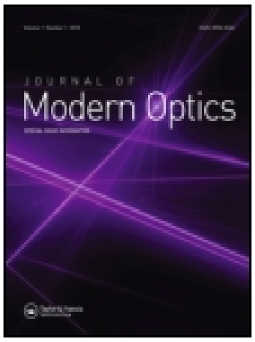

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Stationary discord and non-local correlations via qubit damping

A.-S.F. Obada[a], H.A. Hessian[b], A.-B.A. Mohamed[bc*] and M. Hashem[b]

[a] *Faculty of Science, Al-Azhar University, Nasr City, Egypt;* [b] *Faculty of Science, Assiut University, Assiut, Egypt;* [c] *College of Science & Humanity Studies, Salman Bin Abdulaziz University, Al-Aflaj, Saudi Arabia*

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By using quantum discord (QD), measurement induced non-locality (MIN) and negativity (QE), quantum correlation and entanglement are investigated for two qubits in two different cases for the initial two qubit Werner states, taking into account the influence of qubit damping. It is shown that there is no asymptotic decay for MIN while asymptotic decay exists for QD and QE. Quantum correlations cannot be strengthened by introducing the damping. The appearance time of stationary correlations gets shorter with the increase in the damping parameter. Finally, a uniform damping qubit can affect the stationary correlations when the qubits are initially in an entangled state.

Keywords: quantum discord; measurement induced non-locality; qubit damping

1. Introduction

Entanglement [1] as a kind of quantum correlation has been extensively studied both theoretically and experimentally in the past two decades because it is valuable for understanding the fundamental concepts of quantum mechanics and clarifying the accountability of the orthodox interpretation proposed in quantum mechanics [2]. In particular, concurrence as a type of measurement of entanglement has been widely studied since it was proposed [3,4]. Entanglement is studied under different conditions, such as taking into account decoherence and dissipative effects. Although entanglement is extremely important and even indispensible in the field of quantum information processing [1], it does not contain all of the quantum correlation and is not a unique measure of quantum correlation; there are other quantum correlations, such as quantum non-locality without entanglement [5,6], which has been demonstrated by theoretical [7,8] and experimental [9] results, and they have advantages in the field of quantum information, such as improvement in the efficiency of the Carnot engine.

The quantification of correlations in a bipartite quantum state is a key issue in quantum information theory. Talking about correlations, one usually distinguishes between classical correlations and quantum correlations. The former is related to classical mixing while the latter usually means entanglement. But the classical correlations and quantum correlations are often intertwined in a quantity, which represents the total correlations and it may be difficult to separate them. Therefore, characterizing and qualifying quantum correlations have received much attention. The quantum discord of a general bipartite state is not always larger than the entanglement [10,11], which means that the quantum discord is not simply the sum of entanglement and some other non-classical correlation. The entanglement of two-qubit states has been characterized and qualified completely for any initial state, while the quantum discord has been identified only for particular cases [12–14]. Recently, some results on quantum discord have been obtained for a certain set of a two-qubit X structure density matrix [15,16], and it is an independent measurement of quantum correlation. To further understand the relationships of these correlations, we derive two explicit expressions for quantum discord and classical correlation for any two-qubit X state and generalize previous studies.

Recently, the dynamics of disentanglement of a bipartite qubit system occurred due to spontaneous emission, where the two-level qubits were coupled individually to two cavities (environments). They found that the quantum entanglement may vanish in a finite time, while local decoherence takes an infinite time. They called this phenomenon a Entanglement Sudden Death (ESD) [17]. ESD is not unique to systems of independent atoms. It can also occur for atoms coupled to a common reservoir, in which case we also observe the effect of the revival of the entanglement that has already been destroyed [18]. The effect of global noise on entanglement decay may depend on whether the initial two-party state belongs to a decoherence free subspace or not. As opposed to the ESD and against our intuition, it has been shown that under certain conditions, the process of spontaneous emission can entangle qubits

*Corresponding author. Emails: abdelbastm@yahoo.com, a.mohamed@sau.edu.sa

that were initially unentangled [19], and in some cases the creation of entanglement can occurs sometime after the system–reservoir interaction has been turned on. The authors in Ref. [20] call this phenomenon a delayed sudden birth of entanglement. Motivated by the above topics, quantum discord and measurement-induced nonlocality are used to investigate the quantum correlations for two qubits in two different cases for the initial two-qubit Werner states.

The paper is organized as follows. In Section 2, we formulate a master equation for a two-qubit density matrix describing the processes of interest here. In Section 3, we analyze the quantum correlations a quantum correlation measures. In Section 4, we discuss numerical results. In Section 5, we present conclusions.

2. Tow-qubit model system

Here, two-qubit field system is considered in the dispersive regime with a reservoir. Two initially interacting qubits, labeled by A and B, are chosen as the model. A two-level atom with an excited state $|1\rangle$ and a ground state $|0\rangle$. The standard formalism for the calculations of the time evolution and correlation properties of a collective system of atoms is the master equation method. Suppose, we have two qubits interacting with a quantized field inside a damping cavity. The master equation that governs the dynamics of the whole system can be given as:

$$\begin{aligned}\frac{d\rho}{dt} &= i[\rho, \hat{H}] \\ &\quad + \xi_1[2|0\rangle_{11}\langle 1|\rho|1\rangle_{11}\langle 0| - |1\rangle_{11}\langle 1|\rho - \rho|1\rangle_{11}\langle 1|] \\ &\quad + \xi_2[2|0\rangle_{22}\langle 1|\rho|1\rangle_{22}\langle 0| - |1\rangle_{22}\langle 1|\rho - \rho|1\rangle_{22}\langle 1|],\end{aligned} \tag{1}$$

The Hamiltonian H of the two-qubit system interacting with the field in the dispersive regime is given by [21]:

$$\begin{aligned}\hat{H} = \lambda\Bigg[&\sum_{i=A,B}\left\{|1\rangle_{ii}\langle 1|\hat{a}\hat{a}^\dagger - |0\rangle_{ii}\langle 0|\hat{a}^\dagger\hat{a}\right\} \\ &+ \Big(|1\rangle_{11}\langle 0| \otimes |0\rangle_{22}\langle 1| + |0\rangle_{11}\langle 1| \otimes |1\rangle_{22}\langle 0|\Big)\Bigg],\end{aligned} \tag{2}$$

where $\hat{a}^\dagger$ ($\hat{a}$) is the creation (annihilation) operator and the two eigenstates of the individual qubit ($|0\rangle$,$|1\rangle$) constitute the qubit states and λ is the effective interaction constant. Parameters ξ_1 and ξ_2 are the phase damping constants for the two qubits, $\sigma_z^{(i)} = |1\rangle_{ii}\langle 1| - |0\rangle_{ii}\langle 0|, i = A, B$.

