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Effect of the phase damping of two qubits on both the quantum discord and non-local correlation

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ABSTRACT

An analytical solution of the master equation for two qubits-field system, in the dispersive regime, are investigated. The qubits are initially in Werner states and the field in coherent state. Under the influence of the damping, the geometric measure of quantum discord (GMQD) and the measurement-induced nonlocality (MIN) are investigated. GMQD and MIN are compared and illustrated their different characteristics. It is found that under the influence of damping the phenomenon of the death occurs for GMQD, but this phenomenon does not occur for MIN even when the damping parameter is high. The initial conditions for the qubits play an important role in the phenomenon of collapses and revivals for GMQD and MIN.

1. Introduction

Quantum entanglement (QE) is one of the most remarkable features of quantum mechanics and it plays a central role in quantum information and communication theory [1]. There exists, however, nonclassical correlation, which is more general and more fundamental than entanglement in the sense that separable mixed states can have nonclassical correlation [2,3]. Moreover, nonclassical correlation other than entanglement can be responsible for the quantum computational efficiency of deterministic quantum computation with one qubit. Quantum correlation arises from noncommutativity of operators representing states, observables, and measurements [4]. QE which refers to the separability of the states, is very important in quantum information processing and can be realized in many kinds of physical systems which involve quantum correlation. An alternative classification for quantum correlations, which is based on quantum measurements, has arisen in recent years and also plays an important role in quantum information theory [4–6].

In particular, quantum discord (QD) [2] is introduced to measure these quantum correlations. There exist indeed separable mixed states having nonzero discord and the separable mixed states can be used to perform useful quantum tasks [7]. Evaluation of QD in general requires considerable numerical minimization and analytical expressions are known only for certain classes of states. The authors in [8] evaluated analytically the QD for a large family of two-qubit states, and make a comparative study of the relationships between classical and quantum correlations in terms of the QD.

The authors in [9] propose a geometrical way of quantifying QD, which is termed as GMQD. GMQD can be extended to any number of subsystems, though evaluating the measure of discord becomes progressively more difficult with the increasing of the number of subsystems and that of their dimensionality. Moreover, the authors in [10] evaluate GMQD for an arbitrary state and obtain an explicit and tight lower bound.

Very differently, MIN [11] has been proposed to interpret the maximum global effect caused by locally invariant measurements, the authors claim that MIN is in some sense dual to GMQD. Anyway, both GMQD and MIN may be considered to be the measurement tool of quantum correlation. The interaction of a quantum system with its environment causes the rapid destruction of crucial quantum properties and drives the system to an incoherent state.

It was shown by the authors in [12] that entanglement of a bipartite system decays to zero in a finite time, which is called entanglement sudden death (ESD), while coherence vanishes exponentially with time to zero. Subsequently, ESD in different systems has been made by various groups [13–16]. Another important situation is the dephasing environment, in which energy transfer from the system to the environment does not occur. Some work has been devoted to this issue [17,18]. Recently, by using carefully engineered interactions between system and environments, experimental studies have been carried out to demonstrate ESD, and ESD

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has been observed both in photons [19] and in atomics ensembles [20]. By using GMQD and MIN, the quantum correlation of two qubits is studied in [21,22]. While the quantum correlations and QD with and GMQD is studded in [23]. Also, the pairwise correlations, including QD and GMQD is studded in [24]. The quantumness of the correlations between two qubits by using measurement-induced disturbance is studded [25–27].

Motivated by these premises, here we focus on studying the quantum correlation GMQD and MIN for two qubits-field system in the dispersive reservoir and illustrate their different characteristics. The organization of the article runs as follows: Section 2 presents the model and the dispersive limit is taken and the solution of the master equation is presented. The dynamics of GMQD and MIN are considered for two different initial states of the qubits in Section 3. Followed by discussion of numerical results in Section 4. In Section 5 we draw our conclusions.

