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Quantum entanglement in a system of two moving atoms interacting with a single mode field

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Abstract
We investigate the quantum entanglement in a system of two moving atoms interacting with a single mode field. An analytical solution for this system is obtained when both atoms are initially in the excited state and the field is in a coherent state. We study the effects of atomic motion and other parameters on the entanglement of the system and different bipartite partitions of the system (field–two atoms, atom–(field+atom)) through the tangles. The effect of atomic motion on the amount of entanglement between atoms and the field is also evaluated through the negativity. The results show that atomic motion leads to the periodic death and anabiosis of the entanglement between two moving atoms, and the time of the death and the amplitude of the anabiosis of the entanglement between two moving atoms depend on the coupling constant of two moving atoms and the parameter of the mode field.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Entanglement is the striking feature of quantum mechanics revealing the existence of non-local correlation among different parts of a quantum system. A pair of quantum systems is called entangled if the measurement on one of them cannot be performed independent of that of the other. Quantum entanglement (QE) is one of the most profound features of quantum mechanics. It is a key problem in the Einstein–Podolsky–Rosen (EPR) paradox [1], quantum cryptography [2], quantum teleportation [3], quantum computation [4] and so on, and has been viewed as an elementary resource for quantum information processing. These new aspects have launched intensive experimental efforts to generate entangled states and theoretical efforts to understand their structures. For instance, the entanglement between two atoms in an arbitrary pure state has been quantified by concurrence [5] or negativity [6]; however that of mixed states has been given as the infimum of the average concurrence over all possible pure state ensemble decompositions. Furthermore, the concurrence has been generalized to include a bipartite system AB, with arbitrary dimensions D_A and D_B in an overall pure state [7]. In this case, the concurrence or negativity is simply related to the purity of the marginal density operators. The interaction between the radiation field and the 2-level atom, namely, the Jaynes–Cummings (JC) model, is an important model in quantum optics since it is exactly solvable in the framework of the rotating wave approximation (RWA) and is experimentally implemented [8, 9]. Moreover, the JC model is a rich source for non-classical effects, e.g. the occurrence of the revival collapse phenomenon (RCP) in the evolution of atomic inversion [10] and the generation of cat states at one-half of the revival time [11, 12]. Interest in investigating the JC model has increased as a result of its application in quantum information [13].

The JC model has been generalized and extended in different directions [14]. One of these directions is two 2-level atoms interacting with a single quantized electromagnetic field (TJCM) [15, 16]. In open systems, entanglement can vanish completely in a finite time for certain initial states. This phenomenon is usually called entanglement sudden death (ESD) [17, 18]. While under the same condition, after the entanglement dies, it can revive completely in a finite time; we call this phenomenon entanglement sudden anabiosis (ESA). ESD is an intriguing and potentially very important discovery. Since the first theory of ESD has been demonstrated, further investigations of various systems have been made by different groups [18–27]. By using other entanglement measures [28, 29], ESD has been observed for more complicated systems, and an attempt has also been made to give a geometric interpretation for the phenomenon of ESD [30]. Recently, experimental research has also been carried out to demonstrate ESD by using engineered interactions between systems and environments [31, 32].

The quantized motion of atoms in electromagnetic fields is a subject of much current interest, in particular in the context of atom optics [33], quantum state preparation and detection [34, 35], and possible schemes for quantum computers [36]. Furthermore, the motion of two atoms trapped at distant positions in the field of a driven standing-wave high-Q optical resonator has been studied [37].

In this paper, we study the effects of the atomic motion and the parameter of the mode field on the entanglement of the system and different bipartite partitions of the system (field–two atoms, atom–(field+atom)) through the tangles. In particular, the effect of atomic motion on the amount of entanglement between atoms and the field is evaluated by the negativity. The results show that this system may present not only the periodic sudden death of the entanglement but also the periodic anabiosis of the entanglement, and the periodic sudden death and anabiosis of the entanglement are relative to the coupling constant of moving atoms and the parameter of the mode field. The paper is arranged as follows: section 2 is devoted to the physical system and its dynamics. In section 3, we employ the analytical results obtained in section 2 to discuss the tangles and negativity. Finally, in section 4, we present our conclusion.

2. The model and its solution

We consider two moving 2-level atoms interacting with a single mode field in the cavity. The effective Hamiltonian in the RWA [38] is ($\hbar = 1$)

$$\hat{H} = w\hat{a}^{\dagger}\hat{a} + w_0 \sum_{i=1}^{2} S_3^i + \sum_{i=1}^{2} g_i f(z_i)(\hat{a}^k S_+^i + \hat{a}^{\dagger k} S_-^i), \tag{1}$$

where S_3^i and $S_\pm^i$ are the pseudo-spin operators of the ith atom; $\hat{a}^{\dagger}$ ($\hat{a}$) is the photon creation (annihilation) operator of the mode of the field, and k is the photon multiplicity and is taken equal to 2 afterwards; w_0 and w are the frequencies of the atomic transition and the mode, respectively; g_i is the atoms–field coupling constant. $f(z_i)$ is the shape function of atomic motion along the z-axis so that only the z-dependence of the mode field function would need to be taken into account. Atomic motion can be incorporated in the usual way [39]:

$$f(z_i) \to f(v_i t), \tag{2}$$

where v_i denotes the ith atomic velocity. In order to be specific, we will define the TEM_{mnp} modes:

$$f(z_i) = \cos\left(\frac{\pi p v_i t}{L}\right), \tag{3}$$

where p represents the number of half-wave lengths of the field mode inside a cavity of length L. For simplicity, we consider that the two moving atoms have the same velocity so that $f(z_1) = f(z_2)$ under the resonant case ($w_0 = w$). Here, we assume that two moving 2-level atoms enter the cavity at time $t = 0$ in the state

$$|\Psi(0)\rangle_A = |+,+\rangle, \tag{4}$$

and the mode is in the coherent state

$$|\Psi(0)\rangle_F = \sum_{n=0}^{\infty} q_n |n\rangle, \tag{5}$$

where $q_n = \mathrm{e}^{-|\alpha^2|/2}\frac{\alpha^n}{\sqrt{n!}}$, $\alpha = |\alpha|\ \mathrm{e}^{\mathrm{i}\varphi}$, $\bar{n} = |\alpha|^2$ is the mean photon number of the coherent field and φ is the phase angle of the coherent field (here we take $\varphi = 0$). The initial state of the system is

$$|\Psi(0)\rangle = |\Psi(0)\rangle_F \otimes |\Psi(0)\rangle_A = \sum_{n=0} q_n |+,+,n\rangle. \tag{6}$$

