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A COMMON FIXED POINT THEOREM IN NON-ARCHIMEDEAN MENGER PM-SPACES

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Abstract. In this paper we prove a common fixed point theorem for six compatible self mappings of type (A) in a complete non-Archimedean Menger PM-space.

1. Introduction and preliminaries

Non-Archimedean probabilistic metric spaces and some topological preliminaries on them were first studied by Isrătescu and Crivăt [**7**]. Some fixed point theorems for mappings on non-Archimedean Menger spaces have been proved by Isrătescu [**5, 6**] as a result of the generalization of some of the results of Sehgal and Bharucha-Ried [**9**] and Sherwood [**10**]. Recently, Cho [**2**] introduced the notion of compatible mappings of type (A) in non-Archimedean Menger PM-spaces and proved a common fixed point theorem for four compatible mappings of type (A) in a complete non-Archimedean Menger PM-space.

In this paper we prove a unique common fixed point theorem for six compatible self mappings of type (A) in a complete non-Archimedean Menger PM-space under new contraction condition.

Definition 1.1. [**5, 7**] Let X be any any nonempty set and L be the set of all left-continuous distribution functions. An order pair $(x, \mathbf{F})$ is called a non-Archimedean probabilistic metric space (briefly, a N. A. PM-space) if $\mathbf{F}$ is a mapping from $X \times X$ to L satisfying the following conditions for all $x, y, z \in X$:

(PM-1): $F_{x,y}(t) = 1 \ for\ every\ t > 0\ if\ and\ only\ if\ x = y,$

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(PM-2): $F_{x,y}(0) = 0$,
(PM-3): $F_{x,y} = F_{y,x}$,
(PM-4): if $F_{x,y}(t_1) = 1$ *and* $F_{y,z}(t_2) = 1$, *then* $F_{x,z}(t_1 + t_2) = 1$.

DEFINITION 1.2. [**8**] A T-norm is a function $t : [0,1] \times [0,1] \to [0,1]$ which satisfies:

(T1): $t(a,1) = a$ and $t(0,0) = 0$,
(T2): $t(a,b) = t(b,a)$, (commutativity)
(T3): $t(c,d) \geqslant t(a,b)$, $c \geqslant a, d \geqslant b$, (nondecreasing in each coordinate)
(T4): $t(t(a,b),c) = t(a,t(b,c))$. (associativity)

DEFINITION 1.3. [**7**] A non-Archimedean Menger PM-space is an ordered triplet $(X, \mathbf{F}, t)$, where t is a t-norm and $(X, \mathbf{F})$ is a N. A. PM-space satisfying the following condition:

$$F_{x,z}(\max\{t_1, t_2\}) \geqslant t(F_{x,y}(t_1), F_{y,z}(t_2)) \text{ for all } x, y, z \in X \text{ and } t_1, t_2 \geqslant 0.$$

DEFINITION 1.4. [**2**] A N. A. Menger PM-space $(X, \mathbf{F}, t)$ is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that $g(F_{x,z}(t)) \leqslant g(F_{x,y}(t)) + g(F_{y,z}(t))$ for all $x, y, z \in X$, $t \geqslant 0$, where

$$\Omega = \{g | g : [0,1] \to [0,\infty), \text{is continuous, strictly decreasing with} g(1) = 0 \text{and} g(0) < \infty\}.$$

DEFINITION 1.5. [**2**] A N. A. Menger PM-space $(X, \mathbf{F}, t)$ is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that $g(t(t_1, t_2)) \leqslant g(t_1) + g(t_2)$ for all $t_1, t_2 \in [0,1]$.

REMARK 1.1. [**2**]

(i): If the N. A. Menger PM-space $(X, \mathbf{F}, t)$ is of type $(D)_g$ then it is of type $(C)_g$,
(ii): If $(X, \mathbf{F}, t)$ is N. A. Menger PM-space and $t(r,s) \geqslant t_{max}(r,s) = \max\{r + s - 1, 1\}$, for all $r, s \in [0,1]$, then $(X, \mathbf{F}, t)$ is of type $(D)_g$ for $g \in \Omega$ and $g(t) = 1 - t$.

Throughout this paper $(X, \mathbf{F}, t)$ is a complete N. A. Menger PM-space with a continuous strictly increasing t-norm. Let $\phi : [0,\infty) \to [0,\infty)$ be a function satisfying the condition

ϕ is upper semi-continuous from the right and $\phi(t) < t$ for $t > 0$. (Φ)

DEFINITION 1.6. [**2**] A sequence $\{x_n\}$ in the N. A. Menger PM-space $(X, \mathbf{F}, t)$ converges to a point x in X if and only if for each $\epsilon > 0$, $\lambda > 0$ there exists $M(\epsilon, \lambda)$ such that $g(F_{x_n,x}(\epsilon)) < g(1 - \lambda)$ for all $n > M$.

DEFINITION 1.7. [**2**] A sequence $\{x_n\}$ in the N. A. Menger PM-space is a Cauchy sequence if and only if for each $\epsilon > 0$, $\lambda > 0$ there exists $M(\epsilon, \lambda)$ such that $g(F_{x_n,x_m}(\epsilon)) < g(1 - \lambda)$ for all $m \geqslant n > M$.

EXAMPLE 1.1. [**11**] Let X be any set with at least two elements. If we define $F_{x,x}(t) = 1$ for all $x \in X$, $t > 0$ and $F_{x,y}(t) = \{0 \text{if} t \leqslant 1 \text{and} 1 \text{if} t > 1\}$, where $x, y \in X$, $x \neq y$, then $(X, \mathbf{F}, t)$ is the N. A. Menger PM-space with $t(a,b) = \min\{a,b\}$or $(a.b)$.

