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2009 J. Phys. B: At. Mol. Opt. Phys. 42 175502

(http://iopscience.iop.org/0953-4075/42/17/175502)

IOP PUBLISHING JOURNAL OF PHYSICS B: ATOMIC, MOLECULAR AND OPTICAL PHYSICS

J. Phys. B: At. Mol. Opt. Phys. **42** (2009) 175502 (8pp) doi:10.1088/0953-4075/42/17/175502

Quantum phase properties and Wigner function of two 2-level atoms in the presence of the Stark shift for the Tavis–Cummings model

A-S F Obada[1], H A Hessian[2] and M Hashem[2]

[1] Faculty of Science, Al Azhar University, Nasr City, Cairo, Egypt
[2] Faculty of Science, Assiut University, Assiut, Egypt

Received 25 March 2009, in final form 2 June 2009
Published 12 August 2009
Online at stacks.iop.org/JPhysB/42/175502

Abstract
An analytical solution for two identical 2-level atoms interacting with a single-mode quantized radiation field in the presence of the Stark shift is obtained. Both atoms are prepared initially in the excited state and the field in a coherent state. The phase distribution, phase variance and Wigner function are investigated. The influence of the Stark shift on the Wigner function and the phase properties is analysed.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Quantum information processing has recently attracted much attention in quantum physics and information science [1, 2]. It provides novel information such as quantum computing [3], teleportation [4], cryptography [5], dense coding [6] and entanglement swapping [7]. The system containing two 2-level atoms in an electromagnetic field [8–15] is one of particular interest, since it can represent two qubits, the building blocks of the quantum gates that are essential to implement quantum protocols in quantum information processing.

As a typical model, the Tavis–Cummings model (TCM) [16] provides the simplest example of a collection of 2-level atoms interacting with a common quantized electromagnetic field. A thorough understanding of the dynamical evolution of the TCM has obvious implications for the performance of quantum information processing [1, 17, 18], as well as for our understanding of fundamental quantum mechanics [1, 19]. Two-atom states can be characterized by several parameters such as purity (or mixedness), phase probability, phase variance and the Wigner function.

The quasiprobability distribution defined by Wigner has been one of the main tools to provide insights into the connections between classical and quantum mechanics [20]. Particularly, the negative values of the Wigner (W) function in a two-dimensional phase space of a single-mode light field are seen as a hallmark of the nonclassicality of the dynamical systems. Recently, several schemes for direct measurement of values of the W function have been proposed [21] and performed experimentally [22]. Another important issue related to this negative-valued part is the disappearance of the interference pattern in the mesoscopic quantum superpositions of coherent states [23, 24] (Schrödinger cat states) due to the phenomenon of decoherence [25]. The Wigner representation [20] is a useful tool to express the quantum mechanics in a phase-space formalism. It contains complete information about the state of the system, i.e., it carries the same information as the density operator or the corresponding wavefunction. However, the authors in [26, 27] have shown that the radially integrated Wigner function can be negative and so the polar angle cannot be interpreted as corresponding to a phase angle observable [26].

A phase formalism based on the existence of states with a well-defined phase has been introduced [28–30]. This formalism accommodates a Hermitian phase operator and thus allows one to treat phase properties of the field in fully quantum-mechanical fashion without recourse to semi-classical or phenomenological methods. Quantities such as the phase distribution function and phase variance are now available for investigation. Using this phase formalism, the authors in [31, 32] examined the evolution of the phase of a coherent field interacting with a 2-level atom in an ideal cavity. They found that the time behaviour of the phase probability distribution and the variance of the phase reflect the collapses and revivals of the atomic inversion [33–35] in an interesting way. Also the effects of cooperative atomic interaction, cavity losses and pump fluctuations on quantum phase properties of the field have been studied [36] for different initial states. It is found that the quantum phase properties of the field are highly sensitive to two-atom events and cavity losses and the fluctuation associated with the random injection of the atoms. The phase properties of a field mode interacting with two 2-level atoms with multi-photon transitions were studied [10, 37]. It is found that for symmetric (asymmetric) interaction, the system can generate asymmetric (symmetric) cat states [37]. All the previous studies ignored the Stark shift but when the two atomic levels are coupled with comparable strength to the intermediate relay level, the Stark shift becomes significant and cannot be ignored [38–41].

In the present paper, we study the evolution of two identical 2-level atoms interacting with a single-mode quantized radiation field, taking into account the level shifts produced by the Stark shift. We assume that both the atoms are initially prepared in the excited state and the field in a coherent state. We investigate the phase distribution, phase variance and Wigner function. The paper is arranged as follows: section 2 is devoted to the physical system and its dynamics. In section 3, we employ the analytical results obtained in section 2. Finally, in section 4, we present our conclusion.

2. The model and its solution

We consider the interaction of two identical 2-level atoms with a resonant single-mode quantized electromagnetic field. In this system, either atom can transit from the excited state $|+\rangle$ to the ground state $|-\rangle$ under the driving of a resonant field and emit a photon. Also, either atom that is in state $|-\rangle$ can absorb such a photon and jump to state $|+\rangle$. The authors in [10] discuss a similar model without the Stark shift and in the case of one photon, but our interest lies in the case where the Stark shift and multi-photon are included. The Hamiltonian of the system takes the form

$$\hat{H} = \omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \omega\left(\hat{\sigma}_1^3 + \hat{\sigma}_2^3\right) + g\sum_{i=1}^{2}\left(\hat{a}^l\hat{\sigma}_i^+ + \hat{a}^{\dagger l}\hat{\sigma}_i^-\right) + \hat{a}^\dagger\hat{a}\beta_1\left(\hat{\sigma}_1^-\hat{\sigma}_1^+ + \hat{\sigma}_2^-\hat{\sigma}_2^+\right) + \hat{a}^\dagger\hat{a}\beta_2\left(\hat{\sigma}_1^+\hat{\sigma}_1^- + \hat{\sigma}_2^+\hat{\sigma}_2^-\right), \quad (\hbar = 1), \tag{1}$$