2. Tow-qubit model system

We start the investigation by describing the system under consideration and giving the basic relations and equations, which will be frequently used in this paper. We consider here two-qubit- field system in the dispersive regime with a phase damping reservoir. The two qubits considered here can be impurity atoms in a photonic crystal or single-mode fiber, or atoms in a magnetic trap or microcavity, or atoms in other man-made environments. All of these systems can be accurately described by models of collective decay by interaction with a common heat bath, which can be reduced to models of atom dynamics in one-dimensional resonant fields. Two initially interacting qubits, labeled by A and B, are chosen as the model. A two-level qubit with an excited state $|1\rangle$ and a ground state $|0\rangle$ is considered. The unavoidable interaction between the system and the environment will lead to the loss of coherence, and may also lead to the loss of entanglement and nonclassical correlation. From the quantum master equation:

$$\frac{d\rho}{dt} = i[\rho, \hat{H}] + Ł\rho, \tag{1}$$

where the phase damping reservoir is described by:

$$Ł\rho = \frac{\xi_1}{2}(\sigma_z^{(A)}\rho\sigma_z^{(A)} - \rho) + \frac{\xi_2}{2}(\sigma_z^{(B)}\rho\sigma_z^{(B)} - \rho) \tag{2}$$

Here, the Hamiltonian H of the two-qubit system interacting with the filed in the dispersive regime is given by [28]:

$$\begin{aligned}\hat{H} &= \lambda[\sum_{i=A,B}\{|1\rangle_{ii}\langle 1|\hat{a}\hat{a}^\dagger - |0\rangle_{ii}\langle 0|\hat{a}^\dagger\hat{a}\} + (|1\rangle_{11}\langle 0| \otimes |0\rangle_{22}\langle 1| \\ &\quad + |0\rangle_{11}\langle 1| \otimes |1\rangle_{22}\langle 0|)],\end{aligned} \tag{3}$$

where $\hat{a}^\dagger$ $(\hat{a})$ is the creation (annihilation) operator and the two eigenstates of the individual qubit ($|0\rangle$,$|1\rangle$) constitute the qubit states and λ is the effective interaction constant. Parameters ξ_1 and ξ_2 are the phase damping constants for the two qubits, $\sigma_z^{(i)} = |1\rangle_{ii}\langle 1| - |0\rangle_{ii}\langle 0|$, $i = A, B$.

The density matrix ρ, which describes the state of the system, can be evaluated. From the state, the dynamics of GMQD between the qubits can be investigated. In the expression, $Ł\rho$ includes the effect of the interaction between the environment and the qubits. In the following, the correlated dissipative environments will be investigated. The master equation (Eq. (1)) can be solved to obtain $\rho_{ij}(t)$, $(i, j = 1, 2, 3, 4)$ (for simplicity we will take $\xi_1 = \xi_2 = \xi$). To do this, we suppose the qubits and field are initially in the form:

$$\rho(t) = \sum_{m,n=0} q_n q_m |m\rangle\langle n| \otimes \rho^{AB}(0), \tag{4}$$

with $q_n = e^{\frac{-|\alpha|^2}{2}} \frac{\alpha^n}{\sqrt{n!}}$ and α is a complex number.

Where the field is initially in coherent state $|\alpha\rangle$. While the qubits are initially in Werner states defined by [29]:

$$\rho^{AB}(0) = \mu|\varphi\rangle\langle\varphi| + \frac{1}{4}(1-\mu)I \tag{5}$$

where μ is a real number, which indicates the purity of initial states, I is the identity matrix. We calculate the reduced density matrix ρ^{AB} of two qubits by tracing out the field variables in two cases.

Case 1. We consider the initial state $|\varphi\rangle = sin\theta|11\rangle + cos\theta|00\rangle$. Therefore, the reduced density matrix of the two qubits $\rho^{AB}(t)$ is given by:

$$\rho^{AB}(\tau) = \begin{pmatrix} \frac{1}{4}(1-\mu) + \mu\sin^2\theta & 0 & 0 & \rho_{14}(\tau) \\ 0 & \frac{1}{4}(1-\mu) & 0 & 0 \\ 0 & 0 & \frac{1}{4}(1-\mu) & 0 \\ \rho_{41}(\tau) & 0 & 0 & \frac{1}{4}(1-\mu) + \mu\cos^2\theta \end{pmatrix}, \tag{6}$$

where $\rho_{14}(\tau) = \rho_{41}^*(\tau) = \frac{1}{2}\mu\sin 2\theta \mathbf{e}^{-2\tau(\gamma+i) - |\alpha|^2(1-e^{-4i\tau})}$.