Lemma 1.1. [**1**] *If a function* $\phi : [0,\infty) \to [0,\infty)$ *satisfies the condition* (Φ), *then we get*

- **(i):** *for all* $t \geqslant 0$, $\lim_{n\to\infty} \phi^n(t) = 0$, *where* $\phi^n(t)$ *is the n-th iteration of* $\phi(t)$,
- **(ii):** *if* t_n *is a non-decreasing sequence of real numbers and* $t_{n+1} \leqslant \phi(t_n)$, $n = 1, 2, \ldots$, *then* $\lim_{n\to\infty} t_n = 0$. *In particular, if* $t \leqslant \phi(t)$, *for each* $t \geqslant 0$, *then* $t = 0$.

Lemma 1.2. [**2**] *Let* $\{y_n\}$ *be a sequence in* X *such that* $\lim_{n\to\infty} F_{y_n,y_{n+1}}(t) = 1$ *for each* $t > 0$. *If* $\{y_n\}$ *is not a Cauchy sequence in* X, *then there exist* $\epsilon_0 > 0$, $t_0 > 0$ *and two sequences* $\{m_i\}$ *and* $\{n_i\}$ *of positive integers such that*

- **(i):** $m_i > n_i + 1$ *and* $n_i \to \infty$ *as* $i \to \infty$,
- **(ii):** $F_{y_{m_i},y_{n_i}}(t_0) < 1 - \epsilon_0$ *and* $F_{y_{m_{i-1}},y_{n_i}}(t_0) \geqslant 1 - \epsilon_0$, $i = 1, 2, \ldots$.

Definition 1.8. [**3**] Let $A, S : X \to X$ be mappings. A and S are said to be compatible if

$$\lim_{n\to\infty} g(F_{ASx_n,SAx_n}(t)) = 0$$

for all $t > 0$, when $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Note that commuting and weakly commuting mappings are compatible but the converse is not true (see, [**4**]).

Definition 1.9. [**2**] Let $A, S : X \to X$ be mappings. A and S are said to be compatible of type (A) if

$$\lim_{n\to\infty} g(F_{ASx_n,SSx_n}(t)) = 0 \text{ and } \lim_{n\to\infty} g(F_{SAx_n,AAx_n}(t)) = 0$$

for all $t > 0$, when $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

Now, we give some relations between compatible mappings and compatible mappings of type (A) in non-Archimedean Menger PM-spaces which appears in [**2**].

Proposition 1.1. *Let* $A, S : X \to X$ *be continuous mappings. If* A *and* S *are compatible , then they are compatible of type (A).*

Proposition 1.2. *Let* $A, S : X \to X$ *be compatible mappings of type (A). If one of* A *and* S *is continuous, then they are compatible.*

Proposition 1.3. *Let* $A, S : X \to X$ *be continuous mappings.* A *and* S *are compatible if and only if they are compatible of type (A).*

Proposition 1.4. *Let* $A, S : X \to X$ *be mappings. If* A *and* S *are compatible of type (A) and* $Az = Sz$ *for some* $z \in X$, *then* $SAz = AAz = ASz = SSz$.

Proposition 1.5. *Let* $A, S : X \to X$ *be compatible mappings of type (A) and let* $\{x_n\}$ *be a sequence in* X *such that* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ *for some* $z \in X$. *Then we have the following:*

(1): $\lim_{n\to\infty} ASx_n = Sz$ *if* S *is continuous at* z,
(2): $SAz = ASz$ *and* $Sz = Az$ *if* A *and* S *are continuous at* z.

2. Main Results

In this section, we prove a common fixed point theorem for six self mappings in N. A. Menger PM-space.

Let A, B, S, T, L and M be six self mappings on a N. A. Menger PM-space $(X, \mathbf{F}, t)$ with,

$$L(X) \subseteq ST(X) \qquad \text{and} \qquad M(X) \subseteq AB(X). \tag{2.1}$$

Also, there exists $g \in \Omega$ such that:

$$\begin{aligned} g(F^2_{Lx,My}(t) \leqslant &\phi(\max\{g(F_{ABx,Lx}(t))g(F_{STy,My}(t)), \frac{1}{2}g(F_{ABx,My}(t))g(F_{STy,Lx}(t)), \\ &\frac{1}{2}g(F_{ABx,Lx}(t))g(F_{ABx,My}(t)), g(F_{STy,Lx}(t))g(F_{STy,My}(t)), \\ &g(F_{ABx,Lx}(t))g(F_{STy,Lx}(t)), \frac{1}{2}g(F_{ABx,My}(t))g(F_{STy,My}(t)), \\ &g(F^2_{ABx,Lx}(t)), g(F^2_{STy,My}(t)), g(F^2_{ABx,STy}(t))\}), \end{aligned} \tag{2.2}$$

for every $x, y \in X$ and $t \geqslant 0$, where ϕ satisfies the condition Φ. Then by (2.1), since $L(X) \subseteq ST(X)$, for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Lx_0 = STx_1$. As $M(X) \subseteq AB(X)$, for this point x_1, we can find $x_2 \in X$ such that $Mx_1 = ABx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that

$$\begin{aligned} &y_{2n} = Lx_{2n} = STx_{2n+1} \\ &y_{2n+1} = Mx_{2n+1} = ABx_{2n+2}, \qquad n = 0, 1, 2, ... \end{aligned} \tag{2.3}$$

Before proving our main theorem, we need to prove the following lemma:

Lemma 2.1. *Let* A, B, S, T, L *and* $M : X \to X$ *be mappings satisfying the conditions (2.1) and (2.2), then the sequence* $\{y_n\}$*, defined by (2.3), such that*

$$\lim_{n\to\infty} g(F_{y_n,y_{n+1}}(t)) = 0 \qquad \text{for all } t > 0$$

is a Cauchy sequence in X.