where $\hat{a}$, $\hat{a}^\dagger$ are annihilation and creation operators of the cavity field and the 2-level atoms are described by the atomic pseudospin operators $\hat{\sigma}_i^3$, $\hat{\sigma}_i^\pm$, and l is the photon multiplicity, and it is taken to be equal to 2 hereafter. In addition, β_1 and β_2 are the Stark shift parameters related to the ground–intermediate and intermediate–excited states of the two identical atoms and are defined as

$$\beta_1 = \frac{g_1^2}{\triangle}, \qquad \beta_2 = \frac{g_2^2}{\triangle}, \qquad g = \frac{g_1 g_2}{\triangle},$$

where $g_{1(2)}$ is the coupling strength of the ground–intermediate (intermediate–excited) transition. For simplicity, we consider the case in which the atoms and the field are exactly resonant, i.e., $\triangle_1 = -\triangle_2 = \triangle$, where $\triangle_1 = \omega - (\omega_i - \omega_g)$ and $\triangle_2 = (\omega_e - \omega_i) - \omega$.

The initial state of the total atom–atom field system can be written as

$$|\psi(0)\rangle = |\psi(0)\rangle_f \otimes |\psi(0)\rangle_a = \sum_{n=0} q_n|n, +, +\rangle, \tag{2}$$

where $q_n = \mathrm{e}^{-\bar{n}/2}\frac{\alpha^n}{\sqrt{n!}}$, $\alpha = |\alpha|\,\mathrm{e}^{\mathrm{i}\varphi}$, and $\bar{n} = |\alpha|^2$ is the mean photon number of the coherent field, and φ is the phase angle of the coherent field (here we take $\varphi = 0$). The solution of the Schrödinger wave equation in the interaction picture i.e. the wavefunction of the system at any time $t > 0$ is given by

$$|\Psi(\tau)\rangle = \sum_{n=0}^{\infty} q_n[c_1(n,\tau)|n,+,+\rangle + c_2(n,\tau)|n+l,+,-\rangle + c_3(n,\tau)|n+l,-,+\rangle + c_4(n,\tau)|n+2l,-,-\rangle]. \tag{3}$$

Then we get the explicit forms for the dynamical coefficients $c_j(n,\tau)$, $(j = 1, 2, 3, 4)$ as

$$\begin{aligned} c_1(n,\tau) &= k_{11}\,\mathrm{e}^{\mathrm{i}\gamma_n^1\tau} + k_{12}\,\mathrm{e}^{\mathrm{i}\gamma_n^2\tau} + k_{13}\,\mathrm{e}^{\mathrm{i}\gamma_n^3\tau}, \\ c_2(n,\tau) = c_3(n,\tau) &= k_{21}\,\mathrm{e}^{\mathrm{i}\gamma_n^1\tau} + k_{22}\,\mathrm{e}^{\mathrm{i}\gamma_n^2\tau} + k_{23}\,\mathrm{e}^{\mathrm{i}\gamma_n^3\tau}, \\ c_4(n,\tau) &= k_{31}\,\mathrm{e}^{\mathrm{i}\gamma_n^1\tau} + k_{32}\,\mathrm{e}^{\mathrm{i}\gamma_n^2\tau} + k_{33}\,\mathrm{e}^{\mathrm{i}\gamma_n^3\tau}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} k_{1u} &= \frac{\left(\gamma_n^u + B\right)\left(\gamma_n^u + C\right) - 2\eta^2}{\left(\gamma_n^u - \gamma_n^v\right)\left(\gamma_n^u - \gamma_n^w\right)}, \\ k_{2u} &= \frac{-\zeta\left(\gamma_n^u + C\right)}{\left(\gamma_n^u - \gamma_n^v\right)\left(\gamma_n^u - \gamma_n^w\right)}, \\ k_{3u} &= \frac{2\zeta\eta}{\left(\gamma_n^u - \gamma_n^v\right)\left(\gamma_n^u - \gamma_n^w\right)}, \\ u, v, w &= (1, 2, 3), \qquad u \neq v \neq w, \end{aligned} \tag{5}$$

with

$$\begin{aligned} \gamma_n^1 &= 2\left(-\frac{\lambda_n}{6} + z^{\frac{1}{3}}\cos\left(\frac{\delta}{3}\right)\right), \\ \gamma_n^2 &= 2\left(-\frac{\lambda_n}{6} + z^{\frac{1}{3}}\cos\left(\frac{\delta + 2\pi}{3}\right)\right), \\ \gamma_n^3 &= 2\left(-\frac{\lambda_n}{6} + z^{\frac{1}{3}}\cos\left(\frac{\delta + 4\pi}{3}\right)\right), \\ z &= \sqrt{x^2 + y^2}, \qquad \delta = \tan^{-1}\frac{y}{x}, \\ x &= \frac{27\nu_n - 9\lambda_n\mu_n - 2\lambda_n^3}{54}, \\ y &= \frac{\sqrt{4\left(3\mu_n + \lambda_n^2\right)^3 - \left(27\nu_n - 9\lambda_n\mu_n - 2\lambda_n^3\right)^2}}{54} \end{aligned} \tag{6}$$

and

$$\begin{aligned} \lambda_n &= A + B + C, \\ \mu_n &= 2(\zeta^2 + \eta^2) - (AB + AC + BC), \\ \nu_n &= 2(\zeta^2 A + \eta^2 C) - ABC, \qquad \eta = \sqrt{\frac{(n+2l)!}{(n+l)!}}, \\ \zeta &= \sqrt{\frac{(n+l)!}{(n)!}}, \qquad A = \frac{2n}{r}, \\ B &= \frac{(1+r^2)(n+l)}{r}, \qquad C = 2r(n+2l), \end{aligned} \tag{7}$$

where $r = \sqrt{\frac{\beta_1}{\beta_2}}$, $g = \sqrt{\beta_1 \beta_2}$ and $\tau = gt$. The quantum phase properties and Wigner function for this system are discussed in the following section using the above results. When we ignore the Stark shift, i.e ignore A, B, C, and take $k = 1$ we go back to the case of [18].