The master Equation (1) can be solved to obtain $\rho_{ij}(\tau)$, $(i, j = 1, 2, 3, 4)$ (for simplicity we will take $\xi_1 = \xi_2 = \xi$). To do this, the qubits and field are initially in the form:

$$\rho(\tau) = \sum_{m,n=0} q_n q_m |m\rangle\langle n| \otimes \rho^{AB}(0), \tag{3}$$

where the field is initially in coherent state $|\alpha\rangle\langle\alpha|$ with $q_n = e^{\frac{-|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}}$ and α is a complex number, and, the qubits are initially in Werner states defined by [22]:

$$\rho^{AB}(0) = \mu|\varphi\rangle\langle\varphi| + \frac{1}{4}(1-\mu)I \tag{4}$$

where μ is a real number, which indicates the purity of initial states, I is the identity matrix and $|\varphi\rangle$ is a two-qubit state. The reduced density matrix ρ^{AB} of two qubits is calculated by tracing out the field variables in two cases.

Case 1 One considers $|\varphi\rangle = \sin\theta|11\rangle + \cos\theta|00\rangle$, where θ a parameter weight qubit states (the qubit distribution). For this initial state of the qubits, the reduced density matrix of the two qubits $\rho^{AB}(\tau)$ is given by:

$$\begin{aligned}\rho^{AB}(\tau) &= \rho_{11}|11\rangle\langle 11| + \rho_{22}|10\rangle\langle 10| + \rho_{33}|01\rangle\langle 01| \\ &\quad + \rho_{44}|00\rangle\langle 00| + \rho_{14}|11\rangle\langle 00| + \rho_{14}^*|11\rangle\langle 00|\end{aligned} \tag{5}$$

where

$$\begin{aligned}\rho_{11}(\tau) &= \left[\frac{1}{4}(1-\mu) + \mu\sin^2\theta\right]e^{-4\gamma\tau} \\ \rho_{22}(\tau) &= \rho_{33}(\tau) = \frac{1}{2}\left[(1-\mu)\left(1 - \frac{1}{2}e^{-2\gamma\tau}\right)\right. \\ &\qquad \left. + 2\mu\sin^2\theta\left(1 - e^{-2\gamma\tau}\right)\right]e^{-2\gamma\tau} \\ \rho_{44}(\tau) &= 1 + \left[(1-\mu)\left(-1 + \frac{1}{4}e^{-2\gamma\tau}\right)\right. \\ &\qquad \left. + \mu\sin^2\theta\left(-2 + e^{-2\gamma\tau}\right)\right]e^{-2\gamma\tau} \\ \rho_{14}(\tau) &= \rho_{41}^*(\tau) = \frac{1}{2}\mu\sin 2\theta\mathbf{e}^{-2i\tau - |\alpha|^2(1-e^{-4i\tau})}\end{aligned}$$

Case 2 Here we consider $|\varphi\rangle = sin\theta|10\rangle + cos\theta|01\rangle$. The reduced density matrix of the two qubits $\rho^{AB}(\tau)$ is given by:

$$\begin{aligned}\rho^{AB}(\tau) &= \rho_{11}|11\rangle\langle 11| + \rho_{22}|10\rangle\langle 10| + \rho_{33}|01\rangle\langle 01| \\ &\quad + \rho_{44}|00\rangle\langle 00| + \rho_{23}|01\rangle\langle 10| + \rho_{23}^*|01\rangle\langle 10|\end{aligned} \tag{6}$$

where,

$$\begin{aligned}\rho_{11}(\tau) &= \frac{1}{4}(1-\mu)e^{-4\gamma\tau} \\ \rho_{22}(\tau) &= \frac{1}{4}[(\mu-1)e^{-2\gamma\tau} - 2\mu\cos(2\theta)\cos(2\tau) + 2]e^{-2\gamma\tau} \\ \rho_{33}(\tau) &= \frac{1}{4}[(\mu-1)e^{-2\gamma\tau} + 2\mu\cos(2\theta)\cos(2\tau) + 2]e^{-2\gamma\tau} \\ \rho_{44}(\tau) &= 1 + [(1-\mu)e^{-2\gamma\tau} - 1]e^{-2\gamma\tau} \\ \rho_{23}(\tau) &= \rho_{32}^*(\tau) = \frac{1}{2}\mu[i\cos(2\theta)\sin(2\tau) + \sin(2\theta)]e^{-2\gamma\tau}\end{aligned}$$

with $\tau = \lambda t$ and $\gamma = \frac{\xi}{\lambda}$.

These elements are used to calculate negativity, quantum discord, and Measurement-induced non-locality.

