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Entropy Squeezing of a Qubit Interdicting with Two-Mode Kerr Nonlinear Coupler Due to Intrinsic Damping

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Abstract An analytical description for the dynamical evolution of a qubit interacting with two nonlinear Kerr oscillators pumped by optical parametric process is derived through Su$(1, 1)$-algebraic treatment. The role of intrinsic damping, detuning and Kerr-like Medium on the squeezing phenomenon is elucidated via information entropy squeezing. The evolutions of the interaction of the qubit with two-mode Kerr nonlinear coupler lead to the appearance the regular squeezing phenomenon during the chosen time-interval. The preserving and protecting of the qubit components from the squeezing can be controlled by the intrinsic decoherence, detuning and the Kerr-like medium effects. Where the squeezing phenomenon deteriorates with increasing the decoherence rate, whereas, the Kerr-like medium can not protect some qubit components from the squeezing.

Keywords Intrinsic coherence · Kerr medium · Entropy squeezing

1 Introduction

The squeezing phenomenon of field and atom components is one of the nonclassical phenomena. It has attracted considerable attention due to its potential application in optical communications, [1] detections of weak signals, [2] the high precision atomic fountain clock, [3]. The qubit entropy squeezing (QES) has applications in quantum optic, [4] quantum information, [5] can effectively inhibit quantum noise, [6, 7] quantum optomechanical system [8]. After defining the QES by using the quantum information theory [13] accord-

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ing to entropic uncertainty relation based on position and momentum, more investigations of QES has appeared [14–19].

In the field of optical communication network, the co-directional and contra-directional propagation of light through two adjacent parallel wave guides may produce switching modulation and radiation frequency selection [20–22]. Among the different types of directional couplers, the directional Kerr nonlinear coupler which has received much attention in quantum optics. Recently, it has been broadly explored [23–26]. The parametric amplification interaction is considered as one of the most important nonlinear interactions. Where, squeezed light can be produced through it [27, 28], which could be generated by a non-degenerate parametric down-conversion for excitation of a 2-photon transition in atoms [29].

In real physical models, the decoherence causes decreasing and disappearing of non-classical phenomena as purity, and entanglement, and squeezing which may induce failure of the algorithms and various protocols of quantum information.

In this paper, we explore the decoherence effect on the squeezing phenomenon in two-mode Kerr nonlinear coupler interdicting with Su(2)-system in the physical system of a qubit interacting with two nonlinear Kerr oscillators driven by optical parametric process.

This paper is divided as follow: in Section 2, the physical model and its solution are presented. In Section 3, we recall the entropy squeezing definitions of a qubit. By numerical calculations, we analyze the entropy squeezing under the intrinsic decoherence, detuning and the Kerr-like medium effects. The conclusion is included in Section 4.

2 Hamiltonian and Dynamics

2.1 Hamiltonian

The physical which is formed from a two-level atom interacts with two nonlinear Kerr oscillators mutually coupled by parametric pumping [30, 31]. The Hamiltonian that describes this system is given by

$$\begin{aligned}\hat{H} &= \frac{\omega_0}{2}\hat{\sigma}_z + \sum_{i=1}^{2}\omega_i\left(\hat{\psi}_i^{\dagger}\hat{\psi}_i + \frac{1}{2}\right) + \tilde{\chi}_1\hat{a}^{\dagger}\hat{a}\hat{b}^{\dagger}\hat{b} \\ &+\tilde{\chi}_2\left(\hat{\psi}_1^{\dagger 2}\hat{\psi}_1^2 + \hat{\psi}_2^{\dagger 2}\hat{\psi}_2^2\right) + \lambda\left(\hat{\psi}_1\hat{\psi}_2\hat{\sigma}_+ + \hat{\psi}_1^{\dagger}\hat{\psi}_2^{\dagger}\hat{\sigma}_-\right), \qquad (1)\end{aligned}$$

where $\omega_i (i = 1, 2)$ are the frequencies of the ith-mode with the annihilation operators $\hat{\psi}_i$. The $\bar{\chi}_1$ and $\bar{\chi}_2$ are the nonlinear couplings responsible for the self-action processes and cross-action. The λ is the coupling parameter between the two-level atom and the two-mode parametric process. Here the linear coupling between the two nonlinear oscillators is neglected.