Under this initial condition, the solution of the Schrödinger equation in the interaction picture, i.e. the wave function of the system at any time $t > 0$, is given by

$$\begin{aligned} |\Psi(t)\rangle = \sum_{n=0}^{\infty} q_n [& c_1(n,t)|+,+,n\rangle + c_2(n,t)|+,-,n+k\rangle \\ &+ c_3(n,t)|-,+,n+k\rangle + c_4(n,t)|-,-,n+2k\rangle]. \end{aligned} \tag{7}$$

Then we obtain the explicit forms for the dynamical coefficients $c_j(n,t)$ ($j = 1, 2, 3, 4$) as

$$\begin{aligned} c_1(n,t) &= \frac{1}{\zeta^2 - \eta^2}[(\zeta^2 - \beta_1^2 - \beta_2^2)\cos(\theta\zeta) \\ &\quad + (\beta_1^2 + \beta_2^2 - \eta^2)\cos(\theta\eta)], \\ c_2(n,t) &= \frac{\mathrm{i}[\beta_2(\beta_1\gamma_1 - \beta_2\gamma_2) + \gamma_2\eta^2]\sin(\eta\theta)}{\eta(\zeta^2 - \eta^2)} \\ &\quad - \frac{\mathrm{i}[\beta_2(\beta_1\gamma_1 - \beta_2\gamma_2) + \gamma_2\zeta^2]\sin(\zeta\theta)}{\zeta(\zeta^2 - \eta^2)}, \\ c_3(n,t) &= \frac{\mathrm{i}[\beta_1(\beta_2\gamma_2 - \beta_1\gamma_1) + \gamma_1\eta^2]\sin(\eta\theta)}{\eta(\zeta^2 - \eta^2)} \\ &\quad - \frac{\mathrm{i}[\beta_1(\beta_2\gamma_2 - \beta_1\gamma_1) + \gamma_1\zeta^2]\sin(\zeta\theta)}{\zeta(\zeta^2 - \eta^2)}, \\ c_4(n,t) &= \frac{\gamma_2\beta_1 + \gamma_1\beta_2}{\zeta^2 - \eta^2}[\cos(\zeta\theta) - \cos(\eta\theta)], \end{aligned} \tag{8}$$

where

$$\begin{aligned} \beta_1 &= g_1\sqrt{\frac{(n+2k)!}{(n+k)!}}, \quad \beta_2 = g_2\sqrt{\frac{(n+2k)!}{(n+k)!}}, \\ \gamma_1 &= g_1\sqrt{\frac{(n+k)!}{(n)!}}, \quad \gamma_2 = g_2\sqrt{\frac{(n+k)!}{(n)!}}, \end{aligned} \tag{9}$$