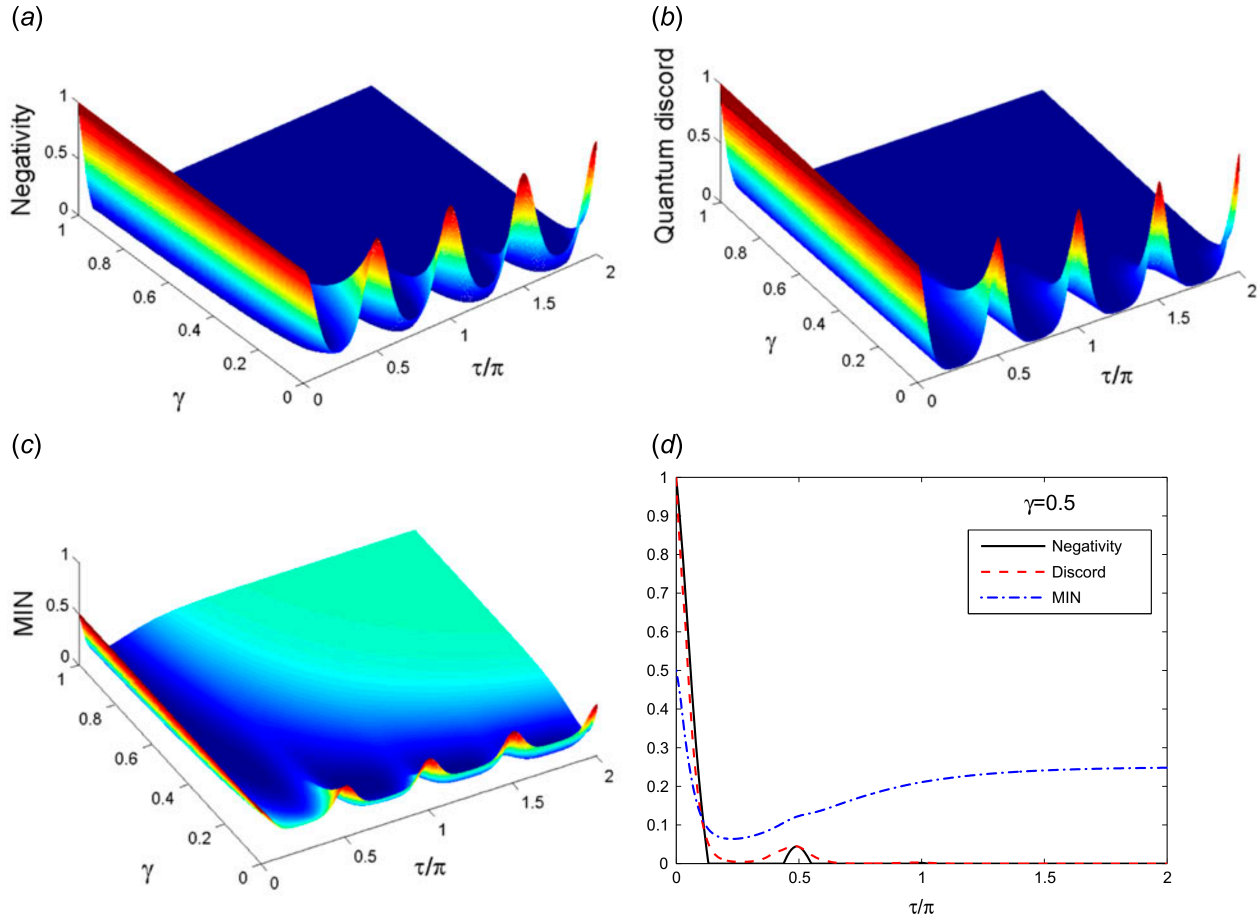

Figure 1. QE, QD and MIN against τ and γ for two qubits prepared initially in Werner state, $|\varphi\rangle = \sin\theta|11\rangle + \cos\theta|00\rangle$ and the field in a coherent state with initial mean photon number $|\alpha|^2 = 4$, $\theta = \frac{\pi}{4}$ and $\mu = 1$. (The colour version of this figure is included in the online version of the journal.)

3. Quantum correlation measures

3.1. Negativity (QE)

The entanglement of the system described by the density operator $\rho(t)$ can be measured by the negativity defined in terms of the negative eigenvalues of the partial transposition of $\rho(t)$ [23,24]

$$E = \max\left(0, -2\sum_i \mu_i\right), \tag{7}$$

where the sum is taken over the negative eigenvalues μ_i of the partial transposition of the density matrix $\rho(t)$ of the system. The value $E = 1$ corresponds to maximum entanglement between the two qubits, while $E = 0$ indicates that the two qubits are separable.

3.2. Quantum discord (QD)

In quantum information theory, quantum discord is a measure of non-classical correlations between two subsystems of a quantum system. It includes correlations that are due to quantum physical effects but do not necessarily involve quantum entanglement.

The notion of quantum discord was introduced [25–27] referred to it also as a measure of quantumness of correlations [26]. From the work of these two research groups, it follows that quantum correlations can be present in certain mixed separable states [28]. In other words, separability alone does not imply the absence of quantum effects. The notion of quantum discord thus goes beyond the distinction which had been made earlier between entangled versus separable (non-entangled) quantum states.

In mathematical terms, quantum discord is defined in terms of the quantum mutual information. More specifically, quantum discord is the difference between two expressions which each, in the classical limit, represent the mutual information. These two expressions are:

$$I(A;B) = H(A) + H(B) - H(A,B)$$
$$J(A;B) = H(A) - H(A|B)$$

where, in the classical case, $H(A)$ is the information entropy, $H(A,B)$ the joint entropy and $H(A|B)$ the