Proof. Since g is continuous and $g(1) = 0$, then $\lim_{n\to\infty} g(F_{y_n,y_{n+1}}(t)) = 0$ implies
$\lim_{n\to\infty} F_{y_n,y_{n+1}}(t) = 1$ for all $t > 0$.
By Lemma 1.2, if $\{y_n\}$ is not Cauchy sequence in X, there exists $\epsilon_0 > 0$, $t_0 > 0$ and two sequences $\{m_i\}$, $\{n_i\}$ of positive integers such that

(A): $m_i > n_i + 1$ and $n_i \to \infty$ as $i \to \infty$;
(B): $g(F_{y_{m_i},y_{n_i}}(t_0)) > g(1-\epsilon_0)$ and $g(F_{y_{m_i-1},y_{n_i}}) \leqslant g(1-\epsilon_0)$, $i = 1, 2, \dots$.

If we define $g(t) = 1 - t$ for all $t \in [0, 1]$, then $(X, \mathbf{F}, t)$ is a N. A. Menger PM-space of type $(D)_g$ for any $t \geqslant t_{max}$.

$$\begin{aligned} g(1-\epsilon_0) &< g(F_{y_{m_i},y_{n_i}}(t_0)) \\ &\leqslant g(F_{y_{m_i},y_{m_i-1}}(t_0)) + g(F_{y_{m_i-1},y_{n_i}}(t_0)) \\ &\leqslant g(F_{y_{m_i},y_{m_i-1}}(t_0)) + g(1-\epsilon_0). \end{aligned} \tag{2.4}$$

Letting $i \to \infty$ in (2.4), we have:

$$\lim_{n\to\infty} g(F_{y_{m_i},y_{n_i}}(t_0)) = g(1-\epsilon_0). \tag{2.5}$$

On the other hand, we have:

$$\begin{aligned} g(1-\epsilon_0) &< g(F_{y_{m_i},y_{n_i}}(t_0)) \\ &\leqslant g(F_{y_{n_i},y_{n_i+1}}(t_0)) + g(F_{y_{n_i+1},y_{m_i}}(t_0)). \end{aligned} \tag{2.6}$$

Now, we consider $g(F_{y_{n_i+1},y_{m_i}}(t_0))$ in (2.6), without loss of generality, assume that both n_i and m_i are even.
Using (2.2) at $x = x_{m_i}$ and $y = x_{n_i+1}$, gets:

$g(F^2_{Lx_{m_i},Mx_{n_i+1}}(t_0)) \leqslant \phi(\max\{g(F_{ABx_{m_i},Lx_{m_i}}(t_0))g(F_{STx_{n_i+1},Mx_{n_i+1}}(t_0)),$
$\frac{1}{2}g(F_{ABx_{m_i},Mx_{n_i+1}}(t_0))g(F_{STx_{n_i+1},Lx_{m_i}}(t_0)),$
$\frac{1}{2}g(F_{ABx_{m_i},Lx_{m_i}}(t_0))g(F_{ABx_{m_i},Mx_{n_i+1}}(t_0)),$
$g(F_{STx_{n_i+1},Lx_{m_i}}(t_0))g(F_{STx_{n_i+1},Mx_{n_i+1}}(t_0)),$
$g(F_{ABx_{m_i},Lx_{m_i}}(t_0))g(F_{STx_{n_i+1},Lx_{m_i}}(t_0)),$
$\frac{1}{2}g(F_{ABx_{m_i},Mx_{n_i+1}}(t_0))g(F_{STx_{n_i+1},Mx_{n_i+1}}(t_0)),\ g(F^2_{ABx_{m_i},Lx_{m_i}}(t_0)),$
$g(F^2_{STx_{n_i+1},Mx_{n_i+1}}(t_0)),\ g(F^2_{ABx_{m_i},STx_{n_i+1}}(t_0))\}),$

$g(F^2_{y_{m_i},y_{n_i+1}}(t_0)) \leqslant \phi(\max\{g(F_{y_{m_i-1},y_{m_i}}(t_0))g(F_{y_{n_i},y_{n_i+1}}(t_0)),$
$\frac{1}{2}g(F_{y_{m_i-1},y_{n_i+1}}(t_0))g(F_{y_{n_i},y_{m_i}}(t_0)),\ \frac{1}{2}g(F_{y_{m_i-1},y_{m_i}}(t_0))g(F_{y_{m_i-1},y_{n_i+1}}(t_0)),$
$g(F_{y_{n_i},y_{m_i}}(t_0))g(F_{y_{n_i},y_{n_i+1}}(t_0)),\ g(F_{y_{m_i-1},y_{m_i}}(t_0))g(F_{y_{n_i},y_{m_i}}(t_0)),$
$\frac{1}{2}g(F_{y_{m_i-1},y_{n_i+1}}(t_0))g(F_{y_{n_i},y_{n_i+1}}(t_0)),$
$g(F^2_{y_{m_i-1},y_{m_i}}(t_0)),\ g(F^2_{y_{n_i},y_{n_i+1}}(t_0)),\ g(F^2_{y_{m_i-1},y_{n_i}}(t_0))\}),$

Letting $i \to \infty$, we have:

$$\begin{aligned} \lim_{i\to\infty} g(F^2_{y_{m_i},y_{n_i+1}}(t_0)) &\leqslant \phi(\max\{0, \frac{1}{2}g(1-\epsilon_0)g(1-\epsilon_0), 0, 0, 0, 0, 0, 0, g(1-\epsilon_0)^2\}), \\ &\leqslant \phi(g(1-\epsilon_0)^2), \\ &< g((1-\epsilon_0)^2). \end{aligned} \tag{2.7}$$