Here, we set $\bar{\chi}_1 = 2\bar{\chi}_2 = \frac{1}{2}\chi$ and $\omega_1 = \omega_2 = \frac{1}{2}\omega + \frac{1}{4}\chi$, and introduce the Su(1, 1) generators $(\hat{R}_{\pm}, \hat{R}_0)$ as follows: $\hat{R}_- = \hat{\psi}_1\hat{\psi}_2 = \hat{K}^{\dagger}{}_+$ and $\hat{R}_0 = \frac{1}{2}(\hat{\psi}_1^{\dagger}\hat{\psi}_1 + \hat{\psi}_2^{\dagger}\hat{\psi}_2 + 1)$, where, $[\hat{R}_0, \hat{R}_{\pm}] = \pm\hat{R}_{\pm}$ and $[\hat{R}_-, \hat{R}_+] = 2\hat{R}_0$ and the Casimier operator $\hat{R}^2 = \hat{R}_0^2 - \frac{1}{2}(\hat{R}_+\hat{R}_- + \hat{R}_-\hat{R}_+) = k(k-1)\hat{I}$ with the Bargmann number k. Therefore, the Hamiltonian is then given by

$$\hat{H} = \frac{\omega_0}{2}\hat{\sigma}_z + \omega\hat{R}_0 + \chi R_o^2 + \lambda\left[\hat{R}_-\hat{\sigma}_+ + \hat{R}_+\hat{\sigma}_-\right] \qquad (2)$$

The relations of the operators $\hat{R}_+$, $\hat{R}_-$ and $\hat{R}_0$ of the Hamiltonian (2) are:

$$\begin{aligned} \hat{R}_-|n,k\rangle &= \sqrt{n(n+2k-1)}\,|n-1,k\rangle, \\ \hat{R}_+|n,k\rangle &= \sqrt{(n+1)(n+2k)}\,|n+1,k\rangle, \\ \hat{R}_0|n,k\rangle &= (n+k)|n,k\rangle, \;\; \hat{R}^2|n,k\rangle = k(k-1)|n,k\rangle, \end{aligned} \tag{3}$$

Here, the main purpose is, to study squeezing properties of the components of the qubit in the presence of intrinsic decoherence. It is found that the dissipation and decoherence [32–37] are the most critical obstacles for the non-classical phenomena as: purity, and entanglement, and squeezing. To derive the time-dependent density matrix form with the effect of the dephasing, the pure phase decoherence mechanism is only taken into account. Therefore the master equation of the systems is given by [38, 39]

$$\frac{d}{dt}\rho(t) = -i[H,\rho] - \gamma[H,[H,\rho]], \tag{4}$$

where γ is the phase decoherence rate.

It can be said that the two Su(2)-system is initially prepared in an upper state, i.e., $\rho^A(0) = |\uparrow\rangle\langle\uparrow|$, while, the Su$(1,1)$-system is initially in Barut-Girardello Su$(1,1)$ coherent state [40], that is defined as:

$$|\mu,k\rangle = \sum_{n=0}^{\infty} P_n|n,k\rangle, \tag{5}$$

with

$$P_n = \sqrt{\frac{|\mu|^{2k-1}}{I_{2k-1}(2|\mu|)}}\frac{\mu^n}{\sqrt{n!\Gamma(2k+n)}}$$

The state $|\mu,k\rangle$ corresponds to the eigenstate of the generator R_- given by: $R_-|\mu,k\rangle = \mu|\mu,k\rangle$. The function $I_\nu(x)$ represents the first kind modified Bessel function.

Here, we use the method of the dressed-state representation to find the solution of the system, where, we write the initial state $\rho(0) = |1\rangle\langle 1| \otimes |\mu,k\rangle\langle\mu,k|$ of the entire system in term the dressed state of the Hamiltonian (2). In the space state of the total, $\left\{|\phi_1^n\rangle = |e,n\rangle, |\phi_2^n\rangle = |g,n+1\rangle\right\}$, the time-evolution of the entire density matrix is given by

