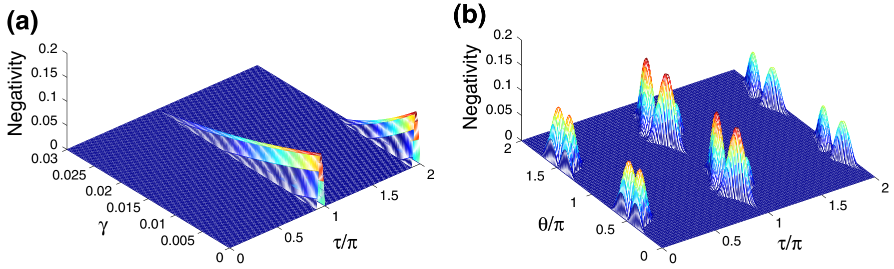

Fig. 5 The negativity when the qubits are initially in a superposition of the excited and the ground state and the field in a coherent state with initial mean photon number $|\alpha|^2 = 4$ and $\theta_1 = \theta_2 = \theta$: **a** negativity against τ and γ with $\theta = \frac{\pi}{4}$, **b** negativity against τ and θ with $\gamma = 0.01$

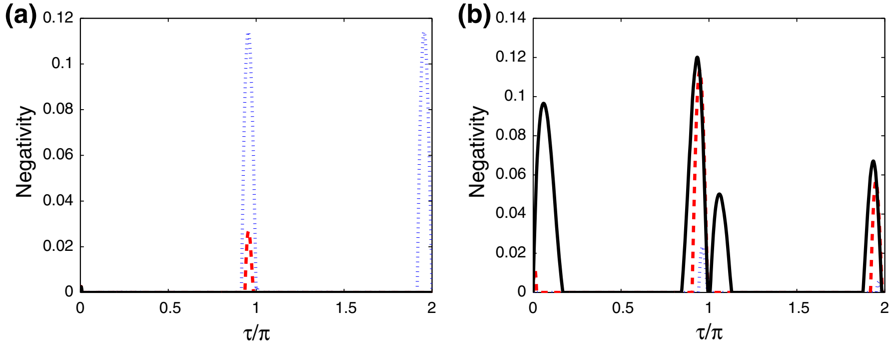

Fig. 6 The negativity as a function of τ, when the qubits are initially in a superposition of the excited and the ground state and the field in a coherent state with initial mean photon number $|\alpha|^2 = 4$ and $\theta_1 = \theta_2 = \theta$: **a** $\theta = \frac{\pi}{4}$ for various values of the damping parameter $\gamma = 0$ (*dot curve*), $\gamma = 0.02$ (*dashed curve*) and $\gamma = 0.03$ (*solid curve*). **b** $\gamma = 0.01$ where $\theta = 0.2\pi$ (*dot curve*), $\theta = 0.3\pi$ (*dashed curve*) and $\theta = 0.4\pi$ (*solid curve*)

In Figs. 5 and 6a, we have plotted the negativity E for two qubits against scaled time τ and γ for the two qubits with the initial mean photon number $|\alpha|^2 = 4$ and $\theta = \frac{\pi}{4}$. From numerical calculations, we find that the degree of the entanglement between the two qubits oscillates at first (i.e., at $\gamma = 0$), and as time evolves it will decrease to zero, which means that the two qubits will become separable eventually. As expected, the small increase in the value of γ leads to a substantial decrease in the value of entanglement and the state finally goes into a pure state and the coherence is lost completely. This means that, after a small value of the damping parameter, the phase damping destroys the entanglement (i.e., the entanglement is very sensitive to the damping parameter).

In Figs. 5 and 6b, the negativity is plotted against scaled time τ and θ with the initial mean photon number $|\alpha|^2 = 4$ and $\gamma = 0.02$. From these figures, it is clearly that the negativity experiences in the case a sudden death, similar to what happens in different systems [35]. That is, the entanglement goes abruptly to zero and remains zero for a finite interval of time before entanglement recovers. This effect has been refereed to as entanglement sudden death (ESD). The entanglement sudden birth phenomenon is observed in Figs. 5 and 6a, b. Entanglement is not presented at earlier times, and suddenly at some finite time an entanglement starts to build up. In other words, for the initial entangled state, there are some intervals of the interaction time where the entanglement reaches its local maxima and drops to zero. Furthermore, we can see that the resurrection of the original negativity takes place periodically and the negativity can remain the original value for a finite interval of time. We can also see from the figures that, apart from the revival peaks of the negativity, a series of peaks appear, and the number of the peaks of the negativity disappears as the damping parameter increases. Physically, the peaks are reflective of the dynamical generation of entanglement between two qubits mediated by the cavity field.

4 Conclusion

The influence of the phase-damped cavity on the mixedness and the amount of entanglement in the dispersive regime is studied. The purity loss of the bipartite partitions of the system [field–two qubits, qubit–(field + qubit)] can be quantified by their tangles. The tangle is found to appear in the model for some initial qubit states and coherent field input with different values of the damping parameter. These two definitions of the tangles are affected by the damping parameter, and the maximum and minimum values of them occur at the same time. It is found that small values of the damping parameter lead to decreasing values of the maximum two-qubit and qubit–field tangles. Increasing the damping parameter not only disturbs the evolution period of the tangles, but also affects their amplitudes. The tangle is used to measure the coherence loss of the qubits and qubit–field states. Under the influence of damping, the degree of tangle for the one qubit-remainder tangle is smaller than that for the field qubit tangle. Also, this shows that the quantum tangle cannot be equally distributed among many different objects in the system. With respect to the distribution angle of the initial qubits states, the time evolution of the tangles is very regular, and it is symmetric and is periodic in time. Furthermore, we discussed the negativity as a measure of entanglement. This study reveals that dispersive regime can be used for generating either the entanglement sudden death or the sudden birth.

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