Since $g \in \Omega$, by (2.7) we have

$$\begin{aligned} \lim_{i\to\infty} F^2_{y_{m_i},y_{n_i+1}}(t_0) &> (1-\epsilon_0)^2, \\ \lim_{i\to\infty} F_{y_{m_i},y_{n_i+1}}(t_0) &> 1-\epsilon_0, \\ g(\lim_{i\to\infty} F_{y_{m_i},y_{n_i+1}}(t_0)) &< g(1-\epsilon_0). \end{aligned}$$

Thus,

$$\lim_{i\to\infty} g(F_{y_{m_i},y_{n_i+1}}(t_0)) < g(1-\epsilon_0). \tag{2.8}$$

Letting $i \to \infty$ in (2.6), substituting by (2.8) gives:

$$\begin{aligned} g(1-\epsilon_0) &< \lim_{i\to\infty} g(F_{y_{n_i},y_{n_i+1}}(t_0)) + \lim_{i\to\infty} g(F_{y_{n_i+1},y_{m_i}}(t_0)), \\ &< 0 + g(1-\epsilon_0), \end{aligned}$$

which is a contradiction. Therefore $\{y_n\}$ is a Cauchy sequence in X. □

THEOREM 2.1. *Let $(X, \boldsymbol{F}, t)$ be a complete non-Archimedean Menger PM-space and A, B, S, T, L and M be mappings from X into itself satisfying the conditions (2.1), (2.2) and the following:*

(i): *$AB = BA$, $ST = TS$, $LB = BL$ and $MT = TM$;*
(ii): *one of the mappings ST, L, AB and M is continuous;*
(iii): *the pairs $\{L, AB\}$ and $\{M, ST\}$ are compatible of type (A).*

Then A, B, S, T, L and M have a unique common fixed point in X.

PROOF. **Step 1**. *We show that* $\lim_{n\to\infty} g(F_{y_n,y_{n+1}}(t)) = 0$ *forall* $t > 0$.
In fact, by (2.2) and (2.3), we have:

$$\begin{aligned} &g(F^2_{Lx_{2n},Mx_{2n+1}}(t)) \leqslant \phi(\max\{g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),\\ &\tfrac{1}{2}g(F_{ABx_{2n},Mx_{2n+1}}(t))g(F_{STx_{2n+1},Lx_{2n}}(t)),\\ &\tfrac{1}{2}g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{ABx_{2n},Mx_{2n+1}}(t)),\\ &g(F_{STx_{2n+1},Lx_{2n}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),\\ &g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STx_{2n+1},Lx_{2n}}(t)),\\ &\tfrac{1}{2}g(F_{ABx_{2n},Mx_{2n+1}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),\\ &g(F^2_{ABx_{2n},Lx_{2n}}(t)), g(F^2_{STx_{2n+1},Mx_{2n+1}}(t)), g(F^2_{ABx_{2n},STx_{2n+1}}(t))\}). \end{aligned}$$

$$\begin{aligned} &g(F^2_{y_{2n},y_{2n+1}}(t)) \leqslant \phi(\max\{g(F_{y_{2n-1},y_{2n}}(t))g(F_{y_{2n},y_{2n+1}}(t)),\\ &\tfrac{1}{2}g(F_{y_{2n-1},y_{2n+1}}(t))g(F_{y_{2n},y_{2n}}(t)),\\ &\tfrac{1}{2}g(F_{y_{2n-1},y_{2n}}(t))g(F_{y_{2n-1},y_{2n+1}}(t)), g(F_{y_{2n},y_{2n}}(t))g(F_{y_{2n},y_{2n+1}}(t)),\\ &g(F_{y_{2n-1},y_{2n}}(t))g(F_{y_{2n},y_{2n}}(t)), \tfrac{1}{2}g(F_{y_{2n-1},y_{2n+1}}(t))g(F_{y_{2n},y_{2n+1}}(t)),\\ &g(F^2_{y_{2n-1},y_{2n}}(t)), g(F^2_{y_{2n},y_{2n+1}}(t)), g(F^2_{y_{2n-1},y_{2n}}(t))\}),\\ &\leqslant \phi(\max\{g(F_{y_{2n-1},y_{2n}}(t))g(F_{y_{2n},y_{2n+1}}(t)), 0,\\ &\tfrac{1}{2}g(F_{y_{2n-1},y_{2n}}(t))[g(F_{y_{2n-1},y_{2n}}(t)) + g(F_{y_{2n},y_{2n+1}}(t))], 0, 0,\\ &\tfrac{1}{2}[g(F_{y_{2n-1},y_{2n}}(t)) + g(F_{y_{2n},y_{2n+1}}(t))]g(F_{y_{2n},y_{2n+1}}(t)),\\ &g(F^2_{y_{2n-1},y_{2n}}(t)), g(F^2_{y_{2n},y_{2n+1}}(t)), g(F^2_{y_{2n-1},y_{2n}}(t))\}). \end{aligned}$$

If $g(F_{y_{2n-1},y_{2n}}(t)) \leqslant g(F_{y_{2n},y_{2n+1}}(t))$ for all $n \in N$ and $t > 0$. Thus,
$g(F^2_{y_{2n},y_{2n+1}}(t)) \leqslant \phi(\max\{g^2(F_{y_{2n},y_{2n+1}}(t)), g(F_{y_{2n},y_{2n+1}}(t)), g(F^2_{y_{2n},y_{2n+1}}(t))\})$.
Since $g(t) \leqslant 1$ for all $t \in [0, 1]$, then

$$g^2(F_{y_{2n},y_{2n+1}}(t)) \leqslant g(F_{y_{2n},y_{2n+1}}(t)) \leqslant g(F^2_{y_{2n},y_{2n+1}}(t)).$$