$$\begin{aligned} \rho(t) = \sum_{i,j=0} P_{i,j} \Big\{ &\left[u_{ij}^{++} + u_{ij}^{+-} + u_{ij}^{-+} + u_{ij}^{--}\right]|\phi_1^i\rangle\langle\phi_1^j \\ &+ \left[u_{ij}^{++} - u_{ij}^{+-} + u_{ij}^{-+} - u_{ij}^{--}\right]\left||\phi_2^i\rangle\langle\phi_2^j\right| \\ &+ \left[u_{ij}^{++} + u_{ij}^{+-} - u_{ij}^{-+} - u_{ij}^{--}\right]\left||\phi_2^i\rangle\langle\phi_1^j\right| \\ &+ \left[u_{ij}^{++} - u_{ij}^{+-} - u_{ij}^{-+} + u_{ij}^{--}\right]\left||\phi_2^i\rangle\langle\phi_2^j\right|\Big\}, \end{aligned} \tag{6}$$

where $u_{ij}^{\pm\pm} = \exp\left[-i\left(\Lambda_i^{\pm} - \Lambda_j^{\pm}\right)t - \gamma\left(\Lambda_i^{\pm} - \Lambda_j^{\pm}\right)^2 t\right]$, the eigenvalues of H are given by

$$\Lambda_n^{\pm} = \omega\left(n+k+\frac{1}{2}\right) + a \pm \sqrt{(b-\delta)^2+\nu^2}, \tag{7}$$

where $\delta = (\omega_0 - \omega)/2$ is the detuning and

$$\begin{aligned} \nu &= \lambda\sqrt{(n+1)(n+2k)}, \quad b = n+k+0.5, \\ a &= (n+k)^2 + n + k + 0.5. \end{aligned} \tag{8}$$

To investigate the entropy squeezing of the Su(2)-system, one has to trace out the states of Su(1, 1)-system, $|m\rangle$, from final states of (6). Then, the final states of Su(2)-system are given by

$$\rho^A(t) = \sum_{m=0}^{\infty} \langle m|\rho(t)|m\rangle. \tag{9}$$

Therefore, we can quantify squeezing properties of the Su(2)-system by the entropy squeezing function. In the next section, we study this function in details.

3 Information Entropy Squeezing

According to previous literatures, the squeezing of entropy squeezing is one of the nonclassical phenomena in the field of quantum optics. Therefore, we investigate the information entropy squeezing of a qubit interdicting with two-mode Kerr nonlinear coupler due to intrinsic damping. According to investigations [9–13], the information entropy of a qubit system is given by:

$$H\left(S_\alpha\right) = \sum_{i=1}^{2} P_i\left(S_\alpha\right) \ln\left[P_i\left(S_\alpha\right)\right], \qquad \alpha = x, y, z. \tag{10}$$

Where S_i $(i = x; y; z)$ are the Pauli matrices, and $P_i(S_\alpha)$ is the probability distribution for two possible outcomes of measurements of an operator S_α. Which are calculated by

$$P\left(S_k\right) = \frac{1}{2}\left(1 + \epsilon\langle S_k\rangle\right), \qquad k = x, y, z, \tag{11}$$

where ϵ takes the values ± 1 for a Su(2)-system, while $\langle S_k\rangle$ is the expectation value of the operators S_k. The information entropies verify [13]

$$\delta H(S_x)\delta H(S_y) \geq \frac{4}{\delta H(S_z)}, \tag{12}$$

Here $\delta H(S_\alpha) \equiv \exp[H(S_\alpha)]$. The fluctuations in the Su(2)-system components S_α are said to be 'squeezed in entropy' if the functions $H(S_\alpha)$ satisfies the condition

$$E(S_\alpha) = \delta H(S_\alpha) - \frac{2}{\sqrt{\delta H(S_z)}} < 0, \alpha \equiv x \ or \ y. \tag{13}$$

With this condition, we can say determine entropy of the qubit is squeezed.

In Fig. 1, we show the time evolution of the entropy squeezing functions $E(S_x)$ (solid plots) and $E(S_y)$ (dashed plots). In Fig. 1a, the temporal evolution of the $E(S_x)$ and $E(S_y)$ for with $(k, \mu) = (1/4, 5)$ for the values of $(\delta, \chi, \gamma) = (0, 0.08, 0)$. We observe that the squeezing occurs only in $E(S_y)$ while the $E(S_x)$ do not satisfies the squeezing condition in the chosen time-interval. Where, the function $E(S_x)$ shows that there is no squeezing in the variable S_x, but under the effect of the Kerr-like medium, the S_x can not protect from the squeezing. The regular fluctuations of $E(S_y)$ shows that the S_y can be preserved and protected from the squeezing due to the evolutions of the interaction of the qubit with two-mode Kerr nonlinear coupler. However, we can see that the squeezing periodically occurs only in $E(S_y)$ with period. In other words, when $t > 0$, one can see only nonclassical negative values for $E(S_y)$, where the maximum magnitude of the squeezing is $\simeq -0.3$ and the minimum values of $E(S_y)$ appears periodically with period π at $\frac{1}{2}(2n = 1)(n = 0, 1, 2, ...)$. The entropy squeezing has two types of regular fluctuations. The first is called *secondary*