Case 2. If the initial state is considered to be $|\varphi\rangle = sin\theta|10\rangle + cos\theta|01\rangle$, then the reduced density matrix of the qubits read as:

$$\rho^{AB}(\tau) = \begin{pmatrix} \frac{1}{4}(1-\mu) & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \frac{1}{4}(1-\mu) \end{pmatrix}, \tag{7}$$

where,

$$\begin{aligned}\rho_{22}(\tau) &= \frac{1}{4}[1 + \mu - 2\mu e^{-\gamma\tau}\cos 2\theta(\cos\sqrt{4-\gamma^2}\tau + \frac{2\gamma\sin\sqrt{4-\gamma^2}\tau}{\sqrt{4-\gamma^2}})],\\ \rho_{23}(\tau) &= \rho_{32}^*(\tau) = [\frac{-2\mu i\cos 2\theta\sin\sqrt{4-\gamma^2}\tau}{\sqrt{4-\gamma^2}} + \mu\sin 2\theta e^{-\gamma\tau}]e^{-\gamma\tau},\\ \rho_{33}(\tau) &= \frac{1}{4}[1 + \mu + 2\mu e^{-\gamma\tau}\cos 2\theta(\cos\sqrt{4-\gamma^2}\tau + \frac{2\gamma\sin\sqrt{4-\gamma^2}\tau}{\sqrt{4-\gamma^2}})]\end{aligned}$$

with $\tau = \lambda t$ and $\gamma = \frac{\xi}{\lambda}$.

These solutions (6) and (7) are used in the following section to study the dynamics of geometric measure of quantum discord and measurement-induced nonlocality of two qubits-field system in the dispersive reservoir.

3. Dynamics of GMQD and MIN

In this section, the dynamics of GMQD and MIN are considered. Geometric measure of quantum discord is introduced by Ref. [30], which measures the quantum correlations through the minimum Hilbert–Schmidt distance between the given state and zero discord state. Generally, GMQD is defined as [31]

$$Q_A(\rho^{AB}) = \min \|\rho^{AB} - \Im\|^2, \tag{8}$$

where the subscript A of Q_A implies that the measurement is taken on the subsystem A. The minimum is over the set of zero-discord states, i.e. $Q_A(\Im)=0$, and the geometric quantity $\|\rho-\Im\|^2=tr(\rho-\Im)^2$ is the square of Hilbert–Schmidt norm of Hermitian operators. A state $\Im$ on $H^A\otimes H^B$ is of zero discord if and only if it is a classical-quantum state [4]. GMQD of the two-qubit general state is evaluated as [31]

$$Q(\rho)=\frac{1}{4}(\|\vec{x}\|^2+\|R\|^2-\lambda_{\max}), \tag{9}$$

where $\vec{x}=(x_1,x_2,x_3)^t$ is a column vector, $\|\vec{x}\|^2=\sum_i x_i^2$. If σ_i are the usual Pauli spin matrices, $x_i=(Tr(\rho^{AB}(\sigma_i\otimes I)))$ is one of the components of the local Boch vector. R is a correlation matrix with the components $r_{ij}=Tr(\rho^{AB}(\sigma_i\otimes\sigma_i))$. $\|R\|^2=tr(R^\dagger R)$ is the Hilbert–Schmidt norm, λ_{max} is the largest eigenvalue of the matrix $\vec{x}\vec{x}^t+RR^t$. Here the superscript t denotes the transpose of vectors or matrices. The two classes of pure states, which are often investigated in research concerning quantum discord [32], are chosen as the initial states of the two interacting qubits. These states and their evolutions in the environment considered in this study belong to the X-form states [33]. The analytic expression of GMQD for the X-form states can be calculated straightforwardly as [34]:

$$Q(\rho^{AB})=\frac{1}{4}\{8(|\rho_{14}|^2+|\rho_{23}|^2)+2[(\rho_{11}-\rho_{33})^2+(\rho_{22}-\rho_{44})^2] - \max(k_1,k_2,k_3)\} \tag{10}$$

where $k_1=4(|\rho_{23}|-|\rho_{14}|)^2$, $k_2=4(|\rho_{23}|+|\rho_{14}|)^2$ and $k_3=2[(|\rho_{11}|-|\rho_{33}|)^2+(|\rho_{22}|-|\rho_{44}|)^2)]$ are the eigenvalues of the matrix xx^t+TT^t. GMQD is used to measure the nonclassical correlation between the interacting qubits in our study of the effects of damping.