Therefore, $g(F^2_{y_{2n},y_{2n+1}}(t)) \leqslant \phi(g(F^2_{y_{2n},y_{2n+1}}(t)))$. If we consider a decreasing sequence $M_{2n} = g(F^2_{y_{2n},y_{2n+1}}(t))$, we have $M_{2n-1} \leqslant M_{2n} \leqslant \phi(M_{2n})$. Therefore, by Lemma (2.1),

$$\lim_{n\to\infty} M_{2n} = \lim_{n\to\infty} g(F^2_{y_{2n},y_{2n+1}}(t)) = 0 \text{ for all } t > 0.$$

On the other hand, if $g(F_{y_{2n-1},y_{2n}}(t)) > g(F_{y_{2n},y_{2n+1}}(t))$, we have:

$$\begin{aligned} g(F^2_{y_{2n},y_{2n+1}}(t)) &< \phi(g(F^2_{y_{2n-1},y_{2n}}(t))), \\ &< \phi(\phi(g(F^2_{y_{2n-2},y_{2n-1}}(t)))), \\ &\vdots \\ &< \phi^{2n}(g(F^2_{y_0,y_1}(t))) \to 0 \text{ as } n \to \infty. \end{aligned}$$

Thus in all cases, we have:

$$\lim_{n\to\infty} g(F^2_{y_{2n},y_{2n+1}}(t)) = 0 \text{ for all } t > 0.$$

Similarly,

$$\lim_{n\to\infty} g(F^2_{y_{2n+1},y_{2n+2}}(t)) = 0 \text{ for all } t > 0.$$

Therefore,

$$\lim_{n\to\infty} g(F^2_{y_n,y_{n+1}}(t)) = 0 \text{ for all } t > 0.$$

By Lemma (2.1), $\{y_n\}$ is Cauchy sequence in X. Since X is complete, the sequence $\{y_n\}$ converges to a point $z \in X$ and so the subsequences $Lx_{2n}, Mx_{2n+1}, ABx_{2n}$ and STx_{2n+1} of $\{y_n\}$ also converge to the limit z.

Step 2. *We show the existence of the common fixed point of the six mappings under consideration at ST be continuous.*

Since M and ST are compatible of type (A), then by proposition (1.5),

$$MSTx_{2n+1}, STSTx_{2n+1} \to STz.$$

Using (2.2) at $x = x_{2n}$ and $y = STx_{2n+1}$, yields

$$\begin{aligned} &g(F^2_{Lx_{2n},MSTx_{2n+1}}(t)) \leqslant \phi(\max\{g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STSTx_{2n+1},MSTx_{2n+1}}(t)),\\ &\tfrac{1}{2}g(F_{ABx_{2n},MSTx_{2n+1}}(t))g(F_{STSTx_{2n+1},Lx_{2n}}(t)),\\ &\tfrac{1}{2}g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{ABx_{2n},MSTx_{2n+1}}(t)),\\ &g(F_{STSTx_{2n+1},Lx_{2n}}(t))g(F_{STSTx_{2n+1},MSTx_{2n+1}}(t)),\\ &g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STSTx_{2n+1},Lx_{2n}}(t)),\\ &\tfrac{1}{2}g(F_{ABx_{2n},MSTx_{2n+1}}(t))g(F_{STSTx_{2n+1},MSTx_{2n+1}}(t)),\\ &g(F^2_{ABx_{2n},Lx_{2n}}(t)), g(F^2_{STSTx_{2n+1},MSTx_{2n+1}}(t)), g(F^2_{ABx_{2n},STSTx_{2n+1}}(t))\}). \end{aligned}$$

Letting $n \to \infty$, we have:

$$\begin{aligned} g(F^2_{z,STz}(t)) &\leqslant \phi(\max\{0, \frac{1}{2}g^2(F_{z,STz}(t)), 0, 0, 0, 0, 0, 0, g(F^2_{z,STz}(t))\}), \\ &\leqslant \phi(g(F^2_{z,STz}(t))). \end{aligned}$$

By Lemma (1.1), we have $g(F^2_{z,STz}(t)) = 0$ for all $t > 0$, that is, $F^2_{z,STz}(t) = 1$ for all $t > 0$. Therefore, $z = STz$.

Again by using (2.2) with $x = x_{2n}$ and $y = z$, we have:

$$\begin{aligned} g(F^2_{Lx_{2n},Mz}(t)) \quad \leqslant \quad & \phi(\max\{g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STz,Mz}(t)), \frac{1}{2}g(F_{ABx_{2n},Mz}(t)) \\ & g(F_{STz,Lx_{2n}}(t)), \frac{1}{2}g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{ABx_{2n},Mz}(t)), \\ & g(F_{STz,Lx_{2n}}(t))g(F_{STz,Mz}(t)), g(F_{ABx_{2n},Lx_{2n}}(t)) \\ & g(F_{STz,Lx_{2n}}(t)), \frac{1}{2}g(F_{ABx_{2n},Mz}(t))g(F_{STz,Mz}(t)), \\ & g(F^2_{ABx_{2n},Lx_{2n}}(t)), g(F^2_{STz,Mz}(t)), g(F^2_{ABx_{2n},STz}(t))\}). \end{aligned}$$