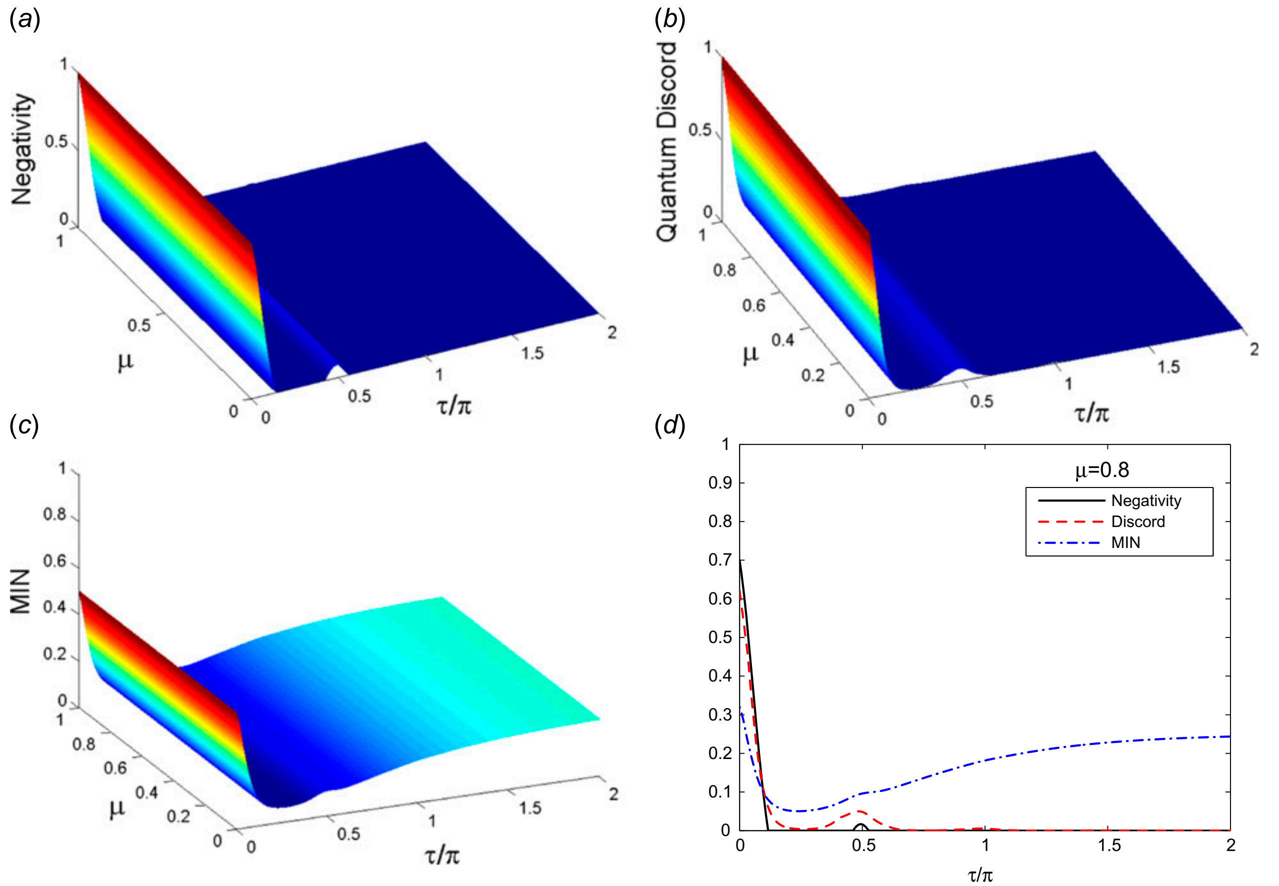

Figure 2. QE, QD and MIN against τ and μ for two qubits prepared initially in Werner state, $|\varphi\rangle = \sin\theta|11\rangle + \cos\theta|00\rangle$ and the field in a coherent state with initial mean photon number $|\alpha|^2 = 4$, $\theta = \frac{\pi}{4}$ and $\gamma = 0.4$. (The colour version of this figure is included in the online version of the journal.)

conditional entropy, and the two expressions yield identical results. In the non-classical case, the quantum physics analogy for the three terms are used $S(\rho^A)$ the von Neumann entropy, $S(\rho)$ the joint quantum entropy, and $S(\rho^A|\rho^B)$ the conditional quantum entropy, respectively, for probability density function ρ;

$$I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho)$$
$$J_A(\rho) = S(\rho^B) - S(\rho^B|\rho^A)$$

The difference between the two expressions $I(\rho) - J_A(\rho)$ defines the basis-dependent quantum discord, which is asymmetrical in the sense that $Q_A(\rho)$ can differ from $Q_B(\rho)$ [29,30]. The notation J represents the part of the correlations that can be attributed to classical correlations and varies in dependence on the chosen eigenbasis, therefore, in order for the quantum discord to reflect the purely non-classical correlations independently of basis, it is necessary that J first be maximized over the set of all possible projective measurements onto the eigenbasis [31]:

$$Q_A(\rho) = S(\rho^A) - S(\rho) + \min_{\prod_J^A} S\left(\rho^B_{|\prod_J^A}\right) \qquad (8)$$

The von Neumann measurement for the system A is written $\prod_J^A = I \otimes |J\rangle\langle J|$ where I is the identity operator for system A, $(j = 1, 2)$. Non-zero quantum discord indicates the presence of correlations that are due to non-commutativity of quantum operators [31]. For pure states, the quantum discord becomes a measure of quantum entanglement [32], more specifically, in that case it equals the entropy of entanglement [28]. The dynamics of the quantum discord and entanglement has been recently compared under the same conditions when entanglement dynamic undergoes a sudden death [33].

Vanishing quantum discord is a criterion for the pointer states, which constitute preferred effectively classical states of a system [26] It could be shown that quantum discord must be non-negative and that states with vanishing quantum discord can in fact be identified with pointer states [34]. Other conditions have been identified which can be seen in analogy to the criterion [35] and in relation to the strong subadditivity of the von Neumann entropy [36].

For a two qubit X state, with computation basis $\{|1\rangle = |\uparrow\rangle_A|\uparrow\rangle_B\}$, $\{|2\rangle = |\uparrow\rangle_A|\downarrow\rangle_B\}$, $\{|3\rangle = |\downarrow\rangle_A|\uparrow\rangle_B\}$, $\{4\rangle = |\downarrow\rangle_A|\downarrow\rangle_B\}$. The density matrix is written as