3. Quantum phase properties and Wigner function

3.1. Phase properties

In this section, we will analyse the phase properties of the field in the presence of the Stark shift. We apply the Hermitian phase operator formalism introduced by Pegg and Barnett [28–30]. This formalism is based on introducing a finite $(s + 1)$-dimensional space spanned by the number states $|0\rangle$, $|1\rangle, \ldots, |s\rangle$ for the given mode of the field. The Hermitian phase operator operates on this finite space and, after all necessary expectation values have been calculated in the finite-dimensional space, the value of s is allowed to tend to infinity. A complete orthonormal basis of $(s + 1)$ states is defined as

$$|\theta_m\rangle = \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} \mathrm{e}^{\mathrm{i}n\theta_m} |n\rangle, \tag{8}$$

where

$$\theta_m = \theta_0 + \frac{2m\pi}{s+1}, \qquad m = 0, 1, \ldots, s. \tag{9}$$

The value of θ_0 is arbitrary and defines a particular basis set of $(s+1)$ mutually orthogonal phase states. The Hermitian phase operator is defined as

$$\hat{\phi}_\theta \equiv \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m|, \tag{10}$$

where the subscript θ indicates the dependence on the choice of θ_0. The phase states (equation (8)) are eigenstates of the phase operator (equation (10)) with the eigenvalues θ_m restricted to lie within a phase window between θ_0 and $\theta_0 + 2\pi$.

The continuum phase distribution $P(\theta)$ is defined as

$$P(\theta) = \lim_{s\to\infty} \frac{s+1}{2\pi} \langle\theta_m|\rho|\theta_m\rangle. \tag{11}$$

In this case, if θ_m has been replaced by the continuous phase variable θ, then

$$P(\theta, \tau) = \frac{1}{2\pi}\left[1 + 2\,\mathrm{Re} \sum_{m,n:m>n} \rho^f_{nm}(\tau)\,\mathrm{e}^{-\mathrm{i}(m-n)\theta}\right], \tag{12}$$

where $\rho^f_{nm}(\tau)$ is the element of the reduced field-mode density matrix ρ, and we can write $\hat{\rho}(\tau)$ in the form:

$$\hat{\rho}(\tau) = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}, \tag{13}$$

where

$$\begin{aligned}
\rho_{11} &= \sum_{n,m=0}^{\infty} q_{n,m} c_1(n,\tau) c_1^*(m,\tau) |n\rangle\langle m|, \\
\rho_{22(33)} &= \sum_{n,m=0}^{\infty} q_{n,m} c_{2(3)}(n+l,\tau) c_{2(3)}^*(m+l,\tau) |n+l\rangle\langle m+l|, \\
\rho_{44} &= \sum_{n,m=0}^{\infty} q_{n,m} c_4(n+2l,\tau) c_4^*(m+2l,\tau) |n+2l\rangle\langle m+2l|, \\
\rho_{12(13)} &= \sum_{n,m=0}^{\infty} q_{n,m} c_1(n+l,\tau) c_{2(3)}^*(m,\tau) |n+l\rangle\langle m| = \rho^*_{21(31)}, \\
\rho_{14} &= \sum_{n,m=0}^{\infty} q_{n,m} c_1(n+2l,\tau) c_4^*(m,\tau) |n+2l\rangle\langle m| = \rho^*_{41}, \\
\rho_{23} &= \sum_{n,m=0}^{\infty} q_{n,m} c_2(n+l,\tau) c_3^*(m+l,\tau) |n+l\rangle\langle m+l| = \rho^*_{32}, \\
\rho_{24(34)} &= \sum_{n,m=0}^{\infty} q_{n,m} c_{2(3)}(n+2l,\tau) c_4^*(m+l,\tau) |n+2l\rangle\langle m+l| \\
&= \rho^*_{42(43)}.
\end{aligned} \tag{14}$$

Now from equation (13) we can write ρ^f_{nm} as

$$\begin{aligned}
\rho^f_{nm}(\tau) &= q_{n,m} c_1(n,\tau) c_1^*(m,\tau) + 2 q_{n-l,m-l} c_2(n,\tau) c_2^*(m,\tau) \\
&\quad + q_{n-2l,m-2l} c_4(n,\tau) c_4^*(m,\tau),
\end{aligned} \tag{15}$$

where $q_{n,m} = q_n\, q_m^*$.

The expectation value of the phase operator moments in the state described by the density operator ρ is given by

$$\langle\theta^k\rangle = \int_{\theta_0}^{\theta_0+2\pi} \theta^k P(\theta)\,\mathrm{d}\theta. \tag{16}$$

Of particular interest in the description of the phase properties of the field for the two photons with the Stark shift is the phase variance that can be calculated according to the formula

$$\begin{aligned}
(\Delta\theta)^2 &= \int_{-\pi}^{\pi} \theta^2 P(\theta)\,\mathrm{d}\theta - (\langle\theta\rangle)^2 \\
&= \frac{\pi^2}{3} + 4\,\mathrm{Re} \sum_{m>n} \frac{(-1)^{m-n} \rho^f_{mn}(\tau)}{(m-n)^2}.
\end{aligned} \tag{17}$$

The phase probability distribution illustrates the phase structure of the field mode while the variance provides a good understanding of the evolution of the field phase fluctuations.