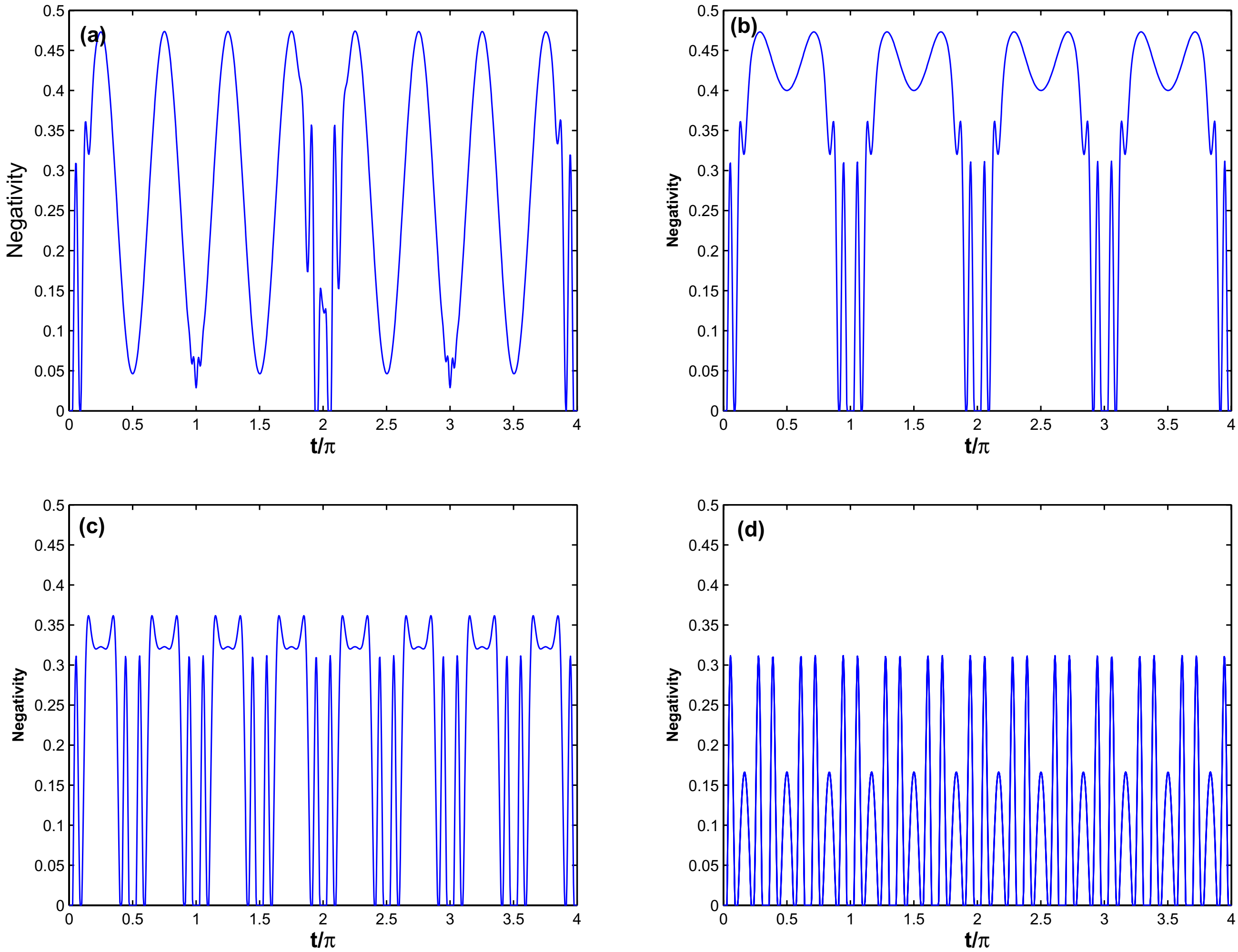

Figure 1. Time evolution of the negativity $\aleph$ for the moving atom–atom entanglement for the two identical atoms prepared initially in the excited state and the field in a coherent state for the two-photon ($k = 2$) process with initial mean photon numbers $|\alpha|^2 = 25$ and $g_1 = g_2 = 0.5(g_1 + g_2)$ for different mode field parameter p: (a) $p = 0$, (b) $p = 1$, (c) $p = 2$ and (d) $p = 3$.

$$\zeta^2 = \tfrac{1}{2}[\beta_1^2 + \beta_2^2 + \gamma_2^2 + \gamma_1^2 - \sqrt{(\beta_1^2 + \beta_2^2 + \gamma_2^2 + \gamma_1^2)^2 - 4(\gamma_2\beta_2 - \beta_1\gamma_1)^2}],$$

$$\eta^2 = \tfrac{1}{2}[\beta_1^2 + \beta_2^2 + \gamma_2^2 + \gamma_1^2 + \sqrt{(\beta_1^2 + \beta_2^2 + \gamma_2^2 + \gamma_1^2)^2 - 4(\gamma_2\beta_2 - \beta_1\gamma_1)^2}], \quad (10)$$

with

$$\theta(t) = \int_0^t f(vt')\mathrm{d}t' = \frac{L}{\pi p v}\sin\left(\frac{\pi \mathrm{pvt}}{L}\right). \quad (11)$$

For a particular choice of atomic motion velocity, we take $v = L/\pi$, and $\theta(t)$ becomes

$$\theta(t) = \frac{1}{p}\sin(pt). \quad (12)$$

Information about the bipartite (i.e. atoms and field) is involved in the wave function (equation (7)) or in the total density matrix $\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)|$. Nevertheless, information on the atomic system solely can be obtained from the atomic reduced density matrix $\hat{\rho}(t)$ having the form

$$\hat{\rho_A}(t) = \mathrm{Tr}_F[\hat{\rho}(t)], \quad (13)$$

and we can write $\hat{\rho_A}(t)$ in the following form:

$$\hat{\rho_A}(t) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}, \quad (14)$$

where

$$\rho_{11} = \sum_n^\infty q(n)q^*(n)c_1(n,t)c_1^*(n,t),$$

$$\rho_{22(33)} = \sum_n^\infty q(n-k)q^*(n-k)c_{2(3)}(n-k,t)c_{2(3)}^*(n-k,t),$$

$$\rho_{44} = \sum_n^\infty q(n-2k)q^*(n-2k)c_4(n-2k,t)c_4^*(n-2k,t),$$

$$\rho_{12(13)} = \sum_n^\infty q(n)q^*(n-k)c_1(n,t)c_{2(3)}^*(n-k,t) = \rho_{21(31)}^*,$$