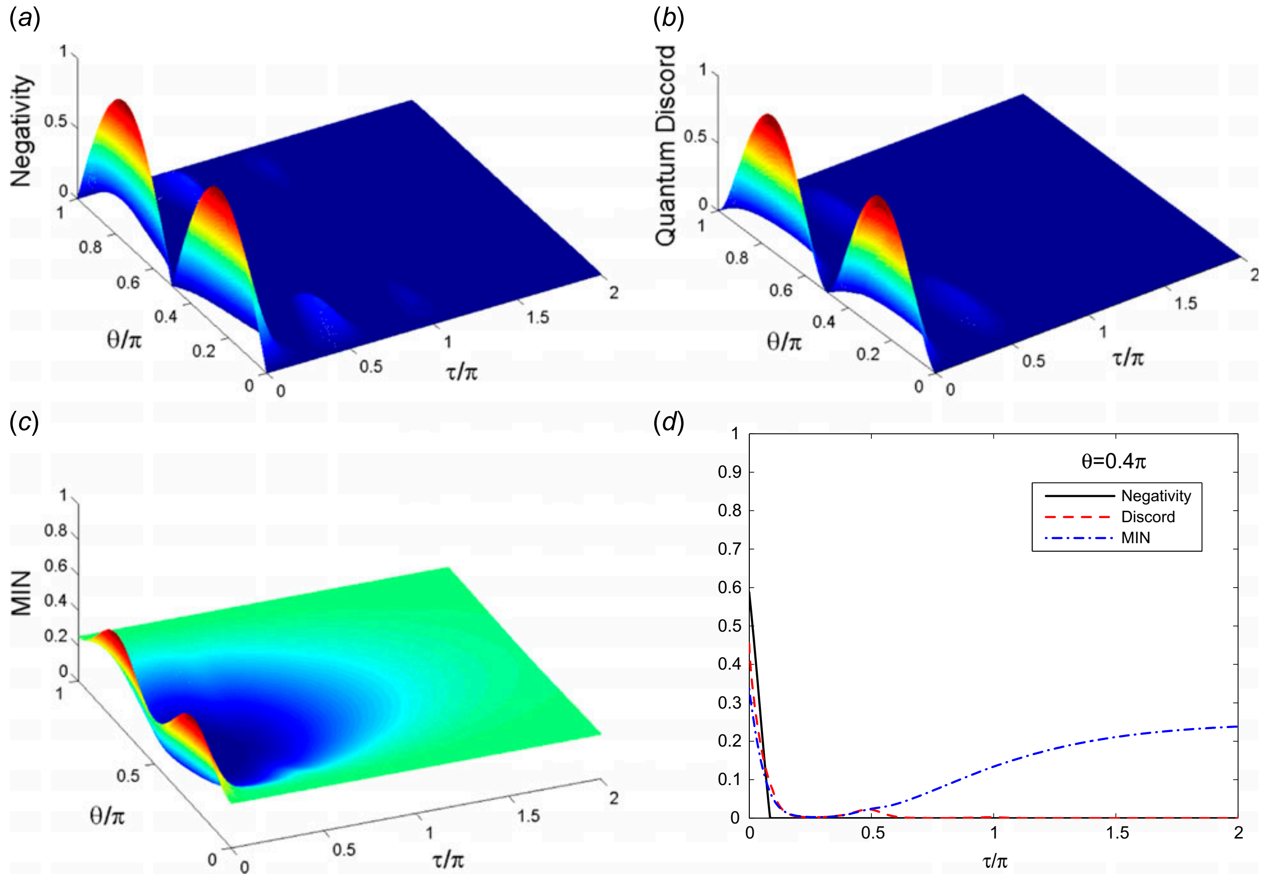

Figure 3. QE, QD and MIN against τ and θ for two qubits prepared initially in Werner state, $|\varphi\rangle = \sin\theta|11\rangle + \cos\theta|00\rangle$ and the field in a coherent state with initial mean photon number $|\alpha|^2 = 4$, $\mu = 1$ and $\gamma = 0.4$. (The colour version of this figure is included in the online version of the journal.)

$$\rho^{AB}(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{9}$$

The quantum discord for measuring system B is given by [37]:

$$Q_B(\rho) = \min\{Q_1(\rho), Q_2(\rho)\} \tag{10}$$

where

$$Q_1(\rho) = S(\rho^B) - S(\rho^{AB}) - \Upsilon,$$
$$Q_2(\rho) = S(\rho^B) - S(\rho^{AB}) - \Lambda.$$

and $S(\rho) = -\mathrm{Tr}\rho\log_2(\rho)$ denotes the von Neumann entropy. Where the two quantities Υ and Λ in the form:

$$\Upsilon = \sum_{m=1}^{2}\left\{\frac{\wp(1+(-1)^m)\Re}{2}\log_2\left[\frac{\wp(1+(-1)^m)\Re}{2}\right] + \frac{\Im(1+(-1)^m)\emptyset}{2}\log_2\left[\frac{\Im(1+(-1)^m)\emptyset}{2}\right]\right\}, \tag{11}$$

$$\Lambda = \sum_{n=1}^{2}\frac{1+(-1)^n\zeta}{2}\log_2\left[\frac{1+(-1)^n\zeta}{2}\right] \tag{12}$$

Also

$$\wp = \rho_{11} + \rho_{33}, \quad \Re = \frac{|\rho_{11} - \rho_{33}|}{\wp},$$
$$\Im = \rho_{22} + \rho_{44}, \quad \emptyset = \frac{|\rho_{22} - \rho_{44}|}{\Im},$$

and

$$\varsigma = \sqrt{(\rho_{11} + \rho_{22} - \rho_{33} - \rho_{44})^2 + 4(|\rho_{14}| + |\rho_{23}|)^2}.$$

These formulas are used in the numerical calculation later on.

3.3. *Measurement-induced non-locality (MIN)*

MIN can be viewed as a kind of non-classical correlation from a geometric perspective based on the local von Neumann measurements from which one of the reduced states is left invariant. Let ρ be any bipartite state shared between two parties A and B, then MIN is defined by [38],

$$M(\rho) = \max_{\Pi^A}\|\rho - \Pi^A(\rho)\|^2 \tag{13}$$

where maximum is over all von Neumann measurements Π^A which do not disturb ρ^A, the local density matrix of A,