In figures 1 and 2, we have plotted the phase probability distribution $P(\theta, \tau)$ and the phase variance $(\Delta\phi)^2$ with respect to time τ for the two identical 2-level atoms in the case of the two photons ($l = 2$) for various values of the parameter of the Stark shift r for the initial mean photon numbers $|\alpha|^2 = 50$. When we neglect the Stark shift (see figure 1(a)) clearly describes the time evolution of the phase probability distribution of the field for the standard two-photon TCM. As the time proceeds, the peaks of the initial coherent state centred at $\theta = 0$ split up into small peaks which move away from each other gradually while there is a strong peak still centred at $\theta = 0$. When $\tau = \frac{\pi}{2}$ the small peaks are much farther apart and meet the boundaries at $\theta = \pm\pi$. Further, when $\tau = \pi$ the small peaks converge until they meet again,

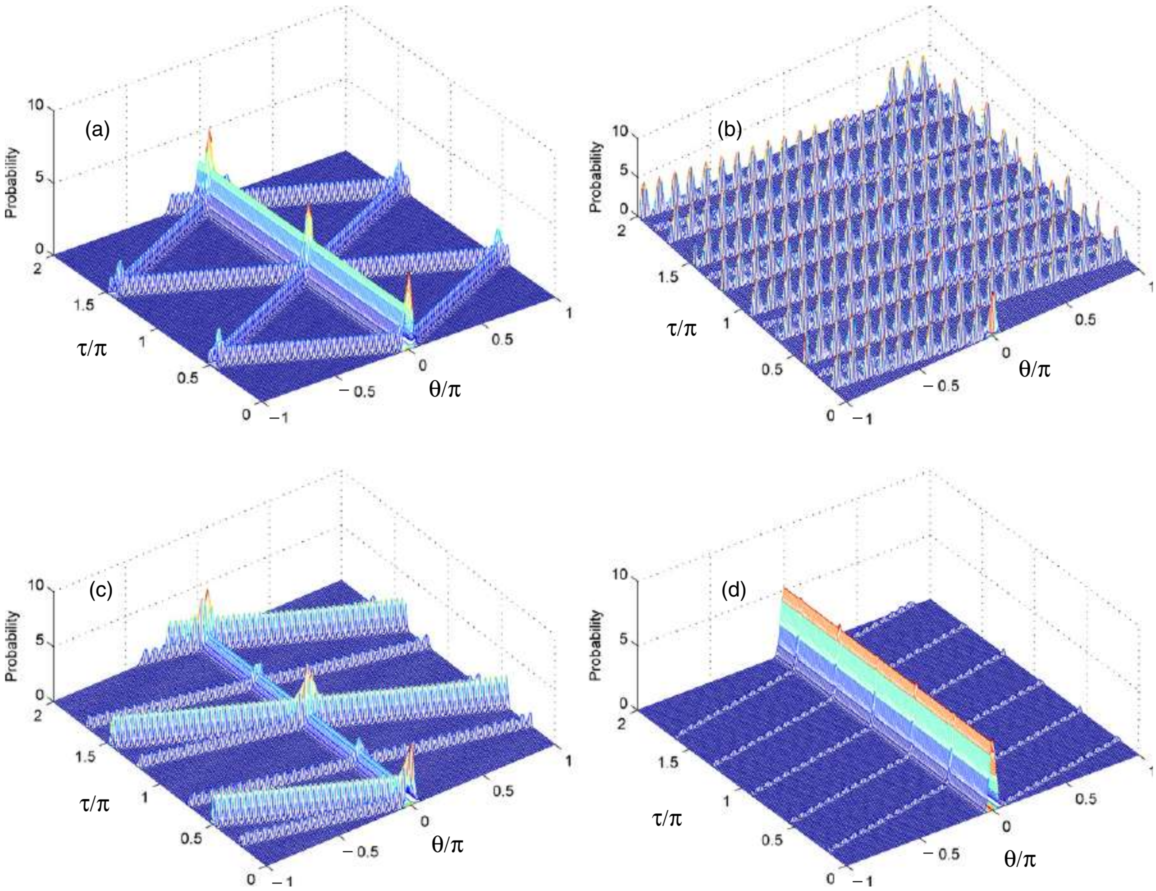

Figure 1. The phase distribution $P(\theta, \tau)$ for the two identical 2-level atoms, when both atoms initially prepared in the excited state and the field in a coherent state for the two photons ($l = 2$) and initial mean photon number $|\alpha|^2 = 50$ for different values of the Stark shift parameter: (a) in the absence of the Stark shift, (b) $r = 0.3$, (c) $r = 1$ ($\beta_1 = \beta_2$) and (d) $r = 5$.

but this time at $\theta = 0$ with the strong peak. Thus the time evolution diagram of the phase probability distribution of the usual two-photon TCM is very regular; it is symmetric about $\theta = 0$ and is periodic in time. This is due to the periodic nature of the interaction phenomenon in the two photons. This can be attributed to the roots γ_n^i which in this case are 0 and $\pm\sqrt{2(\zeta^2 + \eta^2)}$; the amplitudes of the peaks are proportional to the coefficients k of equation (5).

In the presence of the Stark shift (see figures 1(b) and (c)), with a small increase in the parameter of the Stark shift ($r = 0.3$, figure 1(b)) leads to the localization of the peaks. Similar behaviour was reported for a single atom under the Stark shift and Kerr-like medium [42]. The peak centred at $\theta = 0$ disappears due to the intensity-dependent phase shift caused by the Stark shift. For $r = 1$ (see figure 1(c)), at $\theta = 0$ the peak returns to appear again but with small amplitude, and the values of the phase probability distribution are increasing. Analysis of the roots γ_n^i shows that the two roots have the same sign but with different amplitudes. This is the reason of appearance of the two peaks moving away in the same side of the origin but with different rates and amplitudes. This becomes more pronounced at higher values of the Stark shift (figure 1(d) $r = 5$); the phase probability has a maximum value at $\theta = 0$, and the other peaks coincide and almost disappear, as a check of the roots γ_n^i can easily show.