Themeasurement – inducednon – locality, which can be viewed as a kind of nonclassical correlation from a geometric perspective based on the local von Neumann measurements from which one of the reduced states is left invariant. Let ρ be any bipartite state shared between two parties A and B. Then MIN is defined by [11],

$$M(\rho)=\max_{\Pi^A}\|\rho-\Pi^A(\rho)\|^2 \tag{11}$$

where maximum is over all von Neumann measurements Π^A which do not disturb ρ^A, the local density matrix of A, i.e., $\Sigma_k\Pi_k^A\rho^A\Pi_k^A=\rho^A$ and $\Pi^A(\rho)=\Sigma_k(\Pi_k^A\otimes I_B)\rho(\Pi_k^A\otimes I_B)$. By using Ref. [11], MIN for a two-qubit state ρ^{AB} is given by:

$$M(\rho^{AB})=\begin{cases}\frac{1}{4}(TrRR^t-\frac{1}{\|\vec{x}\|}\vec{x}^tRR^t\vec{x}), & \vec{x}\neq 0;\\ \frac{1}{4}(TrRR^t-\lambda_{\min}), & \vec{x}=0.\end{cases} \tag{12}$$

where λ_{min} being minimum eigenvalues of RR^t. σ_i are the usual Pauli spin matrices. In our work one uses $M(t)=2M(\rho)$ to measure MIN. MIN is in some sense, dual to that of geometric measure of discord. Physically, MIN quantifies the global effect caused by locally invariant measurements. MIN has application in general dense coding, quantum state steering etc. MIN vanishes for product state and remains positive for entangled states. For pure states MIN reduces to linear entropy like geometric discord [35]. MIN is invariant under local unitary, i.e., in true sense, it is a non-local correlation measure. The set of all zero MIN states is non-convex. The authors in [36] derived the conditions for the nullity of MIN. They have found that set of states with zero MIN is a proper subset of the set of all classical quantum states, i.e., zero discord states. MIN for classical quantum state vanishes if each eigen-subspace of ρ^A is one dimensional. It therefore reveals that non-commutativity is the cause of these kind of non-locality in quantum states. Recently, in [37], MIN has been quantified in terms of relative entropy to give it another physical interpretation. The results of this quantities for our system can considered in the following section

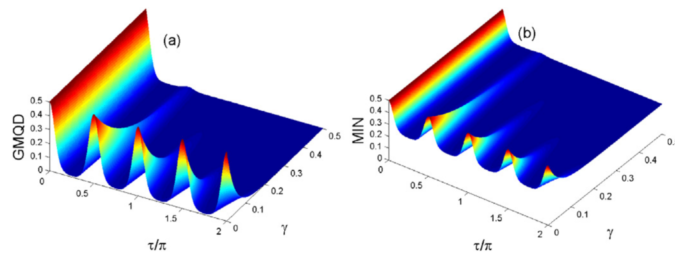

Fig. 1. GMQD and MIN against scaled time and γ for two qubits prepared initially in Werner state, $|\varphi\rangle=sin\theta|11\rangle+cos\theta|00\rangle$ and the field in a coherent state with initial mean photon number $|\alpha|^2=1$, $\theta=\frac{\pi}{4}$ and $\mu=1$.

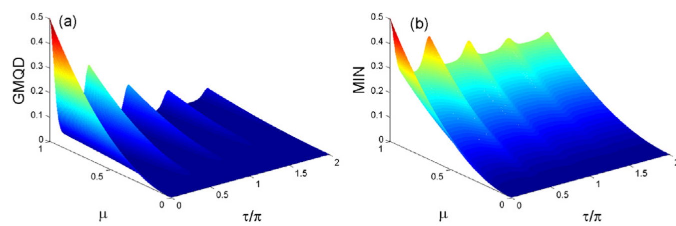

Fig. 2. GMQD and MIN against scaled time and μ for two qubits prepared initially in Werner state, $|\varphi\rangle=sin\theta|11\rangle+cos\theta|00\rangle$ and the field in a coherent state with initial mean photon number $|\alpha|^2=1$, $\theta=\frac{\pi}{4}$ and $\gamma=0.1$.