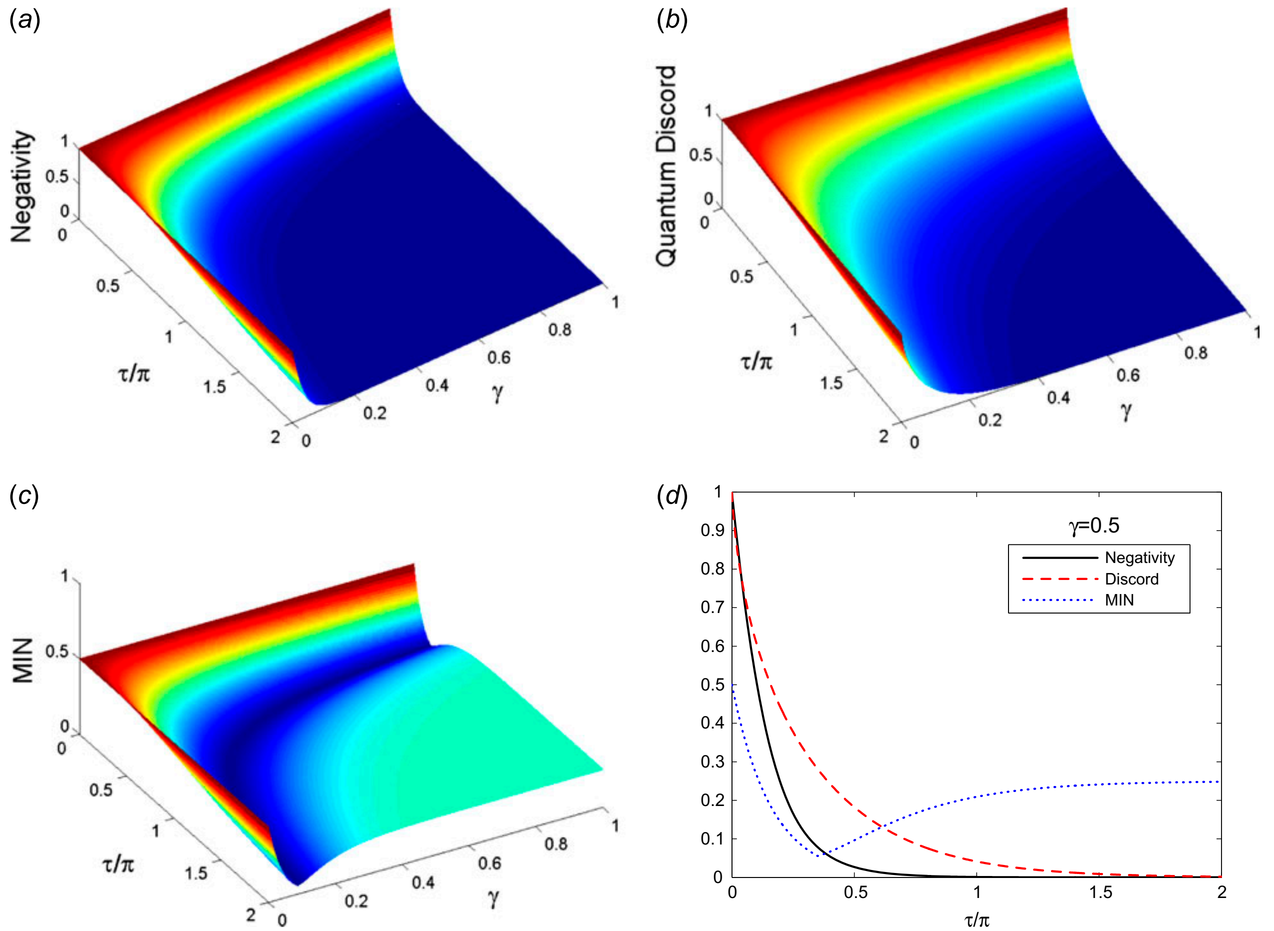

Figure 4. The same as in Figure 1 but the two qubits prepared initially in Werner state, $|\varphi\rangle = \sin\theta|10\rangle + \cos\theta|01\rangle$. (The colour version of this figure is included in the online version of the journal.)

i.e. $\Sigma_k \Pi_k^A \rho^A \Pi_k^A = \rho^A$ and $\Pi^A(\rho) = \Sigma_k(\Pi_k^A \otimes I_B)\rho(\Pi_k^A \otimes I_B)$. By using Ref. [38], MIN for a two-qubit X state ρ^{AB} is given by:

$$M(\rho^{AB}) = \begin{cases} \frac{1}{4}(TrRR^t - \frac{1}{\|\vec{x}\|}\vec{x}^t RR^t \vec{x}), & \vec{x} \neq 0; \\ \frac{1}{4}(TrRR^t - \lambda_{\min}), & \vec{x} = 0. \end{cases} \tag{14}$$

where $\lambda_{\min}$ being minimum eigenvalues of RR^t, $R_{ij} = Tr(\rho^{AB}(\sigma_i \otimes \sigma_j))$ are the components of the correlation matrix [39] and σ_i are the usual Pauli spin matrices. Physically, MIN quantifies the global effect caused by locally invariant measurements. MIN has application in general dense coding, quantum state steering, etc. MIN vanishes for product state and remains positive for entangled states. For pure states, MIN reduces to linear entropy like geometric discord [39]. MIN is invariant under local unitary, i.e. in true sense, it is a non-local correlation measure. The set of all zero MIN states is non-convex. The authors in [40] derived the conditions for the nullity of MIN. They have found that set of states with zero MIN is a proper subset of the set of all classical quantum states, i.e. zero-discord states. MIN for classical quantum state vanishes if each eigen-subspace of ρ^A is one-dimensional. It therefore reveals that noncommutativity is the cause of these kind of non-locality in quantum states. Recently, in [41], MIN has been quantified in terms of relative entropy to give it another physical interpretation. The results of this quantities for our system are considered in the following section.

4. Numerical results

In the first, one considers the time evolution of QE, QD and MIN for $|\varphi\rangle = \sin\theta|11\rangle + \cos\theta|00\rangle$. In Figure 1(*a*)–(*c*), QE, QD, and MIN are plotted against scaled time and γ for the two qubits with the initial mean photon number $|\alpha|^2 = 4$ and $\theta = \frac{\pi}{4}$, $\mu = 1$, which means that ρ^{AB} is pure state. When $\gamma = 0$ (see Figure 1(*a*)), (i.e. in the absence of the damping), this case is corresponding to the evolution of the entanglement in the standard two-photon model. And one can find that QE, QD, and MIN have the same behavior, but they have different amplitude values.