Figure 2 shows the plots of the phase variance $(\Delta\theta)^2$, when we neglect the Stark shift (see figure 2(a)), the phase variance oscillates periodically near $\pi^2/3$ with the period π. In the presence of the Stark shift for $r = 0.3$ (see figure 2(b)), the general frame work of $(\Delta\theta)^2$ is completely changed in the maximum and minimum values, periodic with period $\cong 0.6\pi$ and adds sharpening of the peaks. For $r = 1$ (see figure 2(c)) the phase variance is as in the case of no Stark shift but the difference between them is in the appearance of two shoulders around the peaks. For higher value of the parameter of the Stark shift ($r = 5$) (see figure 2(d)) the peaks are regular, long-lived and periodic with period $\cong 0.4\pi$ for the time. From our results, we note that the Stark shift leads to damp and diffuse the distribution. With the increase in the Stark shift parameter r, the maximum values of $(\Delta\theta)^2$ are decreasing; also the collapse and revival phenomena appear for the phase variance.

3.2. *Wigner function W(X, Y)*

We display the time evolution of the Wigner function $W(X, Y)$, where $X = \text{Re}(\alpha)$, $Y = \text{Im}(\alpha)$ at different times in the exact resonance with $\bar{n} = 4$, We assume that both atoms are initially prepared in the excited state and the field in a coherent state. The $W(X, Y)$ function is informative, sensitive

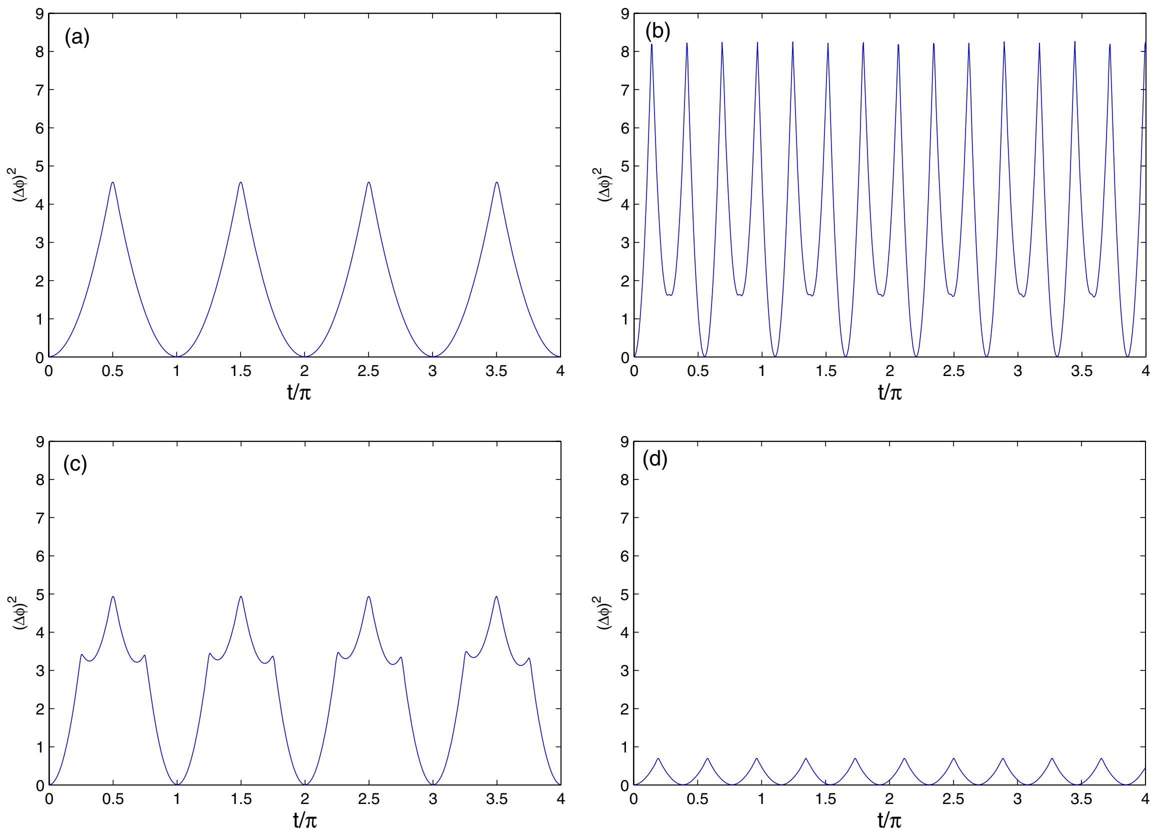

Figure 2. The phase variance $(\Delta\phi)^2$ for the two identical 2-level atoms, when both atoms initially prepared in the excited state and the field in a coherent state for the two photons $(l = 2)$ and initial mean photon number $|\alpha|^2 = 50$ for different values of the Stark shift parameter: (a) in the absence of the Stark shift, (b) $r = 0.3$, (c) $r = 1$ $(\beta_1 = \beta_2)$ and (d) $r = 5$.

to the interference in phase space, and can predict the possible occurrence of the nonclassical effects, which can easily be evaluated as [43]

$$W(\alpha,\tau) = \frac{2}{\pi}\sum_{p=0}^{\infty}\sum_{m,n=0}^{\infty}(-1)^p q_{n,m} \times [c_1(n,\tau)c_1^*(m,\tau)G_{p,n}(\alpha)G_{m,p}(\alpha) + c_2(n,\tau)c_2^*(m,\tau)G_{p,n+l}(\alpha)G_{m+l,p}(\alpha) + c_3(n,\tau)c_3^*(m,\tau)G_{p,n+l}(\alpha)G_{m+l,p}(\alpha) + c_4(n,\tau)c_4^*(m,\tau)G_{p,n+2l}(\alpha)G_{m+2l,p}(\alpha)], \quad (18)$$

where

$$G_{p,n}(\alpha) = \mathrm{e}^{\frac{-|\alpha|^2}{2}}\sum_{j=0}^{\min(p,n)}\frac{(-\alpha)^{p-j}(\alpha^*)^{n-j}\sqrt{p!n!}}{(n-j)!(p-j)!(j)!}, \quad (19)$$

$$G_{m,p}(\alpha) = \mathrm{e}^{\frac{-|\alpha|^2}{2}}\sum_{j=0}^{\min(p,m)}\frac{(-\alpha^*)^{p-j}(\alpha)^{m-j}\sqrt{p!(m)!}}{(m-j)!(p-j)!(j)!} = G^*_{p,m}. \quad (20)$$