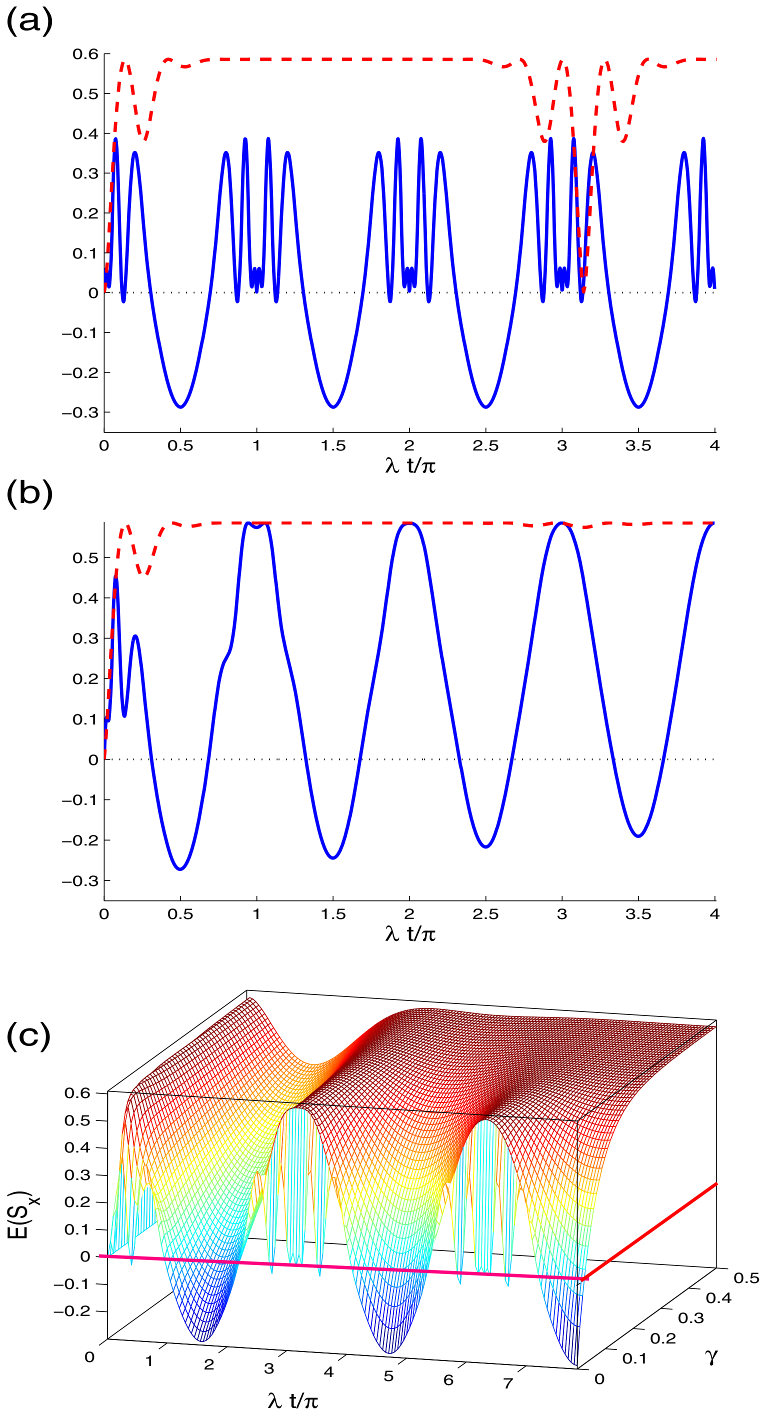

Fig. 1 The time evolution of $E(S_x)$ (dashed plots) and $E(S_y)$ (solid plots), with $(k, \mu) = (1/2, 5)$ for $(\delta, \chi, \gamma) = (0, 0, 0)$ in (**a**), $(\delta, \chi, \gamma) = (0, 0, 0.005)$ in (**a**)

fluctuations, where, $E(S_y)$ oscillates is between the maximum-value 0.4 and zero-value. But, the second type is is called *primary fluctuations*, The entropy squeezing oscillates regular between it extremes values and it leads to the appearance of the squeezing intervals periodically or to the appearance of revivals and collapse of squeezing phenomenon.