$$\rho_{14} = \sum_n^\infty q(n)q^*(n-2k)c_1(n,t)c_4^*(n-2k,t) = \rho_{41}^*,$$

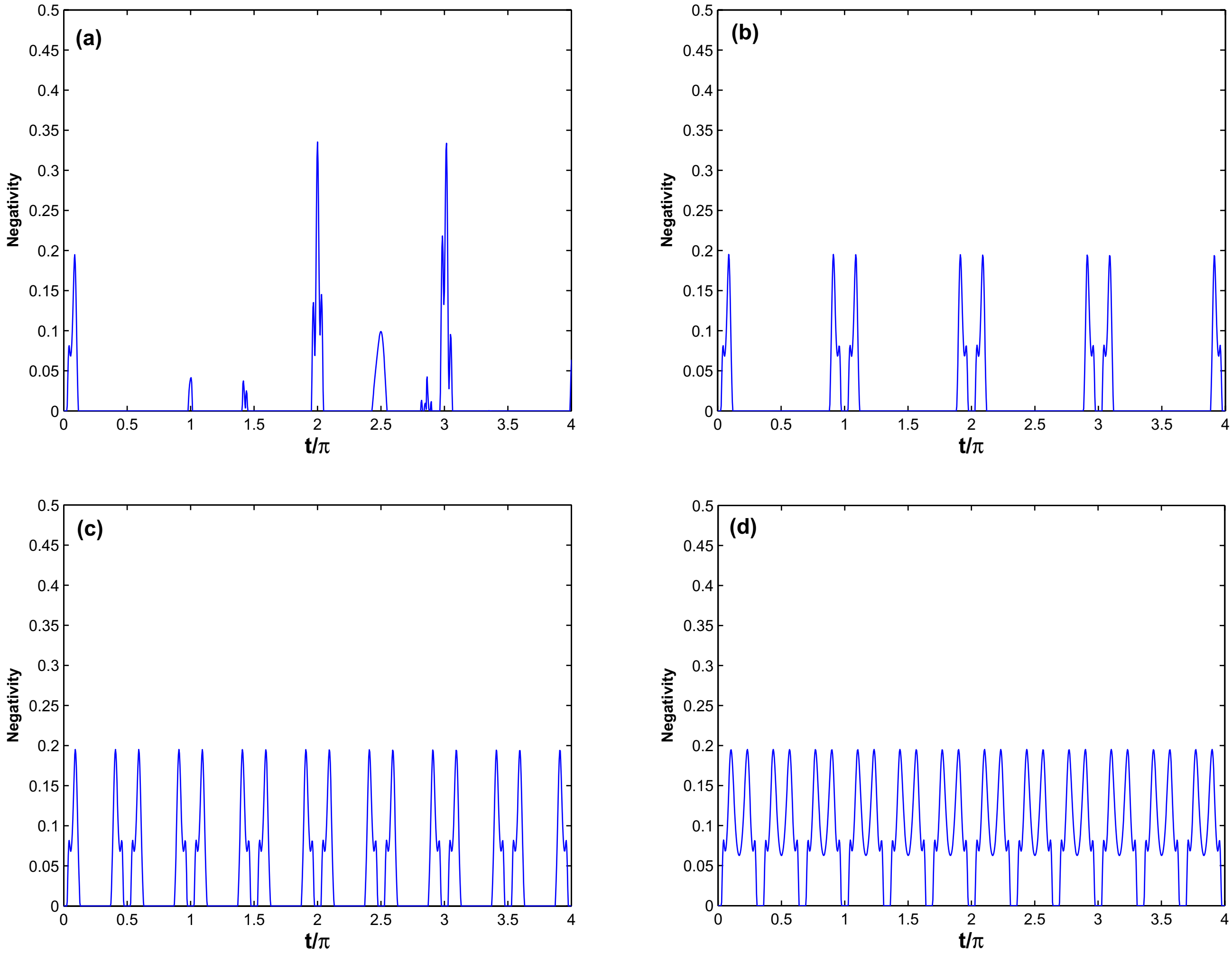

Figure 2. Time evolution of the negativity $\aleph$ for the moving atom–atom entanglement for the two atoms prepared initially in the excited state and the field in a coherent state for the two-photon ($k = 2$) process with initial mean photon numbers $|\alpha|^2 = 25$ and $g_1 = 0.7(g_1 + g_2)$ and $g_2 = 0.3(g_1 + g_2)$ for different mode field parameter p: (a) $p = 0$, (b) $p = 1$, (c) $p = 2$ and (d) $p = 3$.

$$\rho_{23} = \sum_n^{\infty} q(n-k)q^*(n-k)c_2(n-k,t)c_3^*(n-k,t) = \rho_{32}^*,$$

$$\rho_{24(34)} = \sum_n^{\infty} q(n-k)q^*(n-2k)c_{2(3)}(n-k,t)c_4^*(n-2k,t) = \rho_{42(43)}^*. \tag{15}$$

3. Measurement of entanglement degree

3.1. Negativity

In subsequent sections, our goal is to quantify the entanglement of the final state (equation (14)). For pure states, the Bell states represent maximally entangled states, but for mixed states represented by a density matrix there are some difficulties with ordering the states according to various entanglement measures; different entanglement measures can give different orderings of pairs of mixed states and there is a problem of the definition of the maximally entangled mixed state [40, 41]. To assess the amount of entanglement created in a two-atom system, one can use either of two entanglement measures: concurrence or negativity. We use here negativity, which is based on the Peres–Horodecki [42, 43] criterion for entanglement and is defined by the formula

$$\aleph = \max\left(0, -2\sum_i \mu_i\right), \tag{16}$$

where the sum is taken over the negative eigenvalues μ_i of the partial transposition of the density matrix ρ of the system. The value $\aleph = 1$ corresponds to maximum entanglement between the atoms, whereas $\aleph = 0$ describes completely separated atoms. We present numerical results of the negativity $\aleph$ given by equation (16) for two moving atoms when both atoms are initially in the excited state and the field is in a coherent state for the two-photon ($k = 2$) process with the initial mean photon numbers $|\alpha|^2 = 25$ for different parameters of two moving atoms as shown in figures 1 and 2 plotted against $(g_1 + g_2)t/\pi$. Figure 1 shows the influence of the mode field parameter p for the two identical 2-level atoms, i.e. the coupling constant $g_1 = g_2$, while figure 2 illustrates the effects of the mode field parameter p on the negativity $\aleph$ when the atoms can have different coupling constants $g_1 \neq g_2$. In figure 1(a), we have plotted the negativity $\aleph$ for $p = 0$, i.e. in the absence of atomic motion, corresponding to the