In figures 3–5, we have plotted the Wigner function for the cavity field at different times $\left(t = 0, \frac{\pi}{2}, \pi\right)$, with $\bar{n} = 4$, when the field is initially in the coherent state for the two photons $(l = 2)$ for various values of the Stark shift parameter.

For $t = 0$, the Wigner function has a Gaussian shape with a single peak at the point $(X = 2, Y = 0)$ and a round rim (see figure 3).

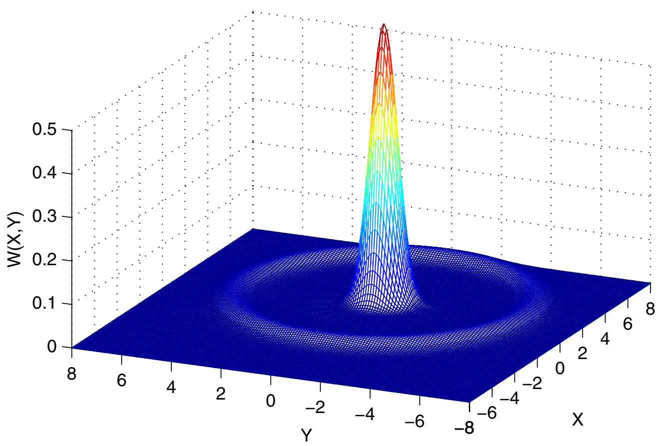

Figure 3. The Wigner function $W(X, Y)$ against X and Y for the two atoms prepared initially in the excited state and the field in a coherent state, for the two photons $(l = 2)$ and $\bar{n} = 4$ when $\tau = 0$.

In figure 4, we see the influence of the Stark shift on the behaviour of the Wigner function at time $\left(t = \frac{\pi}{2}\right)$. When we neglect the Stark shift, we see that the $W(X, Y)$ function splits into two peaks, several small peaks and an interference pattern between them. The $W(X, Y)$ function takes negative values, and the state becomes nonclassical (see figure 4(a)). In the presence of the Stark shift $r = 0.3$, the $W(X, Y)$ has three peaks and interference between them, the state becomes

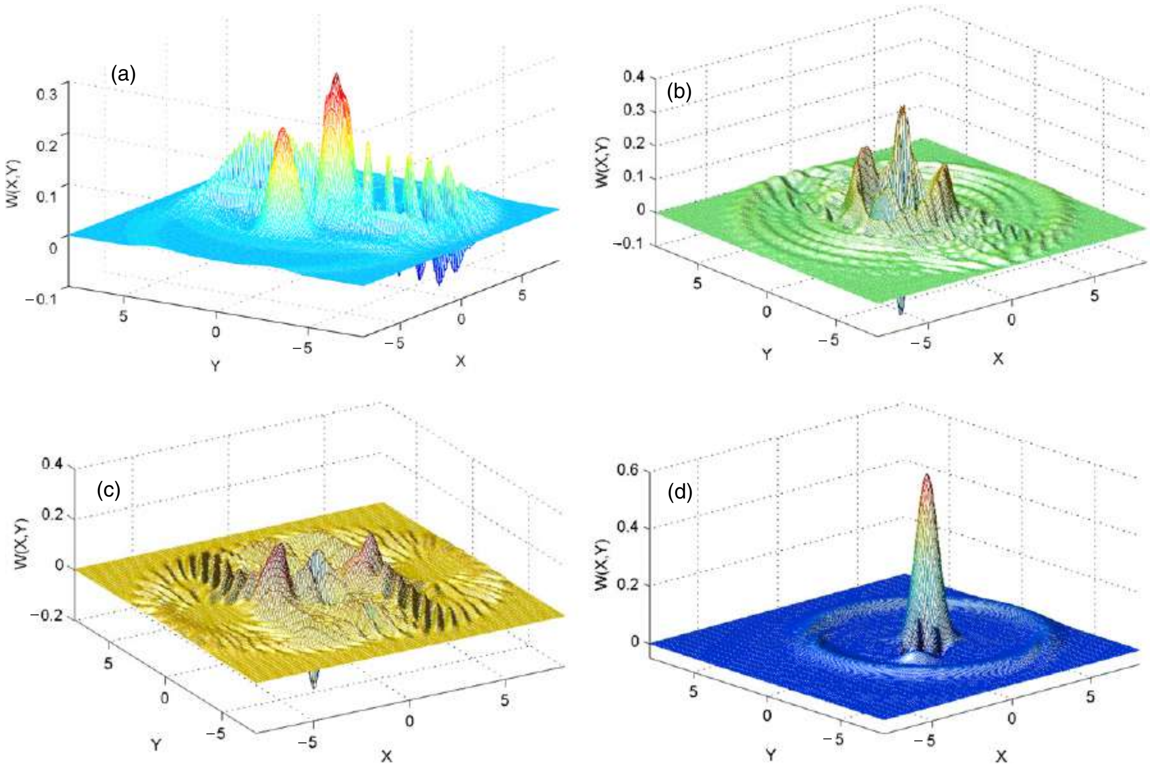

Figure 4. The Wigner function $W(X, Y)$ against X and Y for the two identical 2-level atoms, when both atoms prepared in the excited state and the field in a coherent state for the two photons ($l = 2$) and $\bar{n} = 4$ for different values of the Stark shift parameter: (a) in the absence of the Stark shift, (b) $r = 0.3$, (c) $r = 1$ ($\beta_1 = \beta_2$) and (d) $r = 5$, for $\tau = \frac{\pi}{2}$.