Letting $n \to \infty$, we have

$$g(F^2_{z,Mz}(t)) \leqslant \phi(g(F^2_{z,Mz}(t))).$$

Hence, $z = Mz$. Since $M(X) \subseteq AB(X)$, there exists a point $w \in X$ such that $Mz = ABw = z$. At $x = w$ and $y = z$ in (2.2), we have:

$$\begin{aligned} g(F^2_{Lw,Mz}(t)) \quad \leqslant \quad & \phi(\max\{g(F_{ABw,Lw}(t))g(F_{STz,Mz}(t)), \frac{1}{2}g(F_{ABw,Mz}(t))g(F_{STz,Lw}(t)), \\ & \frac{1}{2}g(F_{ABw,Lw}(t))g(F_{ABw,Mz}(t)), g(F_{STz,Lw}(t))g(F_{STz,Mz}(t)), \\ & g(F_{ABw,Lw}(t))g(F_{STz,Lw}(t)), \frac{1}{2}g(F_{ABw,Mz}(t))g(F_{STz,Mz}(t)), \\ & g(F^2_{ABw,Lw}(t)), g(F^2_{STz,Mz}(t)), g(F^2_{ABw,STz}(t))\}), \\ g(F^2_{Lw,z}(t)) \quad \leqslant \quad & \phi(\max\{0, 0, 0, 0, g^2(F_{z,Lw}(t)), 0, g(F^2_{z,Lw}(t)), 0, 0\}), \\ \leqslant \quad & \phi(g(F^2_{z,Lw}(t))), \end{aligned}$$

which means that $Lw = z$. Since L and AB are compatible of type (A) and $Lw = ABw = z$, by proposition (1.4), $Lz = LABw = ABLw = ABz$. Again by using (2.2), we have $Lz = z$. Therefore, $Lz = ABz = Mz = STz = z$, i.e., z is a common fixed point of the mappings L, AB, M and ST.

Step 3. *We show the existence of the common fixed point at L be continuous.*

As L is continuous and (L, AB) is compatible of type (A), then $L^2x_{2n}, ABLx_{2n} \to Lz$. Putting $x = Lx_{2n}$ and $y = x_{2n+1}$ in (2.2), we have:

$$\begin{aligned} & g(F^2_{LLx_{2n},Mx_{2n+1}}(t)) \leqslant \phi(\max\{g(F_{ABLx_{2n},LLx_{2n}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)), \\ & \tfrac{1}{2}g(F_{ABLx_{2n},Mx_{2n+1}}(t))g(F_{STx_{2n+1},LLx_{2n}}(t)), \\ & \tfrac{1}{2}g(F_{ABLx_{2n},LLx_{2n}}(t))g(F_{ABLx_{2n},Mx_{2n+1}}(t)), \\ & g(F_{STx_{2n+1},LLx_{2n}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)), \\ & g(F_{ABLx_{2n},LLx_{2n}}(t))g(F_{STx_{2n+1},LLx_{2n}}(t)), \\ & \tfrac{1}{2}g(F_{ABLx_{2n},Mx_{2n+1}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)), \\ & g(F^2_{ABLx_{2n},LLx_{2n}}(t)), g(F^2_{STx_{2n+1},Mx_{2n+1}}(t)), g(F^2_{ABLx_{2n},STx_{2n+1}}(t))\}). \end{aligned}$$

Letting $n \to \infty$, we get

$$\begin{aligned} g(F^2_{Lz,z}(t)) &\leqslant \phi(\max\{0, \frac{1}{2}g^2(F_{Lz,z}(t)), 0, 0, 0, 0, 0, 0, g(F^2_{Lz,z}(t))\}), \\ &\leqslant \phi(g(F^2_{Lz,z}(t))). \end{aligned}$$

That is , $Lz = z$.

Since $L(X) \subseteq ST(X)$, there exists a point $w_1 \in X$ such that $Lz = STw_1 = z$. At $x = x_{2n}$ and $y = w_1$ in (2.2), we have:

$g(F^2_{Lx_{2n},Mw_1}(t)) \leqslant \phi(\max\{g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STw_1,Mw_1}(t)),$
$\frac{1}{2}g(F_{ABx_{2n},Mw_1}(t))g(F_{STw_1,Lx_{2n}}(t)),$
$\frac{1}{2}g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{ABx_{2n},Mw_1}(t)),$
$g(F_{STw_1,Lx_{2n}}(t))g(F_{STw_1,Mw_1}(t)),$
$g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STw_1,Lx_{2n}}(t)),$
$\frac{1}{2}g(F_{ABx_{2n},Mw_1}(t))g(F_{STw_1,Mw_1}(t)),$
$g(F^2_{ABx_{2n},Lx_{2n}}(t)), g(F^2_{STw_1,Mw_1}(t)),\ g(F^2_{ABx_{2n},STw_1}(t))\}).$

Letting $n \to \infty$, we obtain

$$\begin{aligned} g(F^2_{z,Mw_1}(t)) &\leqslant \phi(\max\{0, 0, 0, 0, 0, \frac{1}{2}g^2(F_{z,Mw_1}(t)), g(F^2_{z,Mw_1}(t)), 0, 0\}), \\ &\leqslant \phi(g(F^2_{z,Mw_1}(t))). \end{aligned}$$

which means that $Mw_1 = z$. AS M and ST are compatible of type (A) and $Mw_1 = STw_1 = z$, by proposition 2.4, $Mz = MSTw_1 = STMw_1 = STz$. As in step 3 we have $Mz = z$. Therefore, $Lz = Mz = STz = z$.