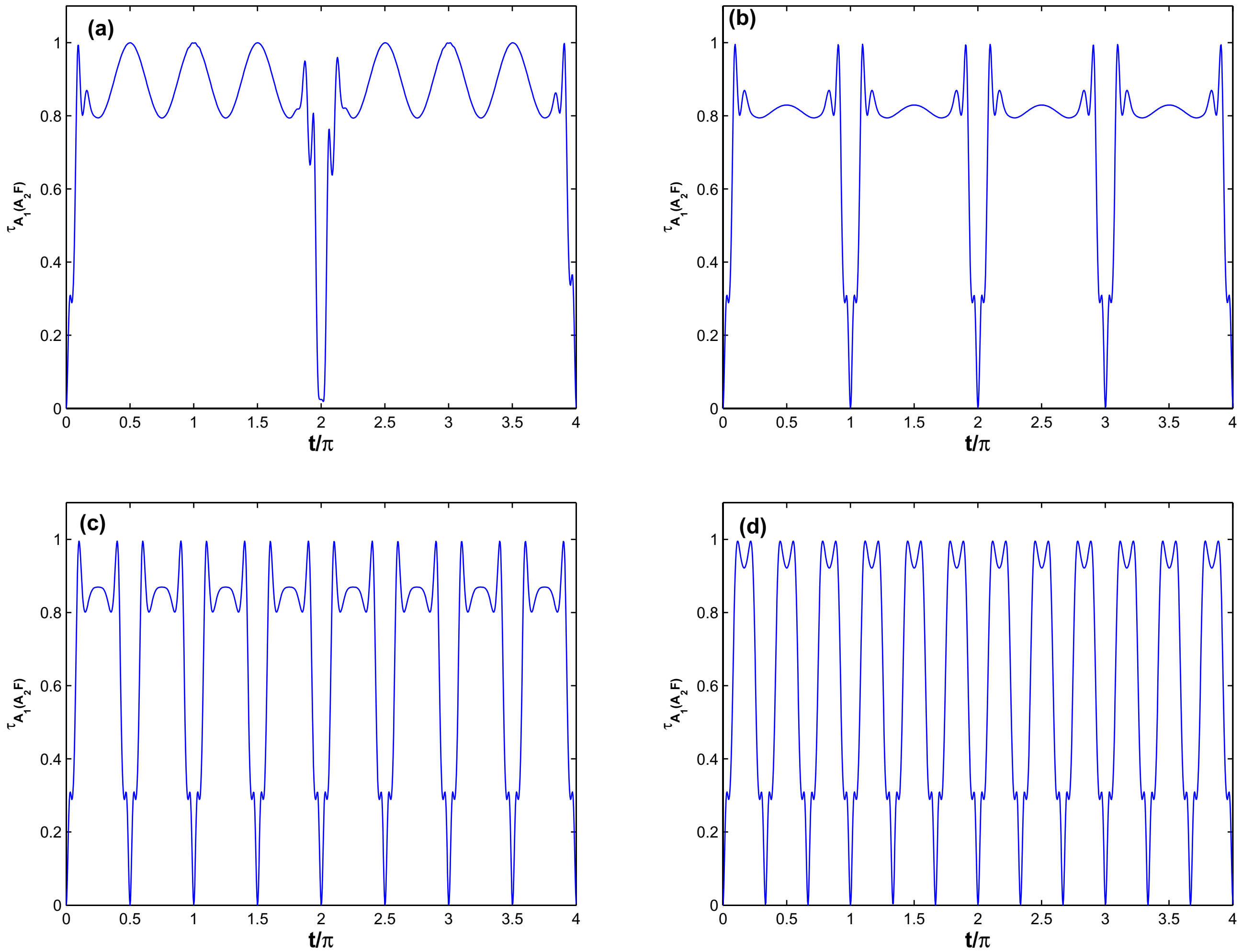

Figure 3. Time evolution of the tangle $\tau_{A_1(A_2F)}$ for the moving atom–atom entanglement for the two identical atoms prepared initially in the excited state and the field in a coherent state for the two-photon ($k = 2$) process with initial mean photon numbers $|\alpha|^2 = 25$ and $g_1 = g_2 = 0.5(g_1 + g_2)$ for different mode field parameter p: (a) $p = 0$, (b) $p = 1$, (c) $p = 2$ and (d) $p = 3$.

evolution of the negativity $\aleph$ in the standard two photon process. It is observed that the negativity $\aleph$ evolves with a period 2π, when $(g_1 + g_2)t = 2n\pi$ $(n = 0, 1, 2, \ldots)$, $\aleph$ evolves to zero and the field is completely disentangled from the atom, while for $(g_1 + g_2)t = (2n - 1)\frac{\pi}{4}$, $S_A(t)$ evolves to the maximum value and the field is entangled with the atoms. In figure 1(b), the motion of two identical 2-level atoms is taken into account, and it is evident that the negativity $\aleph$ of two moving atoms evolves periodically with period π. In the time evolution process, we can see that the death and anabiosis of the entanglement between two moving atoms become periodic. This periodical dynamic behavior of the negativity $\aleph$ depends on the mode field parameter p and can be understood by equation (12). From equation (12), we have $\theta(t) = \frac{1}{p}\sin(pt)$. It is observed that $\theta(t)$ is a periodical function on the scaled time t with period $2\pi/p$. This periodicity of $\theta(t)$ of the scaled time t just leads to the periodicity of evolution of the negativity $\aleph$. When $p = 2$ and 3, the entanglement of the atom–atom evolves in period $\pi/2$ and $\pi/3$, respectively. The influence of parameter p of the mode field on the entanglement between two moving atoms is shown in figures 1(c) and (d) when two moving atoms are initially in their excited states; we show two cases $p = 2$ and $p = 3$, respectively. It is explicit that the period shortens and the amplitude decreases for negativity $\aleph$ as p increases, and the death and anabiosis of the entanglement between two moving atoms become periodic. In figures 2(a), the different coupling constants decrease the amplitude for the negativity $\aleph$. In figures 2(b)–(d), the time of death and the time of anabiosis of the entanglement between two moving atoms rely on the coupling constant of two moving atoms. The atomic motion adds regularity to the negativity $\aleph$. With an increase in the parameter p, the negativity $\aleph$ increases for $p = 3$.

3.2. Bipartite tangles for the two atoms

Entanglement is at the heart of quantum information theory, since it can provide phenomena very different from those of classical correlation [44]. Various efforts are made to characterize qualitatively and quantitatively the entanglement properties of quantum systems. This is motivated by the progress in the experimental techniques in creating entangled states [44]. In this section, we investigate the entanglement property for the system under consideration using the tangle τ defined in [45]. Based on the symmetry in the system, we investigate two types of tangle, which are field–atoms and one