4. Numerical results

This section was devoted to the study the effect of the damping on the geometric measure of quantum discord (GMQD) and the measurement-induced nonlocality (MIN). In Fig. 1 GMQD and MIN are plotted against scaled time and γ when the two qubits prepared initially in Werner state and the field is initially in a coherent state with mean photon number $|\alpha|^2=1$ when, $|\varphi\rangle=sin\theta|11\rangle+cos\theta|00\rangle$ with $\theta=\frac{\pi}{4}$ and $\mu=1$ which means a pure state. When we neglect the damping parameter ($\gamma=0$), the time evolution of GMQD and MIN are clearly described in Fig. 1(a and b) and we note that GMQD and MIN have the same behavior but they have different values. The minimum values of MIN is greater than the minimum values of GMQD. As the time proceeds the peaks of GMQD and MIN have maximum values at $t=0.5n\pi$,i.e. GMQD and MIN oscillate periodically with the period $\frac{\pi}{2}$ if and only if $\gamma=0$ see Fig. 1a and b.

Moreover, for the special initial state $|\varphi\rangle=\frac{1}{\sqrt{2}}(|11\rangle+|00\rangle)$, GMQD and MIN return to their initial value 0.5 of the peaks. There for collapses and revivals phenomenon appears for GMQD and MIN. To investigate the effect of the damping parameter on dynamics of the GMQD and MIN the decay rate γ is assumed to be $\gamma\in[0, 1]$ (see Fig. 1a and b). The time evolution of GMQD changes symmetrically, there for nonclassical of GMQD decreases with oscillation to zero. For the maximally entangled state $\theta=\frac{\pi}{4}$, GMQD decay to zero monotonically. The reason for these results is that GMQD generated by the mechanism of the qubits interaction is destroyed by the damping for the state, while the qubits interaction has no effect on the maximally entangled state and the initial amount of GMQD only dissipates by the environment. We can see that the GMQD fall in sudden death when $\gamma>0.4$. The same for MIN but in this case MIN is immune to sudden death MIN. This can be illustrated as follows, for high value of the scaled time and the damping parameter $\rho_{14}\to 0$ and there for $M(\rho^{AB})\to\frac{\mu}{4}$, i.e., the maximum value of MIN is governed by μ.

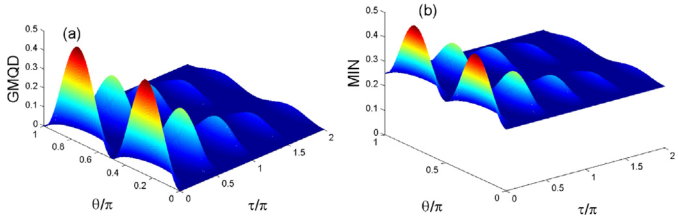

Fig. 3. GMQD and MIN against scaled time and θ for two qubits prepared initially in Werner state, $|\varphi\rangle = sin\theta|11\rangle + cos\theta|00\rangle$ and the field in a coherent state with initial mean photon number $|\alpha|^2 = 1$, $\mu = 1$ and $\gamma = 0.1$.

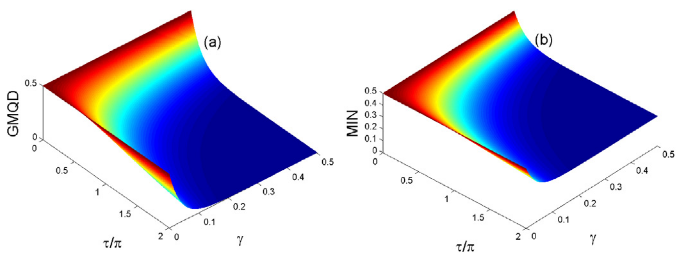

Fig. 4. The same as in Fig. 1 but $|\varphi\rangle = sin\theta|10\rangle + cos\theta|01\rangle$.

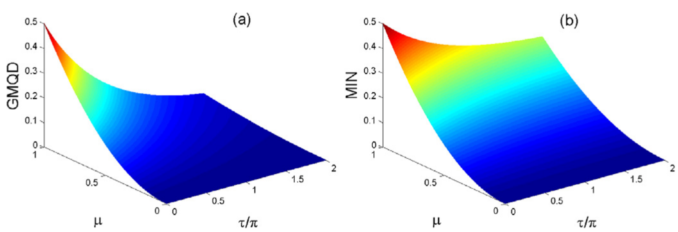

Fig. 5. The same as in Fig. 2 but $|\varphi\rangle = sin\theta|10\rangle + cos\theta|01\rangle$.