As $M(X) \subseteq AB(X)$, there exists a point $w \in X$ such that $Mz = ABw = z$. At $x = w$ and $y = z$ in (2.2), we have:

$$\begin{aligned} g(F^2_{Lw,Mz}(t)) &\leqslant \phi(\max\{g(F_{ABw,Lw}(t))g(F_{STz,Mz}(t)), \frac{1}{2}g(F_{ABw,Mz}(t))g(F_{STz,Lw}(t)), \\ &\quad \frac{1}{2}g(F_{ABw,Lw}(t))g(F_{ABw,Mz}(t)), g(F_{STz,Lw}(t))g(F_{STz,Mz}(t)), \\ &\quad g(F_{ABw,Lw}(t))g(F_{STz,Lw}(t)), \frac{1}{2}g(F_{ABw,Mz}(t))g(F_{STz,Mz}(t)), \\ &\quad g(F^2_{ABw,Lw}(t)), g(F^2_{STz,Mz}(t)), g(F^2_{ABw,STz}(t))\}), \\ g(F^2_{Lw,z}(t)) &\leqslant \phi(\max\{0, 0, 0, 0, g^2(F_{z,Lw}(t)), 0, g(F^2_{z,Lw}(t)), 0, 0\}), \\ &\leqslant \phi(g(F^2_{z,Lw}(t))), \end{aligned}$$

which means that $Lw = z$. Since L and AB are compatible of type (A) and $Lw = ABw = z$, by proposition (1.4), $z = Lz = LABw = ABLw = ABz$. Therefore, $Lz = ABz = Mz = STz = z$, i.e., z is a common fixed point of the mappings L, AB, M and ST.

Step 4. *At the continuity of AB.*

Since L and AB are compatible of type (A), then by proposition (1.5),

$$LABx_{2n}, ABABx_{2n} \to ABz.$$

Using (2.2) at $x = ABx_{2n}$ and $y = x_{2n+1}$, we have:
$g(F^2_{LABx_{2n},Mx_{2n+1}}(t)) \leqslant \phi(\max\{g(F_{ABABx_{2n},LABx_{2n}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$\frac{1}{2}g(F_{ABABx_{2n},Mx_{2n+1}}(t))g(F_{STx_{2n+1},LABx_{2n}}(t)),$
$\frac{1}{2}g(F_{ABABx_{2n},LABx_{2n}}(t))g(F_{ABABx_{2n},Mx_{2n+1}}(t)),$
$g(F_{STx_{2n+1},LABx_{2n}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$g(F_{ABABx_{2n},LABx_{2n}}(t))g(F_{STx_{2n+1},LABx_{2n}}(t)),$
$\frac{1}{2}g(F_{ABABx_{2n},Mx_{2n+1}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$g(F^2_{ABABx_{2n},LABx_{2n}}(t)), g(F^2_{STx_{2n+1},Mx_{2n+1}}(t)), g(F^2_{ABABx_{2n},STx_{2n+1}}(t))\}).$
Letting $i \to \infty$, yields

$$\begin{aligned} g(F^2_{ABz,z}(t)) &\leqslant \phi(\max\{0, \frac{1}{2}g^2(F_{ABz,z}(t)), 0, 0, 0, 0, 0, 0, g(F^2_{ABz,z}(t))\}), \\ &\leqslant \phi(g(F^2_{ABz,z}(t))). \end{aligned}$$

Then, $ABz = z$.

Again by using (2.2) with $x = z$ and $y = x_{2n+1}$, we have:
$g(F^2_{Lz,Mx_{2n+1}}(t)) \leqslant \phi(\max\{g(F_{ABz,Lz}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$\frac{1}{2}g(F_{ABz,Mx_{2n+1}}(t))g(F_{STx_{2n+1},Lz}(t)),$
$\frac{1}{2}g(F_{ABz,Lz}(t))g(F_{ABz,Mx_{2n+1}}(t)), g(F_{STx_{2n+1},Lz}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$g(F_{ABz,Lz}(t))g(F_{STx_{2n+1},Lz}(t)),$
$\frac{1}{2}g(F_{ABz,Mx_{2n+1}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$g(F^2_{ABz,Lz}(t)), g(F^2_{STx_{2n+1},Mx_{2n+1}}(t)), g(F^2_{ABz,STx_{2n+1}}(t))\}).$

Letting $n \to \infty$, we have

$$g(F^2_{Lz,z}(t)) \leqslant \phi(g(F^2_{Lz,z}(t))).$$

Hence, $Lz = z$.

Since $L(X) \subseteq ST(X)$, there exists a point $w_1 \in X$ such that $Lz = STw_1 = z$. At $x = z$ and $y = w_1$ in (2.2), we have:
$g(F^2_{Lz,Mw_1}(t)) \leqslant \phi(\max\{g(F_{ABz,Lz}(t))g(F_{STw_1,Mw_1}(t)),$
$\frac{1}{2}g(F_{ABz,Mw_1}(t))g(F_{STw_1,Lz}(t)),$
$\frac{1}{2}g(F_{ABz,Lz}(t))g(F_{ABz,Mw_1}(t)), g(F_{STw_1,Lz}(t))g(F_{STw_1,Mw_1}(t)),$
$g(F_{ABz,Lz}(t))g(F_{STw_1,Lz}(t)), \frac{1}{2}g(F_{ABz,Mw_1}(t))g(F_{STw_1,Mw_1}(t)),$
$g(F^2_{ABz,Lz}(t)), g(F^2_{STw_1,Mw_1}(t)), g(F^2_{ABz,STw_1}(t))\}).$
$g(F^2_{z,Mw_1}(t)) \leqslant \phi(\max\{0, 0, 0, 0, 0, g^2(F_{z,Mw_1}(t)), 0, g(F^2_{z,Mw_1}(t)), 0\}),$
$\leqslant \phi(g(F^2_{z,Mw_1}(t))).$
which means that $Mw_1 = z$. Since M and ST are compatible of type (A) and $Mw_1 = STw_1 = z$, by proposition 2.4, $Mz = MSTw_1 = STMw_1 = STz$. Again by using (2.2), we have $Mz = z$. Therefore, $Lz = ABz = Mz = STz = z$, i.e., z is a common fixed point of the mappings L, AB, M and ST. By a similar way we can prove the theorem at M continuous.