From Fig. 1b and c, we find that the weak damping, $\gamma = 0.05\lambda$ leads to: (1) the $E(S_x)$ reduces its oscillations, and after a short time, it reaches its maximum stationary value $E(S_x) \simeq 0.588$. (2) The distance between the local maxima and minima decreases, and the squeezing phenomenon deteriorates with increasing the time. However, the qubits lose squeezing phenomenon than they gain due to the intrinsic damping, and finally they completely lose squeezing phenomenon and fall into a non squeezed state. The increase in the intrinsic damping disappears the qubit squeezing phenomenon.

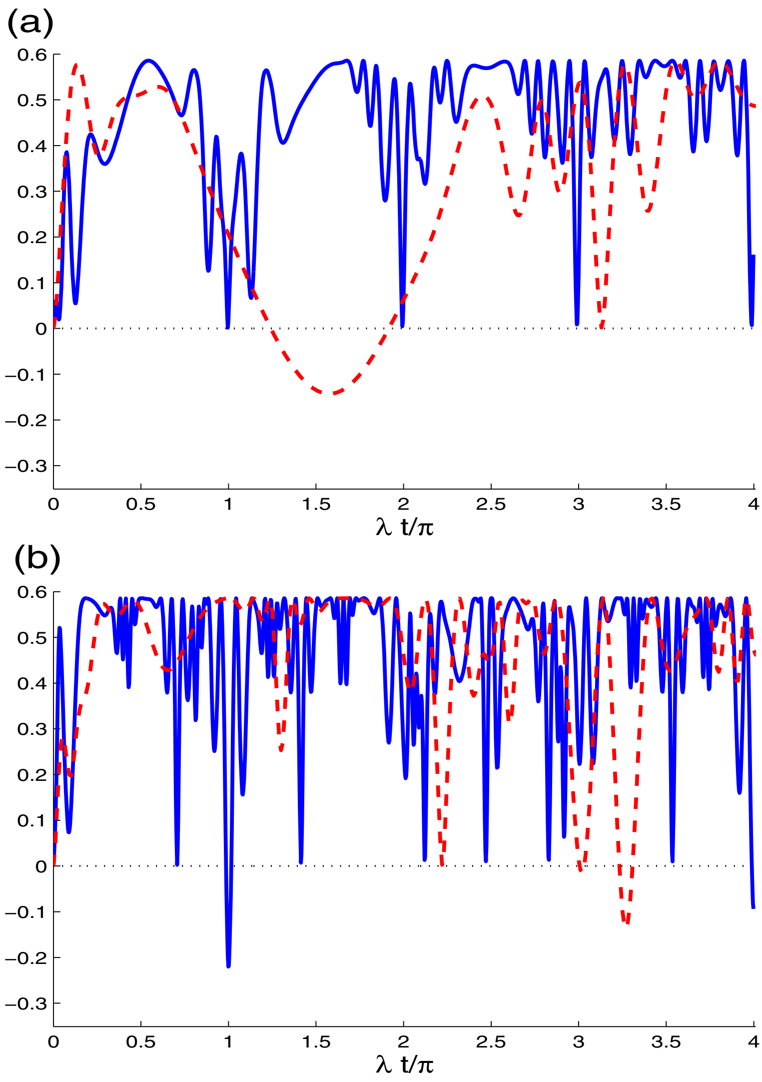

Fig. 2 As Fig. 1a but for different values of χ: $\chi = 0.08$ in (**a**), $\chi = 1$ in (**b**)

In order to show the strong dependence of the $E(S_x)$ and $E(S_y)$ on the Kerr-like medium parameter, they are plotted for different values of χ: $\chi = 0.08$ in Fig. 2a, $\chi = 1$ in Fig. 2b. From Fig. 1a, we observe that: (1) the function $E(S_y)$ shows irregular fluctuations without squeezing in the entropy due the effect of the Kerr-like medium parameter, i.e., the squeezing phenomenon of the variable S_x disappears completely with weak Kerr-like medium. (2) the function $E(S_x)$ shows irregular oscillations and it satisfies the squeezing condition in some time intervals, i. e., the Kerr-like medium parameter leads to appearance of squeezing phenomenon in the entropy and the S_x can not protect from the squeezing. In Fig. 1b, we see notable changes in the behavior of the functions $E(S_x)$ and $E(S_y)$ by increasing the Kerr-like medium parameter, $\chi = 1$. Where, the fluctuations in the Su(2)-system components S_α may be present squeezed in their entropy.

