaces, Appl. Math. Inf. Sci., **7** (2013), 1889-1901.
[11] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl., **62** (2011), 4058-4067.
[12] S. Jafari, Pre-rarely p-continuous function between topological spaces, Far East J. Math.Sci, (2000), Part-1 (Geometry and Topology), 87-96.

[13] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh and A.M.A. El-latif, Soft ideal theory, soft local function and generated soft topological spaces, Appl. Math. Inf. Sci., **8** (4) (2014), 1595-1603.
[14] A. Kandil, O. A. E. Tantawy, S.A. El-Sheikh and A.M. Abd El-latif, Soft connectedness via soft ideals, JNRS, **4** (2014) 90-108.
[15] K. Kannan, Soft generalized closed sets in soft topological spaces, J. Theo. Appl. Inf. Techn., **37** (2012), 17-21.
[16] O. Kazanci, S. Yilmaz and S. Yamak, Soft Sets and Soft BCH-Algebras, Hacettepe Journal of Mathematics and Statistics, **39** (2010), 205-217.
[17] A. Kharal and B. Ahmad, Mappings of soft classes, New. Math. Nat. Comput., **7**(3)(2011), 471-481.
[18] P. K. Maji, A. R. Roy and R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl., **44**(2002), 1077-1083.
[19] P. K. Maji, R. Biswas, and A. R. Roy, Soft set theory, Comput. Math. Appl., **45**(2003), 555-562.
[20] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math., **9**(2001), 589-602.
[21] P. Majumdar and S. K. Samanta, Similarity measure of soft sets, New. Math. Nat. Comput., **4**(2008), 1-12.
[22] W. K. Min, A note on soft topological spaces, Comput. Math. Appl., **62**(2011), 3524-3528.
[23] D. A. Molodtsov, Soft set theory-first results, Comput. Math. Appl., **37**(1999), 19-31.
[24] D. A. Molodtsov, The description of a dependence with the help of soft sets, J. Comput. Sys. Sc. Int., **40** (2001), 977-984.
[25] D. A. Molodtsov, The theory of soft sets (in Russian), URSS Publishers, Moscow, (2004).
[26] D. A. Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nech. Siste. Myagkie Vychisleniya, **1** (2006), 8-39.
[27] D. Pei and D. Miao, From soft sets to information systems, In: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager and B. Zhang, eds., Proceedings of Granular Computing, IEEE, **2**(2005), 617-621.
[28] E. Peyghan, B. Samadi and A. Tayebi, About soft topological spaces, J. New Results in Sci., **2** (2013), 60-75.
[29] O. R. Sayed and A. M Khalil, Some Properties of Soft θ-topology, Hacettepe Journal of Mathematics and Statistics, **44** (5) (2015), 1133–1145.
[30] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl., **61** (2011), 1786-1799.
[31] Y. C. Shao and K. Qin, The lattice structure of the soft groups, Procedia Engineering, **15** (2011), 3621-3625.
[32] P. Zhu and Qiaoyanwen, Operations on soft sets revisited, J. Applied Math., **2013** (2013), 7 pages. http://dx.doi.org/10.1155/2013/105752.
[33] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform., **3** (2012), 171-185.
[34] Y. Zou and Z. Xiao, Data analysis approaches of soft sets under incomplete information, Knowl. Base. Syst., **21** (2008), 941-945.

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