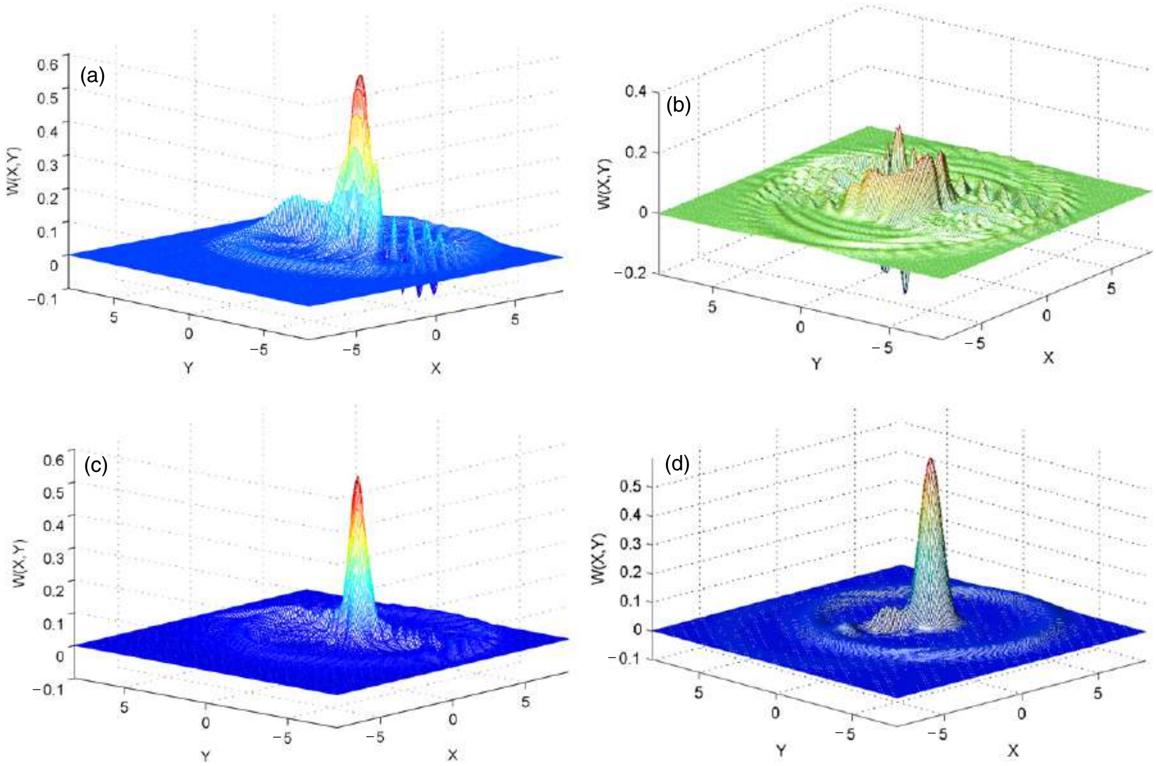

Figure 5. The same as in figure 4, but for $\tau = \pi$.

more nonclassical and the subsidiary peaks nearly disappear (see figure 4(b)). In the case $r = 1$ the $W(X, Y)$ function has two symmetric groups and interference between them; the peaks are nearly distorted and $W(X, Y)$ has a negative value (see figure 4(c)). For a higher value of the parameter of the Stark shift $r = 5$, the $W(X, Y)$ function has one peak, two very small peaks, and the negativity of the $W(X, Y)$ function nearly disappeared (see figure 4(d)).

Finally, for $t = \pi$ the plots are presented in figure 5, where, neglecting the Stark shift, the $W(X, Y)$ function has one peak and several small peaks, an interference pattern between them and negative values (nonclassical) (see figure 5(a). In the presence of the Stark shift, when $r = 0.3$, the nonclassicality (i.e. the negativity) is more pronounced (see figure 5(b)). For the case of $r = 1$, see figure 5(c), it is almost the same for the absence of the Stark shift but with different small peaks. For $r = 5$ we note figure 5(d) is as figure 4(d) but they are out of phase by 90°.

From the above results we note that the state becomes nonclassical for $\left(t = \frac{\pi}{2}, \pi\right)$ with various values of the parameter of the Stark shift r because there is an interaction between the atoms and the field. For higher values of the Stark shift parameter r, the Wigner function nearly returns to Gaussian shape with a very small negative value i.e. the increase of the parameter of the Stark shift leads to the disappearance of the negativity, and the state becomes almost classical.

4. Conclusions

In this paper, we have studied two atoms interacting with a multi-photon single-mode field in the presence of a Stark shift. We obtained an analytical solution for this system when the two atoms are prepared initially in the excited state and the field initially prepared in a coherent state. We have investigated the influence of the Stark shift on the behaviour of the $W(X, Y)$ function, phase distribution and phase variance. From our results, we note that the Stark shift serves to damp and diffuse the distribution. With an increase in the Stark shift parameter (r), the maximum values of $(\Delta\theta)^2$ decrease; also the collapse and revival phenomena appear for the phase variance. It is noted that in $W(X, Y)$ the state becomes nonclassical for $\left(\tau = \frac{\pi}{2}, \pi\right)$ with various values of the Stark shift parameter r. The increase of the Stark shift leads to disappearance of the negativity, and reintroduction of a Gaussian shape with a single peak and the state becomes almost classical.

Acknowledgments

The authors wish to record their thanks to the referees for their valuable comments that resulted in improvements of the paper in many aspects.