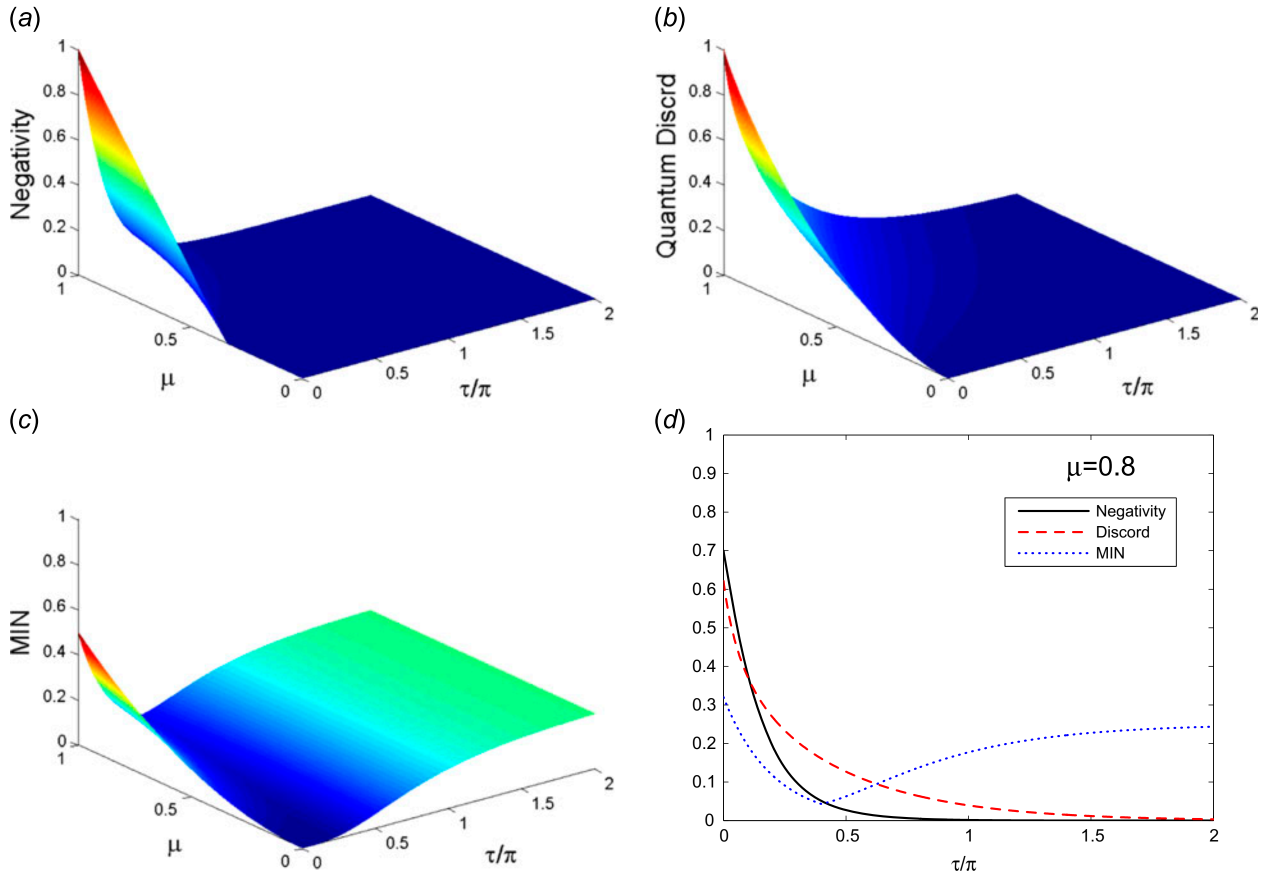

Figure 5. The same as in Figure 2 but the two qubits prepared initially in Werner state, $|\varphi\rangle = \sin\theta|10\rangle + \cos\theta|01\rangle$. (The colour version of this figure is included in the online version of the journal.)

We can observe that QE, QD, and MIN periodically evolve with a period $\frac{\pi}{2}$, and the two qubits are entangled. This evolvement period get shorter with the increasing. Then, one can note that the possibility to revive them periodically into their initial values can be observed if and only if $\gamma = 0$. When $\tau = \frac{\pi}{4}(4n-3)(\ (n = 1, 2, \ldots)$, QE and MIN don't drop to zero but stationary QD drops to zero. In general, the minimum values of MIN is greater than the minimum values of QE and QD but the maximum values of QE and QD is greater than the maximum values of MIN (see Figure 1(c)). Therefore, collapses and revivals phenomenon appears for QE, QD, and MIN.

From Figure 1(a)–(c), one can see the influences of the damping, on QE, QD, and MIN. It is observed that the weak damping parameter leads to the weak decrease in the maximal values. Therefore, in order to prepare states having quantum correlation, the influence of the damping should be taken into account. It is seen that quantum correlations can be disappeared by introducing the damping. The stationary correlations can be increased by increasing γ, after a certain γ, QE and QD vanish, but MIN still exists, as shown in Figure 1(d) i.e. MIN is more robust than QE and QD. Quantum correlations cannot be strengthened by introducing the damping. The appearance time of stationary correlations gets shorter with the increase in the damping parameter [42,43]. This is very easily understood since the period is governed by the decay term ($e^{-\alpha\gamma\tau}$, ($\alpha = 2, 4$)) in the solutions 5,6 of master equation.

In Figure 2 (a)–(c), QE, QD, and MIN are plotted against scaled time and μ for the two qubits with $\theta = \frac{\pi}{4}$ and $\gamma = 0.4$. Clearly, these measures are independent of the cavity decay parameter γ (damping parameter). For $\gamma > 0$, stationary QE and QD present sudden death while MIN presents sudden birth. Looking at the formulas for negativity, we see that whenever the damping occurs ($\gamma > 0, \tau \to \infty$) the elements of the density matrix attend to zero and, therefore, the density matrix is separable, QE and QD vanishes. In this sense stationary MIN is more accurate, because, in this example, is only zero when the matrix is diagonal (in the computational basis) and MIN $\to$ 0.25 see Figure 2(d). In the intervals of the entanglement death, QE and QD equal zero. It is obvious that even for different values of μ QE and QD will vanish. So in this sense, MIN is robust than QE and QD. It is found that the initial state parameter μ leads to the following: decreasing