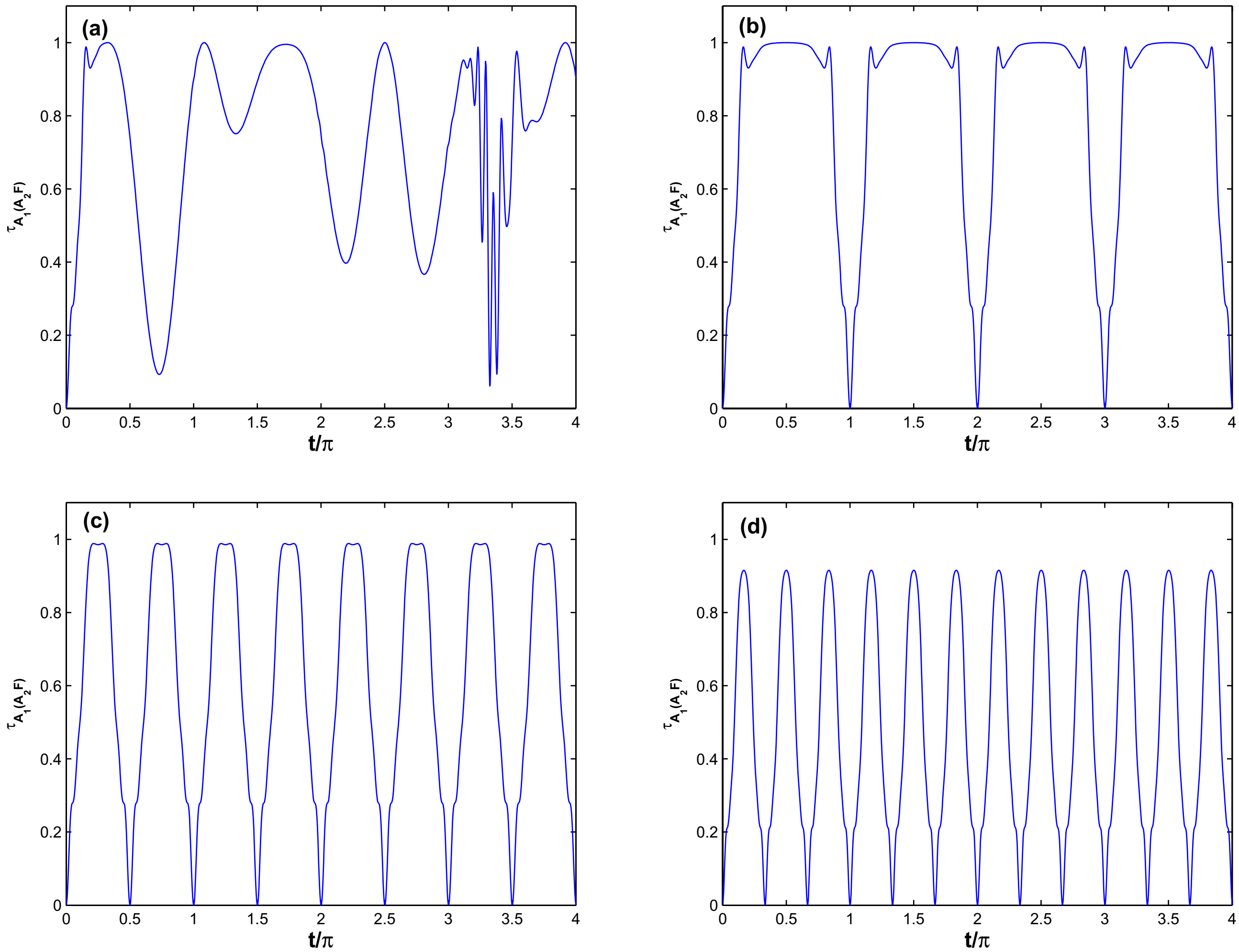

Figure 4. Time evolution of the tangle $\tau_{A_1(A_2F)}$ for the moving atom–atom entanglement for the two atoms prepared initially in the excited state and the field in a coherent state for the two-photon ($k=2$) process with initial mean photon numbers $|\alpha|^2=25$ and $g_1=0.7(g_1+g_2)$ and $g_2=0.3(g_1+g_2)$ for different mode field parameter p: (a) $p=0$, (b) $p=1$, (c) $p=2$ and (d) $p=3$.

atom–remainder. Precisely, assume that F, A_1 and A_2 denote the field, first atom and second atom, respectively. Therefore, the field–atoms tangle (F–A_1A_2) and one atom–remainder tangle (A_1–FA_2) are defined as [46]

$$\tau_{F(A_1A_2)}=2[1-\mathrm{Tr}(\rho_F^2)] \tag{17}$$

and

$$\tau_{A_1(A_2F)}=2[1-\mathrm{Tr}(\rho_{A_2F}^2)], \tag{18}$$

where ρ_F is the reduced density matrix of the field, which can be obtained by tracing the density matrix of the system over the set of atoms A_1A_2. The forms, equations (17) and (18), quantify the degree of entanglement to which the ensembles behave as a collective entity. As mentioned in the introduction, when the system is in an overall pure state the tangle involves the notion of the purity. Generally, when $\tau_{F(A_1A_2)}=0$, say, the parties F and A_1A_2 are completely disentangled, but, of course, they could be in states different from those of the initial ones. Nevertheless, when $\tau_{F(A_1A_2)}=2$ the parties are maximally entangled. In the following, we use the terminologies $\tau_{F(A_1A_2)}\leqslant 1$ for weak entangled parties and $\tau_{F(A_1A_2)}>1$ for strong entangled parties.

- One atom–remainder tangle.

We study the evolution of the tangle $\tau_{A_1(A_2F)}$ for the system under consideration, which can be easily evaluated as [47]

$$\begin{aligned}\tau_{A_1(A_2F)}=2-2\sum_{n,n'}^{\infty}&\{|q(n)q^*(n')|^2[|c_1(n,t)|^2\\&+|c_3(n,t)|^2][|c_1^*(n',t)|^2+|c_3^*(n',t)|^2]\\&+|q(n)q^*(n')|^2[|c_2(n,t)|^2\\&+|c_4(n,t)|^2][|c_2^*(n',t)|^2+|c_4^*(n',t)|^2]\\&+2q(n+k)q(n)q^*(n'+k)q^*(n')\\&\times[c_1(n+k,t)c_2^*(n,t)\\&+c_3(n+k,t)c_4(n,t)][c_1(n'+k,t)c_2^*(n',t)\\&+c_3^*(n'+k,t)c_4^*(n',t)]\}.\end{aligned} \tag{19}$$

We have plotted the tangle $\tau_{A_1(A_2F)}$, i.e. (equation (19)), for two moving atoms when both atoms are initially in the excited state and the field is in a coherent state for the two-photon ($k=2$) process with the initial mean photon number $|\alpha|^2=25$ for different parameters of the two moving atoms, as shown in figures 3 and 4. Figure 3 shows the influence of the mode field parameter p when the atoms can have identical coupling constants $g_1=g_2=0.5(g_1+g_2)$, while figure 4 illustrates the