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
[2] Braunstein S L and Pati A K (ed) 2003 *Quatum Information with Continuous Variables* (Dordrecht: Kluwer)
[3] Benenti G, Casati G and Strini G 2005 *Principle of Quantum Computation and Information* (Singapore: World Scientific)
[4] Benneti C H, Brassard G, Crebeau C, Jozsa R, Peresand A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[5] Ekert A 1991 *Phys. Rev. Lett.* **67** 661
Cirac J I and Gisin N 1997 *Phys. Lett.* L **229** 1
Fuchs C A, Gisin N, Griffiths R B, Niu C-S and Peres A 1997 *Phys. Rev.* A **56** 1163
[6] Ye L and Guo G-C 2005 *Phys. Rev.* A **71** 034304
Mozes S, Oppenheim J and Reznik B 2005 *Phys. Rev.* A **71** 012311
[7] Glöckl O, Lorenz S, Marquardt C, Heersink J, Brownnutt M, Silberhorn C, Pan Q, Loock P V, Korolkova N and Leuchs G 2003 *Phys. Rev.* A **68** 012319
Yang M, Song W and Cao Z-L 2005 *Phys. Rev.* A **71** 034312
Li H-R, snLi F-L, Yang Y and Zheng Q 2005 *Phys. Rev.* A **71** 022314
[8] Iqbal M S, Mahmood S, Razmi M S K and Zubairy M S 1988 *J. Opt. Soc. Am.* B **5** 1312
[9] Sharma M P, Cardimona D A and Gavrielides A 1989 *J. Opt. Soc. Am.* B **6** 1942
[10] Jex I, Matsuoko M and Koashi M 1993 *Quantum Opt.* **5** 275
[11] Jex I 1990 *Qantum Opt.* **2** 443
Kim M S and Agarwal G S 1999 *J. Mod. Opt.* **46** 2111
[12] Abdal-Aty M 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 201
[13] Xu L, Zhang Z-M and Chai J-L 1991 *J. Opt. Soc. Am.* B **8** 1157
[14] Obada A-S F and Omar Z 1995 *J. Egypt. Math. Soc.* **3** 65
[15] El-Orany F A A 2006 *J. Phys. A: Math. Gen.* **39** 3397
[16] Tavis M and Cummings F W 1968 *Phys. Rev.* **170** 379
[17] Alber G *et al* 2001 *Quantum Information* (Berlin: Springer)
[18] Tessier T E, Deutsch I H and Delgado A 2003 *Phys. Rev.* A **68** 062316
[19] Peres A 1995 *Quantum Theory: Concepts and Methods* (Dordrecht: Kluwer)
[20] Buzek V and Hladky B 1993 *J. Mod. Opt.* **40** 1309
Wigner E P 1932 *Phys. Rev.* **40** 749
Hillery M, O'Connell R F, Scully M O and Wigner E P 1984 *Phys. Rep.* **106** 121
Husimi K 1940 *Proc. Phys. Math. Soc. Japan* **22** 264
[21] Lutterbach L G and Davidovich L 1997 *Phys. Rev. Lett.* **78** 2547
Kim M S *et al* 1998 *Phys. Rev.* A **58** R65
[22] Leibfried D *et al* 1996 *Phys. Rev. Lett.* **77** 4281
Nogues G *et al* 2000 *Phys. Rev.* A **62** 054101
[23] Buzek V and Knight P L 1995 *Progress in Optics* vol 34 (Amsterdam: North-Holland) and references therein
[24] Davidovich L *et al* 1996 *Phys. Rev.* A **53** 1295
Brune M *et al* 1996 *Phys. Rev. Lett.* **77** 4887
Földi P *et al* 2001 *Phys. Rev.* A **63** 033807
Benedict M G *et al* 1997 *Acta Phys. Slovaca* **47** 259
[25] Zurek W H 1981 *Phys. Rev.* D **24** 1516
Zeh H D 1970 *Found. Phys.* **1** 69
[26] Garraway B M and Knight P L 1993 *Phys. Scr.* T **48** 66
[27] Yaccaro J A and Pegg D T 1990 *Phys. Rev.* A **41** 5156
[28] Pegg D T and Barnett S M 1988 *Europhys. Lett.* **6** 483
[29] Bernett S M and Pegg D T 1989 *J. Mod. Opt.* **36** 7
[30] Pegg D T and Barnett S M 1989 *Phys. Rev.* A **39** 1665
[31] Dung H T, Tanas R and Shumovsky A S 1990 *Opt. Commun.* **79** 462
[32] Dung H T, Tanas R and Shumovsky A S 1991 *J. Mod. Opt.* **38** 2069
[33] Eberly J H, Narozhny N B and Sanchez-Mondragon J J 1980 *Phys. Rev. Lett.* **44** 1323
[34] Narozhny N B, Sanchez-Mondragon J J and Eberly J H 1981 *Phys. Rev.* A **23** 236

[35] Yoo H I, Sanchez-Mondragon J J and Eberly J H 1981 *Physica* A **14** 1383
[36] Dung H T, Huyen N D and Shumovsky A S 1992 *Physica* A **182** 467
[37] El-Orany F A A 2006 *Phys. Scr.* **74** 563
[38] Alsing P and Zubairy M S 1987 *J. Opt. Soc. Am.* B **4** 177
[39] Puri R R and Bullough R K 1988 *J. Opt. Soc. Am.* B **5** 2021
[40] Joshi A and Puri R R 1989 *J. Mod. Opt.* **36** 215
[41] Nasreen T and Razmi M S K 1991 *J. Opt. Soc. Am.* B **8** 2303
Nasreen T and Razmi M S K 1992 *Phys. Rev.* A **46** 4161
[42] Obada A-S F, Abdel-Hafez A M and Abdel-Aty M 1998 *Eur. Phys. JD* **3** 289
[43] Hessian H A and Mohamed A-B A 2008 *Laser Phys.* **18** 1217