Step 5. Putting $x = Bz$, $y = x_{2n+1}$ in (2.2), we get:
$g(F^2_{LBz,Mx_{2n+1}}(t)) \leqslant$
$\phi(\max\{g(F_{ABBz,LBz}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$\frac{1}{2}g(F_{ABBz,Mx_{2n+1}}(t))g(F_{STx_{2n+1},LBz}(t)),$
$\frac{1}{2}g(F_{ABBz,LBz}(t))g(F_{ABBz,Mx_{2n+1}}(t)),$
$g(F_{STx_{2n+1},LBz}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$g(F_{ABBz,LBz}(t))g(F_{STx_{2n+1},LBz}(t)),$
$\frac{1}{2}g(F_{ABBz,Mx_{2n+1}}(t))g(F_{STx_{2n+1},Mx_{2n+1}}(t)),$
$g(F^2_{ABBz,LBz}(t)), g(F^2_{STx_{2n+1},Mx_{2n+1}}(t)), g(F^2_{ABBz,STx_{2n+1}}(t))\}).$

As $BL = LB$ and $AB = BA$, so $L(Bz) = B(Lz) = Bz$ and $ABBz = B(ABz) = Bz$. Letting $n \to \infty$, we have:

$$\begin{aligned} g(F^2_{Bz,z}(t)) &\leqslant \phi(\max\{0, \frac{1}{2}g^2(F_{Bz,z}(t)), 0, 0, 0, 0, 0, 0, g(F^2_{Bz,z}(t))\}), \\ &\leqslant \phi(g(F^2_{Bz,z}(t))). \end{aligned}$$

Since $ABz = z$ and $Bz = z$, then $Az = z$. Thus,

$$z = Lz = Az = Bz. \tag{2.9}$$

Step 6. Putting $x = x_{2n}$ and $y = Tz$ in (2.2), we get:
$g(F^2_{Lx_{2n},MTz}(t)) \leqslant \phi(\max\{g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STTz,MTz}(t)),$
$\frac{1}{2}g(F_{ABx_{2n},MTz}(t))g(F_{STTz,Lx_{2n}}(t)),$
$\frac{1}{2}g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{ABx_{2n},MTz}(t)), g(F_{STTz,Lx_{2n}}(t))g(F_{STTz,MTz}(t)),$
$g(F_{ABx_{2n},Lx_{2n}}(t))g(F_{STTz,Lx_{2n}}(t)),$
$\frac{1}{2}g(F_{ABx_{2n},MTz}(t))g(F_{STTz,MTz}(t)),$
$g(F^2_{ABx_{2n},Lx_{2n}}(t)), g(F^2_{STTz,MTz}(t)), g(F^2_{ABx_{2n},STTz}(t))\}).$

As $MT = TM$ and $ST = TS$, so $M(Tz) = T(Mz) = Tz$ and $STTz = T(STz) = Tz$. Letting $n \to \infty$, we have:
$g(F^2_{z,Tz}(t)) \leqslant \phi(g(F^2_{z,Tz}(t)))$. Since $STz = z$ and $Tz = z$, then $Sz = z$. Thus,

$$z = Mz = Sz = Tz. \tag{2.10}$$

Combining (2.9) and (2.10), we have, $Az = Bz = Lz = Mz = Tz = Sz = z$. Hence, the six mappings have a common fixed point in X.
Step 7.(Uniqueness)
Let z_1 be another common fixed point of the mappings. Putting $x = z$ and $y = z_1$ in (2.2), yields:
$g(F^2_{Lz,Mz_1}(t)) \leqslant \phi(\max\{g(F_{ABz,Lz}(t))g(F_{STz_1,Mz_1}(t)),$
$\frac{1}{2}g(F_{ABz,Mz_1}(t))g(F_{STz_1,Lz}(t)),$
$\frac{1}{2}g(F_{ABz,Lz}(t))g(F_{ABz,Mz_1}(t)), g(F_{STz_1,Lz}(t))g(F_{STz_1,Mz_1}(t)),$
$g(F_{ABz,Lz}(t))g(F_{STz_1,Lz}(t)), \frac{1}{2}g(F_{ABz,Mz_1}(t))g(F_{STz_1,Mz_1}(t)),$

$g(F^2_{ABz,Lz}(t)), g(F^2_{STz_1,Mz_1}(t)), g(F^2_{ABz,STz_1}(t))\}).$

$$g(F^2_{z,z_1}(t)) \leqslant \phi(\max\{g(F_{z,z}(t))g(F_{z_1,z_1}(t)), \tfrac{1}{2}g(F_{z,z_1}(t))g(F_{z_1,z}(t)),$$
$$\tfrac{1}{2}g(F_{z,z}(t))g(F_{z,z_1}(t)), g(F_{z_1,z}(t))g(F_{z_1,z_1}(t)),$$
$$g(F_{z,z}(t))g(F_{z_1,z}(t)), \tfrac{1}{2}g(F_{z,z_1}(t))g(F_{z_1,z_1}(t)),$$
$$g(F^2_{z,z}(t)), g(F^2_{z_1,z_1}(t)), g(F^2_{z,z_1}(t))\}).$$

$$g(F^2_{z,z_1}(t)) \leqslant \phi(\max\{0, g^2(F_{z,z_1}(t)), 0, 0, 0, 0, 0, 0, g(F^2_{z,z_1}(t))\}),$$
$$\leqslant \phi(g(F^2_{z,z_1}(t))).$$

Thus $z = z_1$ and z is the unique common fixed point of the mappings. □

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