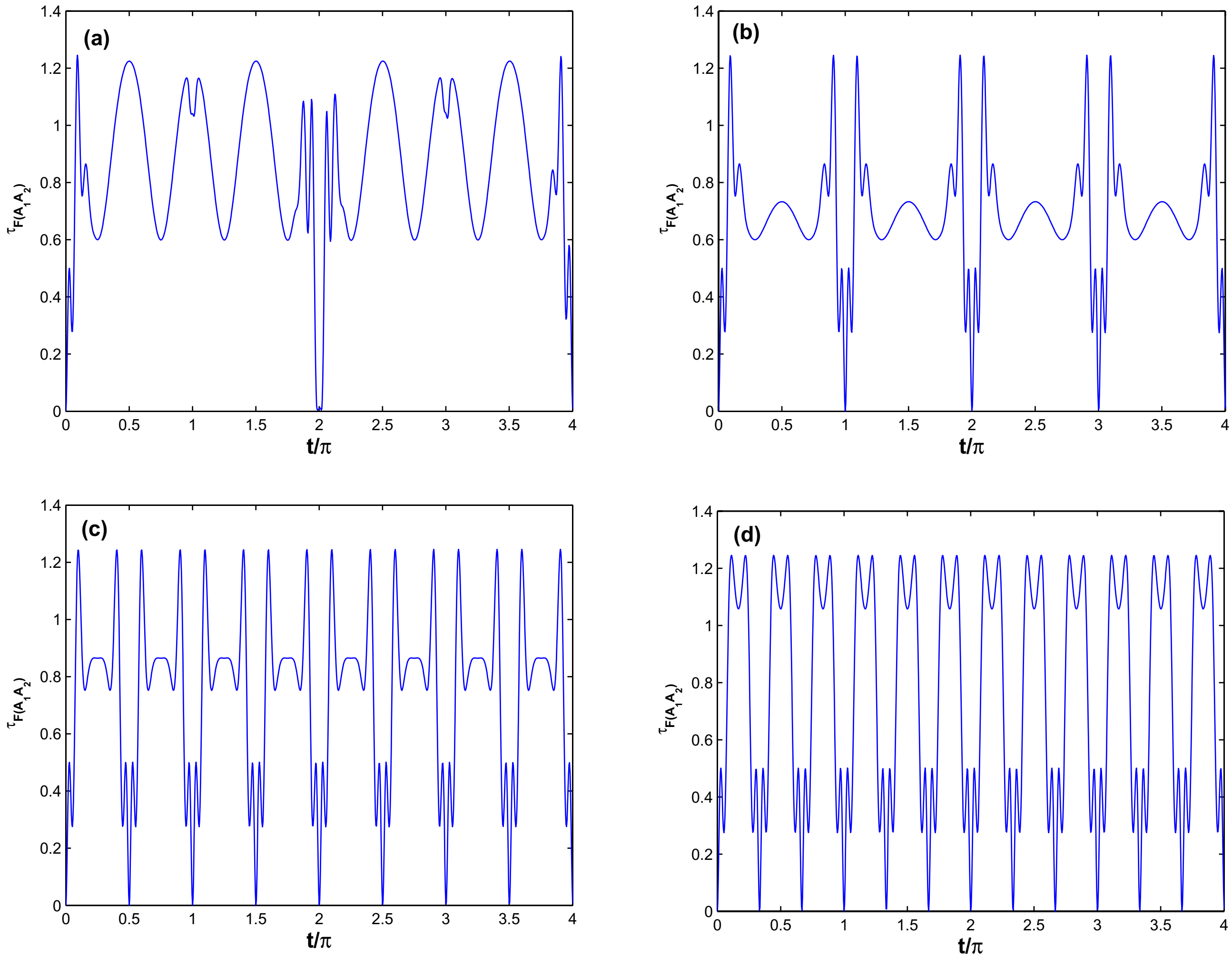

Figure 5. Time evolution of the tangle $\tau_{F(A_1A_2)}$ for the moving atom–atom entanglement for the two identical atoms prepared initially in the excited state and the field in a coherent state for the two-photon ($k = 2$) process with initial mean photon numbers $|\alpha|^2 = 25$ and $g_1 = g_2 = 0.5(g_1 + g_2)$ for different mode field parameter p: (a) $p = 0$, (b) $p = 1$, (c) $p = 2$ and (d) $p = 3$.

effects of the mode field parameter p on the tangle $\tau_{A_1(A_2F)}$ when the two atoms have different coupling constants $g_1 \neq g_2$.

As seen from these figures, we can conclude that atomic motion leads to periodic evolution of the tangle $\tau_{A_1(A_2F)}$ and entanglement. This periodical dynamic behavior of the tangle $\tau_{A_1(A_2F)}$ depends on the mode field parameter p and the increase in the parameter p results in not only a shortening of the evolution period of the tangle $\tau_{A_1(A_2F)}$ but also affects the amplitude of the tangle $\tau_{A_1(A_2F)}$. Physically, all these features can be attributed to the change in the atom–field interaction time due to atomic motion (see figure 3). To show the influence of coupling constants g_1 and g_2, we see that when $g_1 = g_2$ the tangle $\tau_{A_1(A_2F)}$ has the same amplitude along the t-axis. But when $g_1 \neq g_2$, the amplitude decreases for the tangle $\tau_{A_1(A_2F)}$ as p increases (see figure 4).

- Field–atoms tangle.

We study the evolution of the tangle $\tau_{F(A_1A_2)}$ for the system under consideration, which can be easily evaluated as

$$\tau_{F(A_1A_2)} = 2 - 2[\rho_{11}^2 + \rho_{22}^2 + \rho_{33}^2 + \rho_{44}^2 + 2\rho_{12}\rho_{21} + 2\rho_{13}\rho_{31} + 2\rho_{14}\rho_{41} + 2\rho_{23}\rho_{32} + 2\rho_{24}\rho_{42} + 2\rho_{14}\rho_{41}. \quad (20)$$

Under the influence of atomic motion, we have plotted $\tau_{F(A_1A_2)}$ in figures 5 and 6 for given values of the interaction parameters. The obvious remark from figures 3 and 4 is that $0 \leqslant \tau_{A_1(A_2F)} \leqslant 1$. This behavior is completely different from that of $\tau_{F(A_1A_2)}$. This means that the degree of entanglement for the one atom–remainder tangle is smaller than that of the field–atoms tangle. Also, this shows that QE cannot be equally distributed among many different objects in the system. This can be explained as follows. Entanglement is a direct consequence of the energy flow between different components and parties of the system. As we have an isolated TJCM, i.e. the interaction with the environment is neglected, the energy is periodically interchanged between the field and the two-atom system. More illustratively, when the k photons are annihilated from the field, they are created, i.e. equally distributed, in the two-atom party and vice versa. This means that the energy involved in the FA_2 party is more than that in the A_1 party. In this regard, the rate of flow of energy between the parties F and A_1A_2 is greater than that between A_1 and FA_2, and this is the origin of the difference between the evolution of $\tau_{F(A_1A_2)}$ and $\tau_{A_1(A_2F)}$ [47].