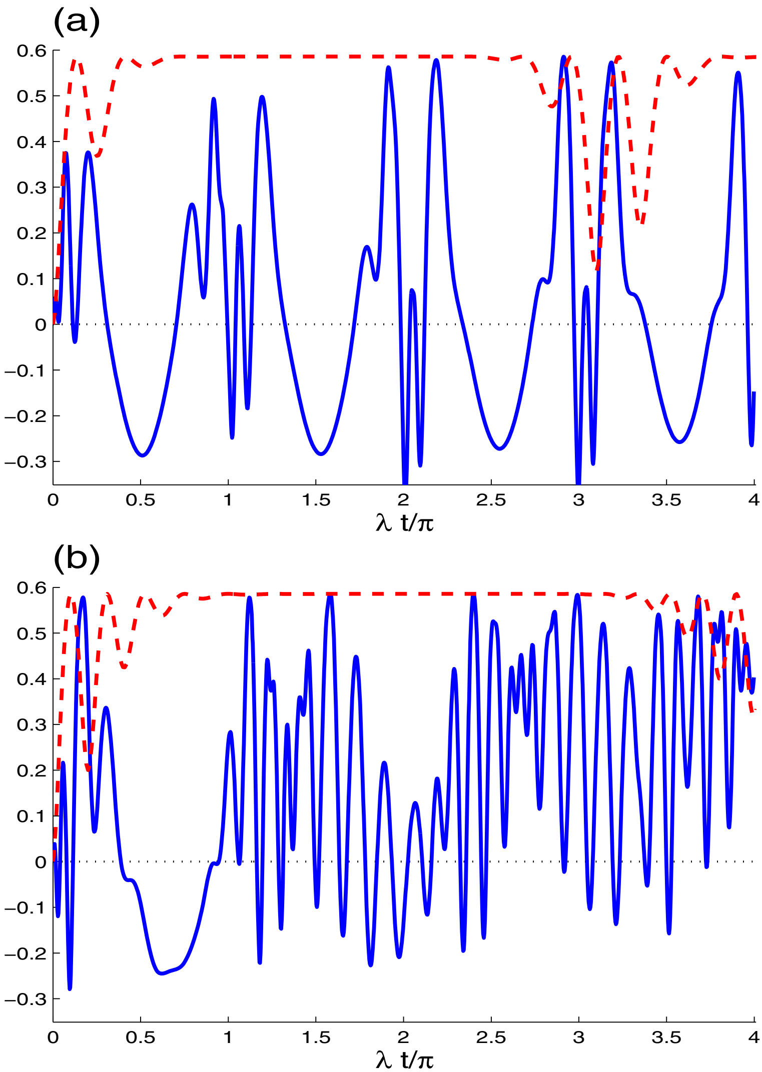

Fig. 3 As Fig. 1a but for different values of δ: $\delta = 1$ in (**a**), $\delta = 5$ in (**b**)

In Fig. 3, the dependence of the the entropy squeezing function $E(S_x)$ and $E(S_y)$ on the detuning parameter is showed in Fig. 1d, where $E(S_x)$ and $E(S_y)$ are depicted for different values of δ: $\delta = 1$ in (a), $\delta = 5$ in (b). From Fig. 3a, we find that the weak off-resonant case, $\delta = 1$, leads to increasing the time intervals of squeezing phenomenon in the component S_x. The secondary fluctuations disappearance and their positive values decrease and tend to become negative values. As for large values of the detuning parameter $\delta = 5$, the function $E(S_y)$ shows irregular fluctuations with squeezing in the entropy during small time intervals comparing with the case of $\delta = 1$ and resonance case $\delta = 0$. The oscillatory behavior of the functions $E(S_x)$ and $E(S_y)$ appears clearly by increasing the detuning parameter, see Fig. 3b.