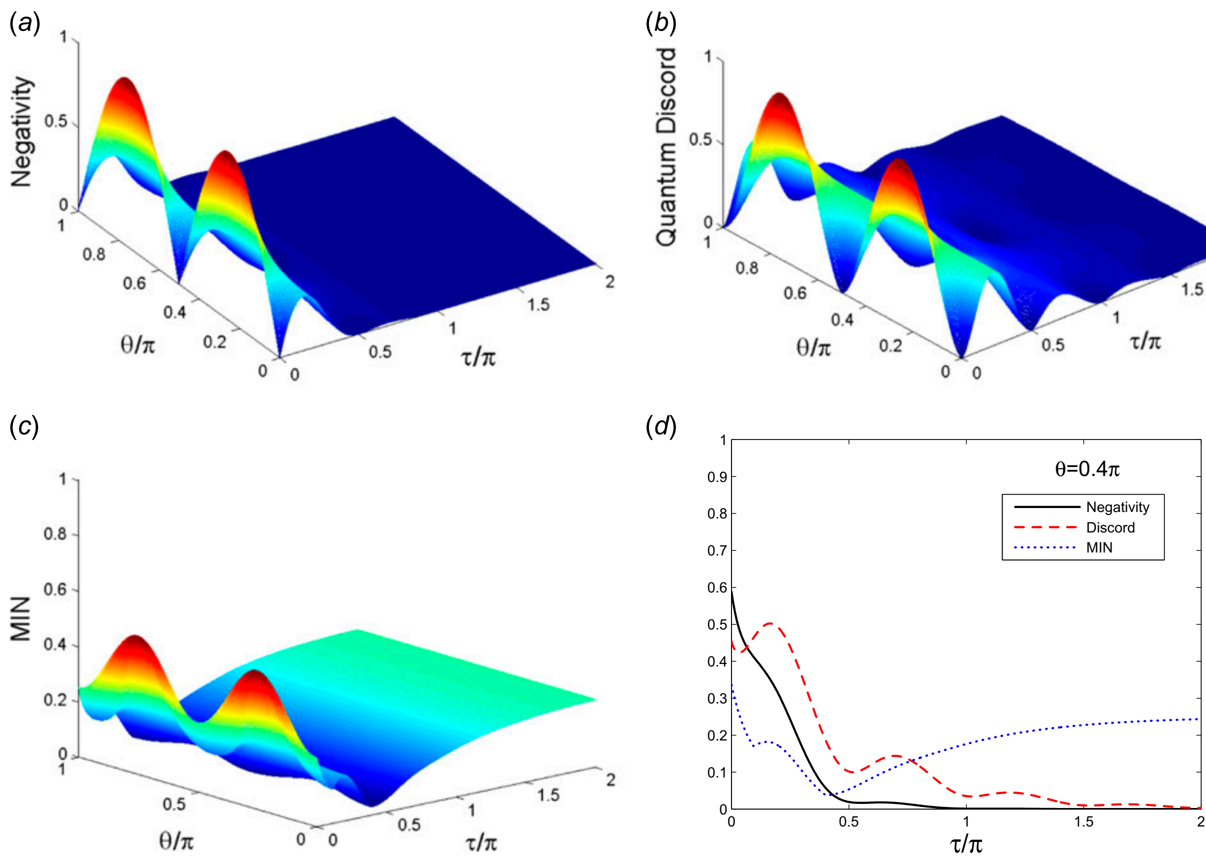

Figure 6. The same as in Figure 1 but the two qubits prepared initially in Werner state, $|\varphi\rangle = \sin\theta|10\rangle + \cos\theta|01\rangle$. (The colour version of this figure is included in the online version of the journal.)

the amplitudes of the QE and QD and the disappearance of the phenomenon of entanglement sudden birth.

In Figure 3(*a*)–(*c*), QE, QD, and MIN are plotted against scaled time and θ for the two qubits with $\mu = 1$ and $\gamma = 0.4$. When we neglect the damping parameter clearly describes the time evolution for the standard two photon. Thus, the time evolution diagram for stationary values QE, QD, and MIN of the usual two-photon is very regular; it is symmetric about $\theta = \frac{\pi}{2}$. This is due to the periodic nature of the interaction phenomenon in the two photons. This can be attributed to the weighted qubit states (the qubit distribution θ). In the presence of the damping with a small increase in the parameter of the damping leads to the localization of the peaks. The peak centered at $\theta = \frac{\pi}{2}$ disappears due to the damping.

To see the effect of the initial state on QE, QD, and MIN these measures are plotted in Figures 4–6 for the initial state, $|\varphi\rangle = \sin\theta|10\rangle + \cos\theta|01\rangle$ where Figure 4 the same as in Figures 1 and 5 the same as in Figures 2 and 6 the same as in Figure 3. From Figure 4(*a*) and (*b*), we note that collapses and revivals phenomenon disappears for QE, QD and MIN. The maximum values for QE and QD occur when $\gamma = 0$. We find that QE and QD evolve with respective to damping parameter γ. There is no entanglement for $\gamma > 0.2$. It is shown that stationary QE and QD experiences a sudden transition when entanglement changes from a finite value to zero, while quantum correlation evolves continuously with respective to damping even it tends to be zero. After a particular value for γ, stationary MIN has a sudden transition to a fixed value (see Figures 4–6(*d*)). In Figures 4–6, also shows that there is no asymptotic decay for MIN while asymptotic decay exists for QD and QE. The inevitable onset of the sudden decrease in the quantum discord can be substantially delayed by the decrease in the noise γ defines the environment.

5. Conclusions

In this paper, quantum correlation is investigated, using QD and MIN, in a two-qubit system. It is seen that quantum correlation cannot be strengthened by introducing the damping interaction. It is found that there exists not only quantum non-locality without entanglement but also quantum non-locality without quantum discord. Also with weak initial entanglement, there are QE and QD in an interval of death quantum discord. After a certain γ, QE and QD vanish, but MIN still exists, MIN is more robust than QE and QD. It is found that the stationary correlations QE and QD present sudden death, while MIN presents sudden birth. Finally, a uniform damping qubit can affect the stationary correlations when the qubits are initially in an entangled state.

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