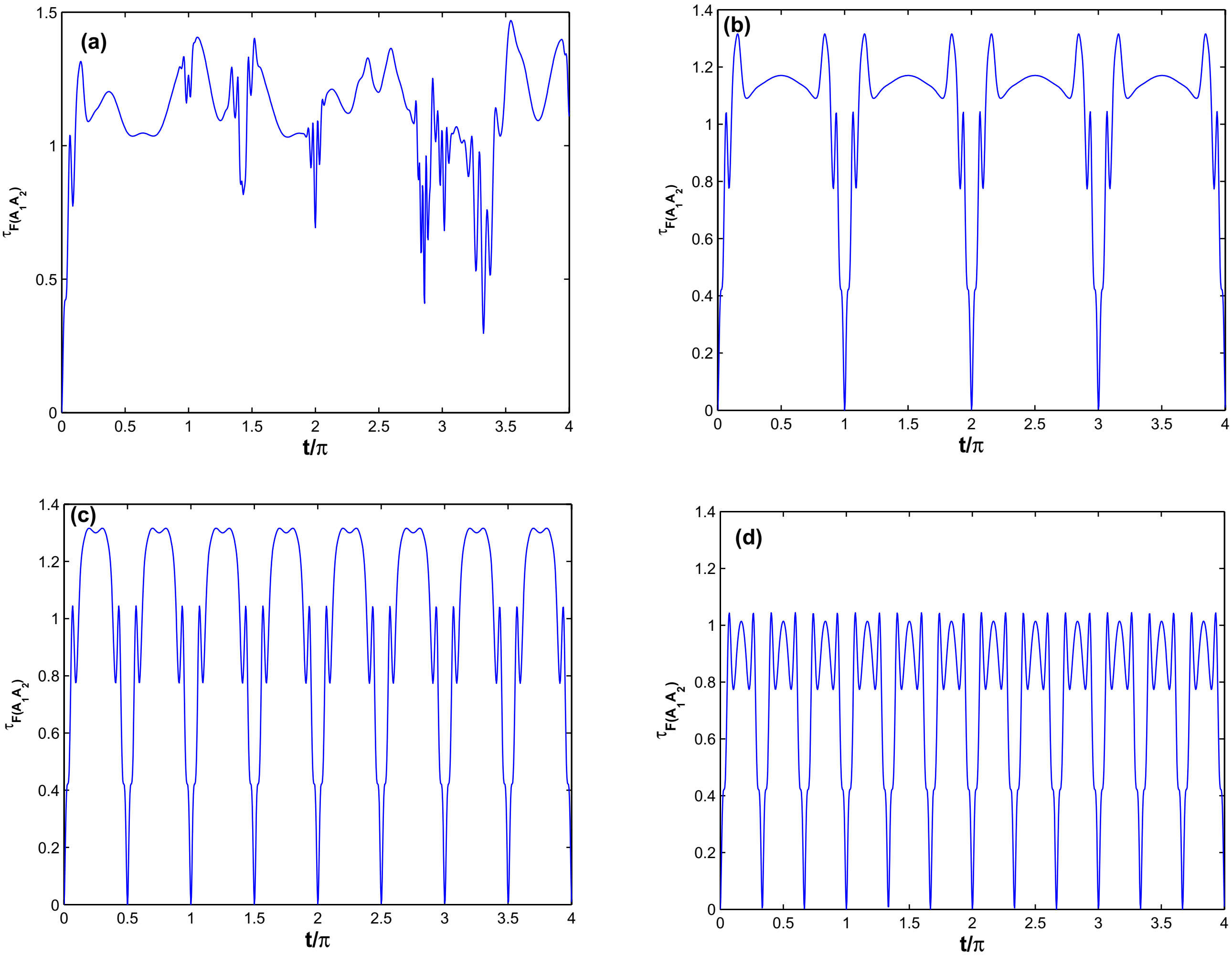

Figure 6. Time evolution of the tangle $\tau_{F(A_1A_2)}$ for the moving atom–atom entanglement for the two atoms prepared initially in the excited state and the field in a coherent state for the two-photon ($k = 2$) process with initial mean photon numbers $|\alpha|^2 = 25$ and $g_1 = 0.7(g_1 + g_2)$ and $g_2 = 0.3(g_1 + g_2)$ for different mode field parameter p: (a) $p = 0$, (b) $p = 1$, (c) $p = 2$ and (d) $p = 3$.

4. Conclusions

In this paper, we have studied the QE for two moving atoms interacting with a multi-photon single mode field. We have investigated the influence of atomic motion on the behavior of tangles and negativity. Atomic motion taken as a trigonometric function leads to the periodic evolution of entanglement between two moving atoms. The period depends on the parameter p. The time of the death and the amplitude of the anabiosis of the entanglement between two moving atoms depend on the coupling constant. The death of the entanglement still exists with the change of the parameter p. We can conclude that atomic motion leads to the periodic evolution of tangle and negativity. This periodical dynamic behavior of the tangle depends on the mode field parameter p, and an increase in the parameter p results is not only a shortening of the evolution period of the tangle but also affects the amplitude of the tangle. Under the influence of atomic motion, the degree of entanglement for the one atom–remainder tangle is smaller than that of the field–atoms tangle. Also, this shows that the QE cannot be equally distributed among many different objects in the system.

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