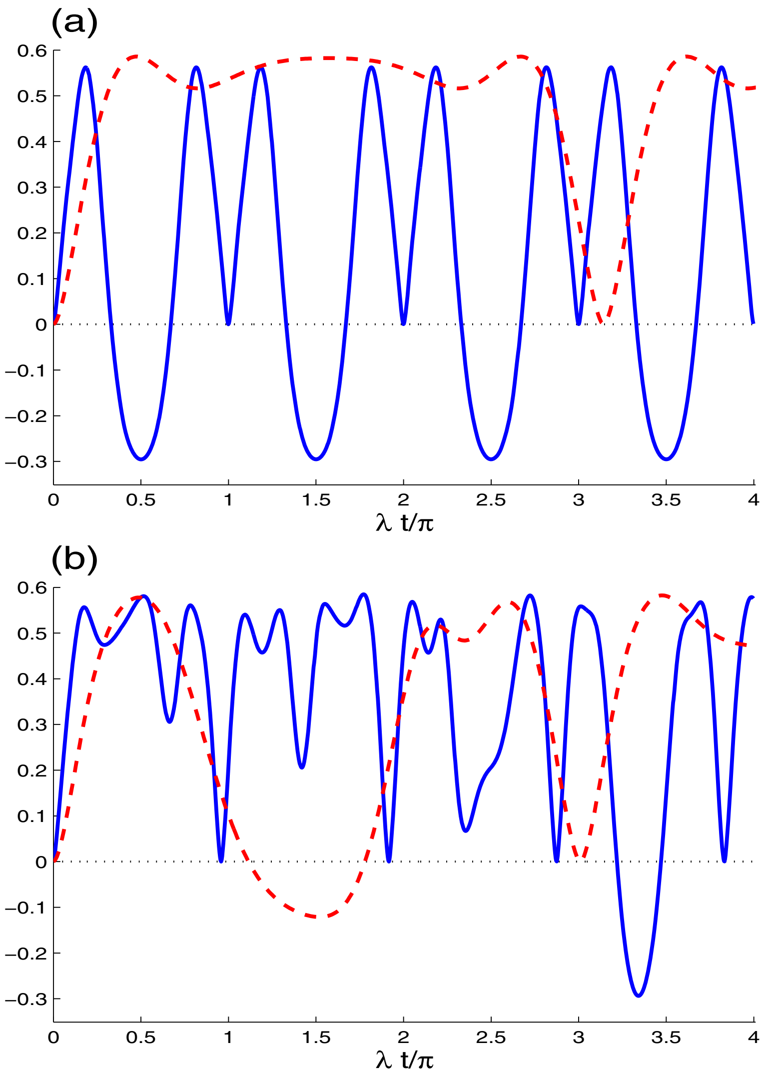

Fig. 4 As Fig. 1a but for $\mu = 1$ and different values of χ: $\chi = 0.0$ in (**a**), $\chi = 0.3$ in (**b**)

In Fig. 4, the effect of the initial coherence intensities, μ, on the entropy squeezing functions $E(S_x)$ and $E(S_y)$ are shown, where they plotted for small initial coherence intensity, $\mu = 1$ with different values of χ: $\chi = 0.0$ in (a), $\chi = 0.3$ in (b). From In Fig. 4a, we note that the initial coherence intensity can be changing the behavior of the secondary and primary fluctuations of $E(S_y)$. In Fig. 4b, the Kerr-like medium parameter, $\chi = 0.3$, shows that the S_y can protect and S_x can not protect from the squeezing.

4 Conclusions

In this paper, we have explored the intrinsic decoherence, detuning, and Kerr-like medium effects on the squeezing phenomenon of Su(2)-system interdicting with a two-mode Kerr nonlinear coupler Su(2)-system. In absence of the intrinsic decoherence, the evolutions of the interaction of the qubit with two-mode Kerr nonlinear coupler leads to generation of regular squeezing phenomenon during the chosen time-interval. The interaction of the qubit with two-mode Kerr nonlinear coupler leads to appearance the regular squeezing phenomenon during the chosen time-interval. The appearance and disappearance of the qubit components from the squeezing can be controlled by the choice of the parameters. Where the squeezing deteriorates with increasing the decoherence parameter, whereas, the Kerr-like medium can not protect some qubit components from from the squeezing. The results show that the squeezing depends, not only on the detuning and the Kerr-like medium, but also on intrinsic decoherence and initial coherence-intensity parameters.

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