In Fig. 2, GMQD and MIN are presented against scaled time and μ with $|\varphi\rangle = sin\theta|11\rangle + cos\theta|00\rangle$, $\theta = \frac{\pi}{4}$ and $\gamma = 0.1$. GMQD is periodic in the time for $\gamma = 0$ and its value increases with the increase of μ then it reach its maximum value at $\mu = 1$ and vanishes at $\mu = 0$ for the classical state as would be expected. For the influence of the damping parameter we take $\gamma = 0.1$, it is clear that for $\mu = 1$ the maximum value of GMQD decrease with increasing the time. The influence of the damping for MIN affect for a short time and then MIN goes back to the fixed evaluated.

In Fig. 3 we present GMQD and MIN against scaled time and θ/π with $|\varphi\rangle = sin\theta|11\rangle + cos\theta|00\rangle$, $\gamma = 0.1$ and $\mu = 1$. The time evolution of GMQD and MIN of the two qubits is very regular, it is symmetric at $\theta = \frac{\pi}{2}$ where the states as separable at $\theta = 0$, $\frac{\pi}{2}$ and is periodic in time for $\gamma = 0$, i.e., collapses and revivals phenomenon appears for GMQD and MIN. This is due to the periodic nature of the interaction phenomenon in the two qubits. The amplitudes of the local extrema of GMQD and MIN decrease with increasing the time τ. This becomes more pronounced at higher values of the damping parameter. GMQD quite vanishes and the peaks centered at $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ disappears due to the phase damping. This means that, after a particular value for the damping parameter, the phase damping destroys GMQD. But MIN still retains its fixed value.

To see what happen for GMQD and MIN for $|\varphi\rangle = sin\theta|10\rangle + cos\theta|01\rangle$, we compare Fig. 4 with Fig. 1 and Fig. 5 with Fig. 2. We find that GMQD and MIN have the same maximum and minimum value, but, the phenomenon of collapses and revivals disappears in Figs. 4 and 5 while it appears in Figs. 1 and 2. This means that there is no oscillating decay for the combination of $|01\rangle$ and $|10\rangle$ in contrast of the combination of $|00\rangle$ and $|11\rangle$. And if we compare Fig. 6 with Fig. 3. we note that the phenomenon of collapses and revivals disappear in Fig. 6 at $\tau = n\frac{\pi}{2}$ but appear in Fig. 3 at the same value.

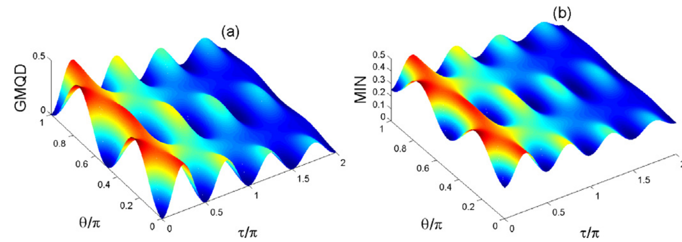

Fig. 6. The same as in Fig. 3 but $|\varphi\rangle = sin\theta|10\rangle + cos\theta|01\rangle$.

5. Conclusions

GMQD and MIN under the influence of the phase damping are studied for qubits prepared in Werner state and the field in coherent state. When the effect of the damping parameter is neglected, We note that GMQD and MIN have the same behavior but they have different values. Therefore, oscillatory phenomenon appears for GMQD and MIN and return its initial value 0.5. Under the influence of the damping the phenomenon of death occurs for GMQD, but this phenomenon does not occur for MIN even when the damping parameter is high. The maximum value of MIN is governed by μ. The maximum value of GMQD and MIN increase by increasing the parameter μ and the amplitude of GMQD decrease faster than MIN. The time evolution of GMQD and MIN of the two qubits is very regular, it is symmetric at $\theta = \frac{\pi}{2}$. The initial conditions for the atoms play an important role in the amplitudes of the local extreme, phenomenon of collapses and revivals for GMQD and MIN.

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